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COMPARATIVE STUDY OF THE SUBSYSTEMS SUBJECTED TO INDEPENDENT AND SIMULTANEOUS FAILURE

BADANIA PORÓWNAWCZE PODSYSTEMÓW ULEGAJĄCYCH USZKODZENIOM NIEZALEŻNYM I JEDNOCZESNYM

The paper discusses the comparison between availability of a pipe manufacturing industry when the sub systems are subjected to simultaneously and independent failure. The failure rates of the sub-systems are constant and the repair rates are variable. The governing differential equations of the system are solved using Lagrange's Method. The performance evaluation of system is done by means of long run availability making use of software package Matlab 7.0.4. The tables for various parameters are given which can be useful to the plant management for improving and planning the maintenance schedule.

Keywords: *Supplementary Variable Technique, Chapman- Kolmogorov, MAT-LAB, Lagrange's Method, Steady State Availability.*

W artykule porównano dostępność zakładu przemysłowego produkującego rury w przypadkach występowania jednoczesnych i niezależnych uszkodzeń podsystemów. Badania prowadzono przy stałych intensywnościach uszkodzeń podsystemów i zmiennych intensywnościach napraw . Konstytutywne równania różniczkowe systemu rozwiązano przy użyciu metody Lagrange'a. Oceny wydajności systemu dokonano na podstawie długotrwałej dostępności z wykorzystaniem pakietu oprogramowania Matlab 7.0.4. Przedstawiono tabele dla różnych parametrów, które mogą być wykorzystywane przez osoby zarządzające produkcją przy poprawianiu i planowaniu harmonogramów przeglądów.

Słowa kluczowe: *technika dodatkowej zmiennej, Chapman–Kołmogorow, MAT-LAB, metoda Lagrange'a, gotowość stacjonarna.*

1. Introduction

Availability analysis of a system can benefit the industry in terms of higher productivity and low maintenance cost. It is possible to improve the availability of the plant with proper maintenance planning and monitoring. Reliability analysis helps us to obtain the necessary information about the control of various parameters. The polytube industry involves many processes i.e. Mixture, Extruder, Die and Cutter. The Die and Cutter machine can also work in reduced state. The process starts from the Mixture section in which pipe mixture is prepared with the help of PVC rising, $CaCo_3$, citric acid and wax which is heated up to 130^0 C. The heated material is then cooled up to 100^0 C and transported to the Extruder by conveyors. With the help of Die and Extruder, pipe is prepared. After smoothing the pipe, sorting process take place. In this process, the pipe carried to Cutter is cut into different sizes as per the need and requirement.

The mechanical systems have attracted the attention of several researchers in the area of reliability theory. Singh [7] first time applied reliability technology to analyze the working of process industries(sugar, fertilizer and paper industries). Zhang [8] studied the stochastic behavior of an (N+1) standby system under preemptive priority repair and obtain the expression of transient and steady state of the system using supplementary variable and Laplace Transform. Dyal and Singh [1] studied reliability analysis of a system in a fluctuating environment. Singh and Mahajan [6] examined the reliability and long run availability of a Utensils Manufacturing Plant using Laplace transforms. Gupta et. al. [2] studied the behavior of Cement manufacturing plant. Kiureghian and Ditlevson [3] analyzed the availability, reliability and downtime of system with repairable components. Kumar et.al. [4] discussed the behavior analysis of Urea decomposition in the Fertilizer industry under general repair policy. Kumar et.al. [5] analyzed the designed and cost of a refining system in the sugar industry using supplementary variable technique.

2. System descriptions

The Polytube industry consists of four subsystems, namely:

Sub-system A (Mixture): It mixes raw material such as PVC rising, calcium carbonate, wax and other chemicals in appropriate proportion for manufacturing pipe. It consists of a heater by which the raw material is heated up to 130^0 C and transported to the extruder by conveyors. It consists of blades and a motor whose failure cause complete failure of the system.

Sub-system B (Extruder): Raw material obtained from mixer is heated in this section. It consists of a heater to heat the raw material at different temperatures. The quality of the product depends upon heating process. Its failure causes the complete failure of the system.

Sub-system C (Die): It is used to make different sizes of pipe. Minor failure of the sub-system reduced the capacity of the system and hence loss in production. Major failure results in complete failure of the system.

Sub-system D (Cutter): This sub-system has two units arranged in series. First unit is blade which cut the pipe and the second unit is motor which cut the pipe in different size. Failure of blade reduces the capacity of the system while the failure of motor causes the complete failure of the system.

3. Sub-systems subjected to independent failures

3.1. Notations

A, B, C, D	: Indicates that the sub-system is working in full capacity.
$\bar{C}, \bar{D}$	: Indicate the reduced state of the sub-system C and D.
a, b, c, d	: Indicate the failed state of the sub-system.
λ_1, λ_2	: Transition rate of subsystem C and D.
α_i	: Failure rate of the sub-system A, B, C, D.
$\phi(x)\psi(y)$ $\mu(z)\sigma(s)$	: General repair rates of A, B, C, D respectively.
$P_o(t)$	: The system is working in full capacity.
$P_i(x,t)$	: Probability that there is failure in subsystem A at time‘t’ and has an elapsed repair time x. i = 5,7,10,1317
$P_j(y,t)$	: Probability that there is failure in subsystem B at time‘t’ and has an elapsed repair time y. j = 6,8,11,14,18.
$P_2(s,t)$	: Probability that there is failure in subsystem D at time‘t’ and system remains in reduced state and has an elapsed repair time s.
$P_1(z,t)$	: Probability that there is failure in subsystem C at time ‘t’ and system remains in reduced state and has an elapsed repair time z.
$P_4(z,t)$	: Probability when the sub system D is in reduced state and has an elapsed repair time s then the sub system C comes in reduced state and has an elapsed repair time z.
$P_{3(1)}(s,t)$	: Probability when the sub system C comes firstly in reduced state and has an elapsed repair time z then the sub system D comes in failed state and has an elapsed repair time s.

3.2. Assumptions

The assumptions used in developing the performance model are as follows [5, 6]:

1. Failure rates are constant over time and statistically independent.
2. A repaired unit as good as new, performance wise, for a specified duration.
3. Sufficient repair facilities are provided.
4. Service includes repair and/or replacement.

4. Mathematical formulations

The differential-difference equations obtained from the state transition is as follows (Fig. 1):

$$\left[\frac{d}{dt} + S_0(t)\right] P_{0=}M_0(t) \tag{1}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + S_2(z)\right] P_1(z,t) = M_1(z,t) \tag{2}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial s} + S_2(s)\right] P_2(s,t) = M_2(s,t) \tag{3}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial s} + S_{3(1)}(s)\right] P_{3(1)}(s,t) = M_3(s,t) \tag{4}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + S_4(z)\right] P_{4(2)}(z,t) = M_4(z,t) \tag{5}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \phi(x)\right] P_5(x,t) = 0 \tag{6}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \psi(y)\right] P_6(y,t) = 0 \tag{7}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \phi(x)\right] P_7(x,t) = 0 \tag{8}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \psi(y)\right] P_8(y,t) = 0 \tag{9}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \mu(z)\right] P_9(z,t) = 0 \tag{10}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \phi(x)\right] P_{10}(x,t) = 0 \tag{11}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \psi(y)\right] P_{11}(y,t) = 0 \tag{12}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial s} + \sigma(s)\right] P_{12}(s,t) = 0 \tag{13}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \phi(x)\right] P_{13}(x,t) = 0 \tag{14}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \psi(y)\right] P_{14}(y,t) = 0 \tag{15}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \mu(z)\right] P_{15}(z,t) = 0 \tag{16}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial s} + \sigma(s)\right] P_{16}(s,t) = 0 \tag{17}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \phi(x)\right] P_{17}(x,t) = 0 \tag{18}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \psi(y)\right] P_{18}(y,t) = 0 \tag{19}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \mu(z)\right] P_{19}(z,t) = 0 \tag{20}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial s} + \sigma(s)\right] P_{20}(s,t) = 0 \tag{21}$$

Where,

$$S_0(t) = \sum_{i=1}^{2} \alpha_i + \lambda_i$$

$$M_0(t) = \int P_5(x,t)\phi(x)dx + \int P_6(y,t)\psi(y)dy + \\ + \int P_2(s,t)\sigma(s)ds + \int P_1(z,t)\mu(z)dz$$

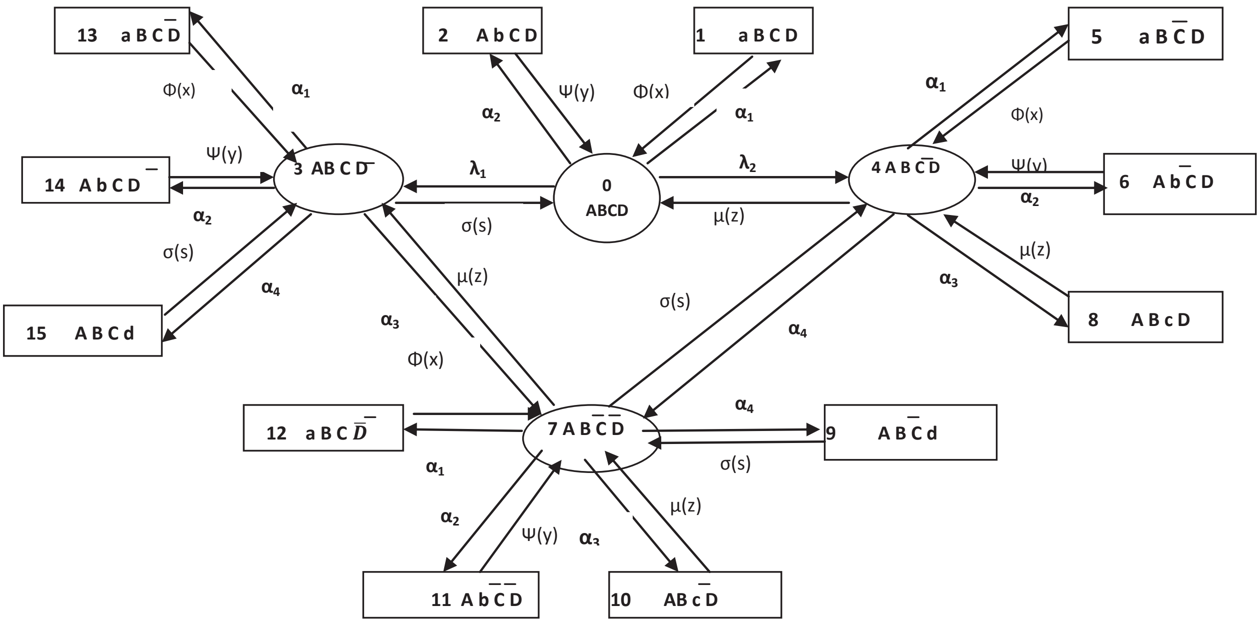

Fig. 1. Transition diagram of Poly Tube Industry when Sub-System simultaneously

$$S_2(s) = \sum_{i=1}^{4} \alpha_i + \sigma(s) \quad M_2(s,t) = \lambda_2 P_0(t) + \int P_{10}(x,t)\phi(x)dx \; +$$
$$+\int P_{11}(y,t)\psi(y)dy + \int P_{12}(s,t)\sigma(s)ds + \int P_{4(2)}(z,t)\mu(z)dz$$

$$S_{3(1)}(s) = \sum_{i=1}^{4} \alpha_i + \sigma(s)$$

$$M_3(s,t) = \int \alpha_4 P_1(z,t)dz + \int P_{20}(s,t)\sigma(s)ds \; +$$
$$+\int P_{19}(z,t)\mu(z)dz + \int P_{18}(y,t)\psi(y)dy + \int P_{17}(x,t)\phi(x)dx$$

$$S_{4(2)}(z) = \sum_{i=1}^{4} \alpha_i + \mu(z)$$

$$M_4(z,t) = \int \alpha_3 P_2(s,t)ds + \int P_{16}(s,t)\sigma(s)ds \; +$$
$$+\int P_{15}(z,t)\mu(z)dz + \int P_{14)}(y,t)\psi(y)dy + \int P_{13}(x,t)\phi(x)dx$$

4.1. Initial conditions:

P_0 (0)=1 otherwise 0

$P_i(x,0) = 0 \; For \; i = \; 5,7,10,13,17$

$P_j(y,0) = 0 \; For \; j = \; 6,8,11,14,18$

$P_l(z,0) = 0 \; For \; l \; = 1,9,15,17$

$P_r(s,0) = 0 \; For \; r = 2,12,16,20.$

$P_{3(1)}(s,0) = 0$

$P_{4(2)}(z,0) = 0$

4.2. Boundary conditions:

$$P_1(0,t) = \lambda_1 P_0(t) \qquad P_2(0,t) = \lambda_2 P_0(t)$$
$$P_5(0,t) = \alpha_1 P_0(t) \qquad P_6(0,t) = \alpha_2 P_0(t)$$
$$P_{3(1)}(0,t) = \int \alpha_4 P_1(z,t)\,dz \qquad P_{4(2)}(0,t) = \int \alpha_2 P_2(s,t)ds$$
$$P_7(0,t) = \int \alpha_1 P_1(z,t)dz \qquad P_8(0,t) = \int \alpha_2 P_1(z,t)\,dz$$
$$P_9(0,t) = \int \alpha_3 P_1(z,t)dz \qquad P_{10}(0,t) = \int \alpha_1 P_2(s,t)\;ds$$
$$P_{11}(0,t) = \int \alpha_2 P_2(s,t)\,ds \qquad P_{12}(0,t) = \int \alpha_1 P_2(s,t)\,ds$$
$$P_{13}(0,t) = \int \alpha_1 P_{4(2)}(z,t)\;dz \qquad P_{14}(0,t) = \int \alpha_2 P_{4(2)}(z,t)\,dz$$
$$P_{15}(0,t) = \int \alpha_3 P_{4(2)}(z,t)\,dz \qquad P_{16}(0,t) = \int \alpha_4 P_{4(2)}(z,t)dz$$
$$P_{17}(0,t) = \int \alpha_1 P_{3(1)}(s,t)ds \qquad P_{18}(0,t) = \int \alpha_2 P_{3(1)}(s,t)ds$$
$$P_{19}(0,t) = \int \alpha_3 P_{3(1)}(s,t)ds \qquad P_{20}(0,t) = \int \alpha_4 P_{3(1)}(s,t)ds$$

Equation (1) is linear differential equation of first order and other equations (2-21) are partial differential equations of first order. Using the boundary and initial conditions, the equations (1-21) are solved to give the following solution:

$$P_0(t) = e^{-\int S_0(t)dt}\left[1 + \int M_0(t)e^{\int S_0(t)dt}dt\right]$$

$$P_1(z,t) = e^{-\int S_1(z)dz}\left[\lambda_1 P_0(t-z) + \int M_1(z,t)e^{\int S_1(z)dz}dz\right]$$

$$P_2(s,t) = e^{-\int S_2(s)ds}\left[\lambda_2 P_0(t-s) + \int M_2(s,t)e^{\int S_2(s)ds}ds\right]$$

$$P_{3(1)}(s,t) = e^{-\int S_{3(1)}(s)ds}\left[\int \alpha_4 P_1(z,t-s)dz + \right.$$
$$\left. + \int M_3(s,t)e^{\int S_{3(1)}(s)ds}ds\right]$$

$$P_{4(2)}(z,t) = e^{-\int S_{4(2)}(z)dz}\left[\int \alpha_3 P_2(s,t-z)ds + \right.$$
$$\left. + \int M_4(z,t)e^{\int S_{4(2)}(z)dz}dz\right]$$

$P_5(x,t)=\alpha_1 P_0(t-x)e^{-\int \phi(x)dx}$
$P_6(y,t)=\alpha_2 P_0(t-y)e^{-\int \psi(y)dy}$
$P_7(x,t)=e^{-\int \phi(x)dx}\int \alpha_1 P_1(z,t-x)dz$
$P_8(y,t)=e^{-\int \psi(y)dy}\int \alpha_2 P_1(z,t-y)dz$
$P_9(z,t)=e^{-\int \mu(z)dz}\int \alpha_3 P_1(z,t-z)\,dz$
$P_{10}(x,t)=e^{-\int \phi(x)dx}\int \alpha_1 P_2(s,t-x)ds$
$P_{11}(y,t)=e^{-\int \phi(x)dx}\int \alpha_2 P_2(s,t-y)ds$
$P_{12}(s,t)=e^{-\int \sigma(s)ds}\int \alpha_4 P_2(s,t-s)ds$
$P_{13}(x,t)=e^{-\int \phi(x)dx}\int \alpha_1 P_{4(2)}(z,t-x)dz$
$P_{14}(y,t)=e^{-\int \psi(y)dy}\int \alpha_2 P_{4(2)}(z,t-y)dz$
$P_{15}(z,t)=e^{-\int \mu(z)dz}\int \alpha_3 P_{4(2)}(z,t-z)dz$
$P_{16}(s,t)=e^{-\int \sigma(s)ds}\int \alpha_4 P_{4(2)}(z,t-s)dz$
$P_{17}(x,t)=e^{-\int \phi(x)dx}\int \alpha_1 P_{3(1)}(s,t-x)ds$
$P_{18}(y,t)=e^{-\int \psi(y)dy}\int \alpha_2 P_{3(1)}(s,t-y)ds$
$P_{19}(z,t)=e^{-\int \mu(z)dz}\int \alpha_3 P_{3(1)}(s,t-s)ds$
$P_{20}(s,t)=e^{-\int \sigma(s)ds}\int \alpha_4 P_{3(1)}(s,t-s)ds$

It is evident that all probabilities are obtained in terms of $P_0(t)$ which is given by (1). The time dependent availability $A(t)$ is:

$$A(t)=P_0(t)+\int P_2(s,t)ds+\int P_1(z,t)\,dz+ \\ +\int P_{4(2)}(z,t)dz+\int P_{3(1)}(s,t)ds \quad (22)$$

4.3. Steady State Availability

In the process industry, we require long run availability of the system, which is obtained by putting $\frac{d}{dt}$=0, $\frac{\partial}{\partial t}$=0 and taking probabilities independent of "t" then limiting probabilities from (1-21) are:

$[(\alpha_1+\alpha_2+\lambda_1+\lambda_2]P_0=\phi P_5+\psi P_6+\sigma P_2+\mu P_1$
$[(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\mu]P_1=\lambda_1 P_0+\sigma P_{3(1)}+\mu P_9+\psi P_8+\phi P_7$
$[(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\sigma]P_2=\lambda_2 P_0+\phi P_{10}+\psi P_{11}+\sigma P_{12}+\mu P_{4(2)}$
$[(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\sigma]P_{3(1)}=\alpha_4 P_1+\sigma P_{20}+\mu P_{19}+\psi P_{18}+\phi P_{17}$
$[(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\mu]P_{4(2)}=\alpha_3 P_2+\phi P_{13}+\psi P_{14}+\sigma P_{16}+\mu P_{15}$

$\phi P_5=\alpha_1 P_0$ $\quad \psi P_6=\alpha_2 P_0$
$\phi P_7=\alpha_1 P_1$ $\quad \psi P_8=\alpha_2 P_1$
$\mu P_9=\alpha_3 P_1$ $\quad \phi P_{10}=\alpha_1 P_1$
$\psi P_{11}=\alpha_2 P_2$ $\quad \sigma P_{12}=\alpha_4 P_2$
$\phi P_{13}=\alpha_1 P_{4(2)}$ $\quad \psi P_{14}=\alpha_2 P_{4(2)}$
$\mu P_{15}=\alpha_3 P_{4(2)}$ $\quad \sigma P_{16}=\alpha_4 P_{4(2)}$
$\phi P_{17}=\alpha_1 P_{3(1)}$ $\quad \psi P_{18}=\alpha_2 P_{3(1)}$
$\mu P_{19}=\alpha_3 P_{3(1)}$ $\quad \sigma P_{20}=\alpha_4 P_{3(1)}$

With initial conditions $P_i(0)=\begin{cases}1, i=0\\ 0, i\#0\end{cases}$

On solving these equations recursively, we have the steady state probabilities:

$P_2=T_2P_0$ $\quad P_1=T_1P_0$
$P_{4(2)}=M_2P_0$ $\quad P_{3(1)}=M_1P_0$
$A_v=[1+M_1+M_2+T_1+T_2]P_0$

Now using normalizing conditions

$\sum_{i=0}^{20} P_i=1$
$A_v=[1+M_1+M_2+T_1+T_2]P_0$

Where

$$P_0=\left[1+\frac{\alpha_1}{\phi}+\frac{\alpha_2}{\psi}+\left(1+\frac{\alpha_1}{\phi}+\frac{\alpha_2}{\psi}+\frac{\alpha_3}{\mu}+\frac{\alpha_4}{\sigma}\right)M_1+ \right. \\ +\left(1+\frac{\alpha_1}{\phi}+\frac{\alpha_2}{\psi}+\frac{\alpha_4}{\sigma}+\frac{\alpha_3}{\mu}\right)M_2+\left(1+\frac{\alpha_1}{\phi}+\frac{\alpha_4}{\sigma}+\frac{\alpha_3}{\mu}\right)T_1+ \\ \left. +\left(1+\frac{\alpha_1}{\phi}+\frac{\alpha_2}{\psi}+\frac{\alpha_4}{\sigma}\right)T_2\right]^{-1}$$

$M_1=\frac{\lambda_1\alpha_4}{\sigma\mu}$ $\quad M_2=\frac{\lambda_2\alpha_3}{\sigma\mu}$ $\quad T_1=\frac{\lambda_1}{\mu}$ $\quad T_2=\frac{\lambda_2}{\sigma}$

5. Sub- system subjected to simultaneously failure

5.1. Notations

A, B, C, D : Indicates that the sub-system is working in full capacity.
$\overline{C},\overline{D}$: Indicate the reduced state of the sub-system *C* and *D*.
a,b,c,d, : Indicate the failed state of the sub-system.
α_i : Failure rate of the sub-system *A,B,C,D*.
$\phi(x)\psi(y)$ $\mu(z)\sigma(s)$: General repair rates of *A,B,C,D* respectively.
$P_o(t)$: The system is working in full capacity.
$P_i(x,t)$: Probability that there is failure in subsystem A at time't' and it is repaired within time interval $(x,x+\Delta)$. For i = 1,5,12,13.
$P_j(y,t)$: Probability that there is failure in subsystem *B* at time't' and it is repaired within time interval $(yyx+\Delta)$. For j = 2,6,11,14. $_3(s,t)$: Probability that there is failure in subsystem *D* at time't' and system remains in reduced state till it is repaired within time interval $(s,s+\Delta)$.
$P_4(z,t)$: Probability that there is failure in subsystem *C* at time 't' and system remains in reduced state till it is repaired within time interval $(z,z+\Delta)$.
$P_7(s,z,t)$: Probability that there is failure in subsystem *C* and *D* at time 't' and system remains in reduced state till it is repaired within time interval $(z,z+\Delta)$ and $(s,s+\Delta)$ respectively.
$P_k(z,t)$: Probability that there is failure in subsystem *C* at time't' and it is repaired within time interval $(z,z+\Delta)$. For k = 8,10.
$P_l(z,t)$: : Probability that there is failure in subsystem *D* at time 't' and it is repaired within time interval $(s,s+\Delta)$. For l = 9,15.

5.2. Assumptions

1. Failure rates are constant over time and statistically independent.

2. A repaired unit as good as new, performance wise, for a specified duration.
3. Sufficient repair facilities are provided.
4. Service includes repair and/or replacement.
5. System may work at reduced capacity.
6. There are simultaneous failures

6. Mathematical formulations

The differential-difference equations obtained from the state transition is as follows (figure 2):

$$\left[\frac{d}{dt} + N_0(t)\right] P_0(t) = C_0(t) \quad (23)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial s} + N_1(s)\right] P_3(s,t) = C_1(\mathrm{s},\mathrm{t}) \quad (24)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + N_2(z)\right] P_4(z,t) = C_2(z,t) \quad (25)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial s} + \frac{\partial}{\partial z} + N_3(s,z)\right] P_7(s,z,t) = C_3(s,z,t) \quad (26)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \phi(x)\right] P_1(x,t) = 0 \quad (27)$$

$$\left[\frac{\partial}{\partial \mathrm{t}} + \frac{\partial}{\partial \mathrm{y}} + \psi(\mathrm{y})\right] \mathrm{P}_2(\mathrm{y},\mathrm{t}) = 0 \quad (28)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \phi(x)\right] P_5(x,t) = 0 \quad (29)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \psi(y)\right] P_6(y,t) = 0 \quad (30)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \mu(z)\right] P_8(z,t) = 0 \quad (31)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial s} + \sigma(s)\right] P_9(s,t) = 0 \quad (32)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \mu(z)\right] P_{10}(z,t) = 0 \quad (33)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \psi(y)\right] P_{11}(y,t) = 0 \quad (34)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \phi(x)\right] P_{12}(x,t) = 0 \quad (35)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \phi(x)\right] P_{13}(x,t) = 0 \quad (36)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \psi(y)\right] P_{14}(y,t) = 0 \quad (37)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial s} + \sigma(s)\right] P_{15}(s,t) = 0 \quad (38)$$

Where,

$$N_0(t) = \sum_{i=1}^{2} \alpha_i + \sum_{i=1}^{2} \lambda_i$$

$$C_0(t) = \int P_1(x,t)\phi(x)dx + \int P_2(y,t)\psi(y)dy + \\ + \int P_3(s,t)\sigma(s)ds + \int P_4(z,t)\mu(z)dz$$

$$N_1(s) = \sum_{i=1}^{4} \alpha_i + \sigma(s) \;\; C_1(\mathrm{s},\mathrm{t}) = \lambda_1 P_0(t) + \\ + \int P_{13}(x,t)\phi(x)dx + \int P_{14}(y,t)\psi(y)dy + \\ + \int P_{15}(s,t)\sigma(s)ds + \int P_7(s,z,t)\mu(z)dz\,ds$$

$$N_2(z) = \sum_{i=1}^{4} \alpha_i + \mu(z) \;\; C_2(z,t) = \lambda_2 P_0(t) + \\ + \int P_5(x,t)\phi(x)dx + \int P_6(y,t)\psi(y)dy + \\ + \int P_7(s,z,t)\sigma(s)ds\,dz + \int P_8(z,t)\mu(z)dz$$

$$N_3(s,z) = \sum_{i=1}^{4} \alpha_i + \sigma(s) + \mu(z) \; C_3(s,z,t) = \int \alpha_3 P_3(s,t)ds + \\ + \int \alpha_4 P_4(z,t)dz + \int P_9(s,t)\sigma(s)ds + \int P_{10}(z,t)\mu(z)dz + \\ + \int P_{11}(y,t)\psi(y)dy + \int P_{12}(x,t)\phi(x)dx$$

6.1. Initial conditions:

$P_0\ (0) = 1$ otherwise 0
$P_i(x,0) = 0$ For i=1,5,12,13
$P_j(y,0) = 0$ For j=2,6,11,14
$P_k(z,0) = 0$ For k =4,7,8,10
$P_l(s,0) = 0$ For l=3,7,9,15
$P_0(0) = 1$ otherwise 0
$P_i\ (x,0) = 0$ For i=1,5,12,13
$P_j\ (y,0) = 0$ For j=2,6,11,14
$P_k\ (z,0) = 0$ For k =4,7,8,10
$P_l\ (s,0) = 0$ For l=3,7,9,15

6.2. Boundary Conditions:

$$P_1(0,t) = \alpha_1 P_0(t)$$

$$P_2(0,t) = \alpha_2 P_0(t)$$

$$P_3(0,t) = \alpha_4 P_0(t)$$

$$P_4(0,t) = \alpha_3 P_0(t)$$

$$P_5(0,t) = \int \alpha_1 P_4(z,t)\,dz$$

$$P_6(0,t) = \int \alpha_2 P_4(z,t)dz$$

$$P_7(0,0,t) = \int \alpha_3 P_3(s,t)ds + \int \alpha_4 P_4(z,t)dz$$

$$P_8(0,t) = \int \alpha_3 P_4(z,t)\,dz\,dz$$

$$P_9(0,t) = \int \alpha_4 P_7(s,z,t)ds\;\;dz$$

$$P_{10}(0,t) = \int \alpha_3 P_7(s,z,t)\;\;ds\,dz$$

$$P_{11}(0,t) = \int \alpha_2 P_7(s,z,t)\,ds\,dz$$

$$P_{12}(0,t) = \int \alpha_1 P_7(s,z,t)\,dsdz$$

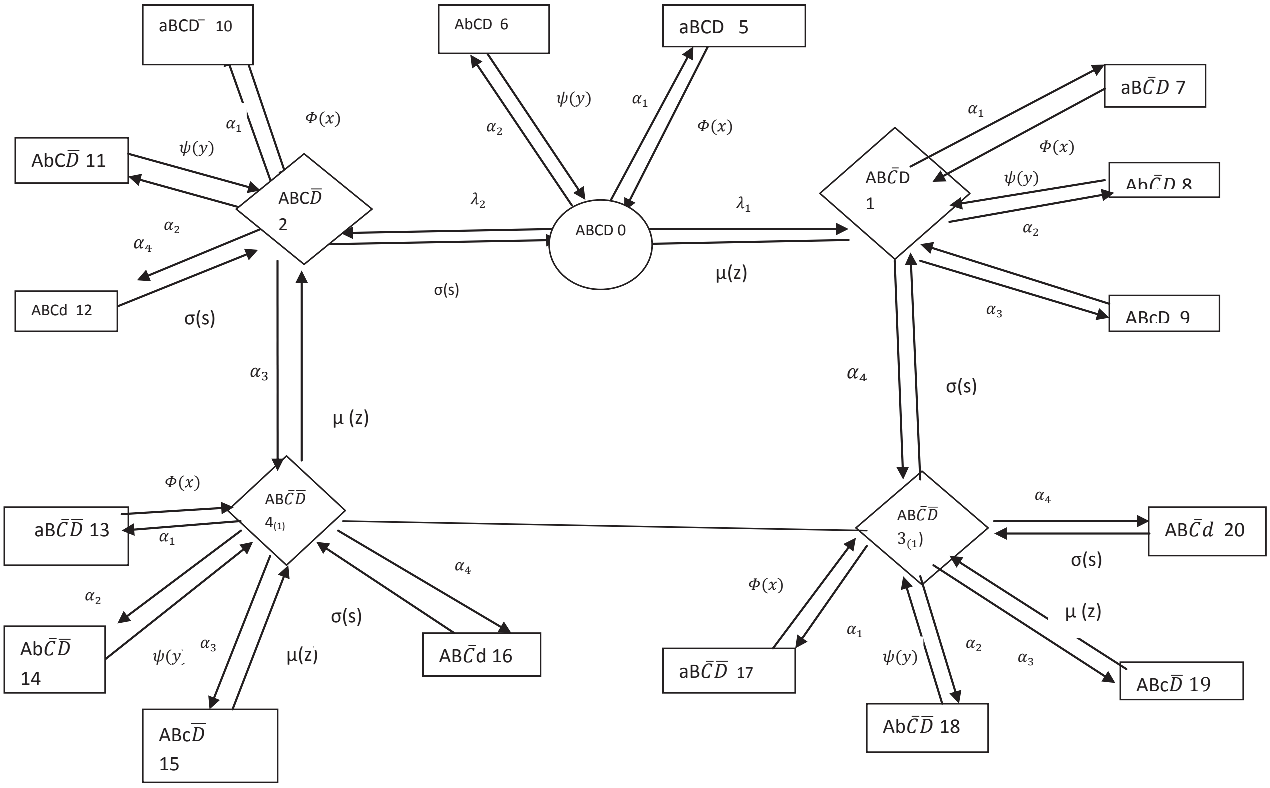

Fig. 2. Transition diagram of Poly Tube Industry when Sub-System failed independently

$$P_{13}(0,t) = \int \alpha_1 P_3(s,t)$$

$$P_{14}(0,t) = \int \alpha_2 P_3(s,t)ds$$

$$P_{15}(0,t) = \int \alpha_4 P_3(s,t)ds$$

Equation (23) is linear differential equation of first order and other equations (24-38) are partial differential equations of first order. Using the boundary and initial conditions, the equations (23-38) are solved to give the following solution:

$$P_0(t) = e^{-\int N_0(t)dt}\left[1 + \int C_0(t)e^{\int N_0(t)dt}dt\right]$$

$$P_1(x,t) = \alpha_1 P_0(t-x)e^{-\int \phi(x)dx}$$

$$P_2(y,t) = \alpha_2 P_0(t-y)e^{-\int \psi(y)dy}$$

$$P_3(s,t) = e^{-\int N_1(s)ds}\left[\alpha_4 P_0(t-s) + \int C_1(s,t)e^{\int N_1(s)ds}ds\right]$$

$$P_4(z,t) = e^{-\int N_2(z)dz}\left[\alpha_3 P_0(t-z) + \int C_2(z,t)e^{\int N_2(z)dz}dz\right]$$

$$P_5(x,t) = e^{-\int \phi(x)dx}\int \alpha_1 P_4(z,t-x)dz$$

$$P_6(y,t) = e^{-\int \psi(y)dy}\int \alpha_2 P_4(z,t-y)dz$$

$$P_7(s,z,t) = e^{-\int N_3(s,z)ds}\Big[\int \alpha_3 P_3(s,t-z)ds + \int \alpha_4 P_4(z,t-s)dz + \int C_3(s,z,t)e^{\int N_3(s,z)dsds}\Big]$$

$$P_8(z,t) = e^{-\int \mu(z)dz}\int \alpha_3 P_4(z,t-z)\,dz$$

$$P_9(s,t) = e^{-\int \sigma(s)ds}\int \alpha_4 P_7(s,z,t-s)ds\ dz$$

$$P_{10}(z,t) = e^{-\int \mu(z)dz}\int \alpha_3 P_7(s,z,t-z)ds\ dz$$

$$P_{11}(y,t) = e^{-\int \psi(y)dy}\int \alpha_2 P_{7(3)}(s,z,t-y)ds\ dz$$

$$P_{12}(x,t) = e^{-\int \phi(x)dx}\int \alpha_1 P_{7(3)}(s,z,t-x)ds\ dz$$

$$P_{13}(x,t) = e^{-\int \phi(x)dx}\int \alpha_1 P_3(s,t-x)ds$$

$$P_{14}(y,t) = e^{-\int \psi(y)dy}\int \alpha_2 P_3(s,t-y)ds$$

$$P_{15}(s,t) = e^{-\int \sigma(s)ds}\int \alpha_4 P_3(s,t-s)ds$$

It is evident that all probabilities are obtained in terms of $P_0(t)$ which is given by (23).

The time dependent availability $A(t)$ is:

$$A(t) = P_0(t) + \int P_3(s,t)ds + \int P_4(z,t)\,dz + \int P_7(s,z,t)dz\,ds \qquad (39)$$

6.3. Steady state availability

In the process industry, we require long run availability of the system, which is obtained by putting $\frac{d}{dt}=0$, $\frac{\partial}{\partial t}=0$ and taking probabilities independent of "t" then limiting probabilities from (23-38) are :

$$[(\alpha_1+\alpha_2+\lambda_1+\lambda_2)]P_0=\phi P_1+\psi P_2+\sigma P_3+\mu P_4$$

$$[(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\sigma]P_3=\lambda_1 P_0+\phi P_{13}+\psi P_{14}+\sigma P_{15}+\mu P_7$$

$$[(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\mu]P_4=\lambda_2 P_0+\phi P_5+\psi P_6+\sigma P_7+\mu P_8$$

$$[(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\sigma+\mu]P_7=\alpha_3 P_3+\alpha_4 P_4+\sigma P_9+\mu P_{10}+\psi P_{11}+\phi P_{12}$$

$$\phi P_1=\alpha_1 P_0 \qquad \psi P_2=\alpha_2 P_0$$

$$\phi P_5=\alpha_1 P_4 \qquad \psi P_6=\alpha_2 P_4$$

$$\mu P_8=\alpha_3 P_4 \qquad \sigma P_9=\alpha_4 P_7$$

$$\mu P_{10}=\alpha_3 P_7 \qquad \psi P_{11}=\alpha_2 P_7$$

$$\phi P_{12}=\alpha_1 P_7 \qquad \phi P_{13}=\alpha_1 P_3$$

$$\psi P_{14}=\alpha_2 P_3 \qquad \sigma P_{15}=\alpha_4 P_3$$

On solving these equations recursively, we get:

$$P_7=\frac{\alpha_3}{s_1}P_3+\frac{\alpha_4}{s_1}P_4$$

$$P_4=\frac{\lambda_2}{s_2}P_0+\frac{\sigma\alpha_3}{s_1 s_2}P_3$$

$$P_3=\left(\frac{\lambda_1}{s_3}+\frac{\alpha_3\mu\alpha_4}{s_1 s_2 s_3}\right)P_0$$

Now using normalizing conditions:

$$\sum_{i=0}^{15}P_i=1$$

$$A_v=[1+M_1+M_2+M_3]P_0 \tag{40}$$

Where:

$$P_0=\left[1+\frac{\alpha_1}{\phi}+\frac{\alpha_2}{\psi}+\left(1+\frac{\alpha_1}{\phi}+\frac{\alpha_2}{\psi}+\frac{\alpha_4}{\sigma}\right)M_1+\right.$$
$$\left.+\left(1+\frac{\alpha_1}{\phi}+\frac{\alpha_2}{\psi}+\frac{\alpha_3}{\mu}\right)M_2+\left(1+\frac{\alpha_1}{\phi}+\frac{\alpha_2}{\psi}+\frac{\alpha_4}{\sigma}+\frac{\alpha_3}{\mu}\right)M_3\right]^{-1}$$

$$s_1=\sigma+\mu$$

$$s_2=\alpha_4+\mu-\frac{\sigma\alpha_4}{s_1}$$

$$s_3=\alpha_3+\sigma-\frac{\mu\alpha_3}{s_1}-\frac{\sigma\mu\alpha_4\alpha_3}{s_1 s_2 s_1}$$

$$M_1=\left(\frac{\lambda_1}{s_3}+\frac{\alpha_3\mu\alpha_4}{s_1 s_2 s_3}\right)$$

$$M_2=\frac{\lambda_2}{s_2}+\frac{\sigma\alpha_3}{s_1 s_2}M_1$$

$$M_3=\frac{\alpha_3}{s_1}M_1+\frac{\alpha_4}{s_1}M_2$$

Under the available facilities the concern management minimize the failure time of each sub-system adopting the following measures

I. Getting the information of failure of each equipment at the earliest moment.
II. Starting the repair work at the earliest moment.
III. Providing trained workers.
IV. Providing the special tools required.
V. Making available the spare parts and special parts

7. Performance analysis

Table 1 shows that with the increase in failure rate of mixture α_1 from 0.0057 to 0.0063, by keeping other parameters constant (λ_1=0.001, λ_2=0.002, α_2=0.007, α_3=0.0133, α_4=0.015, ψ=2, μ=0.33, σ=0.02), the availability of system decreases approximately 1.13% . Whereas, the availability increases approximately 0.63% with the increase repair rate of mixture from 0.5 to 1.1. Similarly, table 5 shows that availability decreases approximately by 1.12% with the increase of failure rate of mixture α_1 from 0.0057 to 0.0063 keeping other parameters constant (λ_1=0.001, λ_2=0.002, α_2=0.007, α_3=0.01, α_4=0.015, ψ =2, μ =0.33, σ =0.02) whereas, the availability increases approximately 0.63% with the increase repair rate of mixture ϕ from 0.5 to 1.1. Tables 2, 3, 4, 6, 7 and 8 shows that there is a almost negligible change in the availability of the Polytube extrusion system with the increase the repair rate of Extruder, Die and Cutter Subsystems.

8. Conclusions

The comparative study of the table 1 and table 5 shows that when the Sub-Systems Die and Cutter failed simultaneously or independently, then in both cases, the Sub- System A i.e. Mixture affects the Availability appreciably. We, thus make an inference that management should take paramount care of sub system A in order to improve overall reliability.

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Tab. 1. Effect of failure and repair rate of the Sub-System Mixture (A) on Availability when both Sub-Systems failed independently.

ϕ \ α_1	0.0057	0.0059	0.0061	0.0063	Constant values
0.5	0.9769	0.9732	0.9694	0.9656	α_2=0.007, ψ=2, λ_1=0.001, λ_2=0.002, α_3=0.01, μ =0.33, α_4=0.015, σ =0.02
0.7	0.9802	0.9775	0.9748	0.9721	
0.9	0..9821	0.9796	0.9770	0.9757	
1.1	0.9832	0.9815	0.9797	0.9780	

Tab. 2. Effect of failure and repair rate of the Sub-System Extruder (B) on Availability when both sub-systems failed independently.

ψ \ α_2	0.007	0.009	0.011	0.013	Constant values
2	0.9769	0.9760	0.9751	0.9741	α_1=0.0057, φ =0.5
4	0.9786	0.9781	0.9777	0.9772	λ_1=0.001, λ_2=0.002
6	0..9792	0.9789	0.9785	0.9782	α_3=0.01, μ =0.33
8	0.9795	0.9792	0.9790	0.9786	α_4=0.015,σ =2

Tab. 3. Effect of failure and repair rate of the Sub-System Die (C) on Availability when both sub-systems failed independently.

μ \ α_3	0.01	0.02	0.03	0.04	Constant values
0.33	0.9769	0.9621	0.9382	0.9067	α_1=0.0057, ϕ=0.5
0.53	0.9788	0.9673	0.99486	0.9236	α_2=0.007, ψ =2
0.73	0..99799	0.9705	0.9552	0.9344	α_4=0.015, σ =2
0.93	0.9807	0.9728	0.9597	0.9420	λ_1=0.002, λ_2=0.002

Tab. 4. Effect of failure and repair rate of the sub-system Cutter (D) on Availability when both sub-systems failed independently.

σ \ α_4	0.015	0.030	0.045	0.060	Constant values
2	0.9769	0.9763	0.9756	0.9749	α_1=0.0057, ϕ=0.5
4	0.9812	0.9809	0.9809	0.9802	α_2=0.007, ψ =0.04
6	0..9825	0.9823	0.9821	0.9818	α_3=0.33, μ =0.02
8	0.9831	0.9829	0.9827	0.9826	λ_1=0.001, λ_2=0.002

Tab. 5. Effect of failure and repair rate of the Sub-System Mixture (A) on Availability when both Sub-Systems failed simultaneously.

ϕ \ α_1	0.0057	0.0059	0.0061	0.0063	Constant values
0.5	0.9853	0.9849	0.9845	0.9841	α_2=0.007, ψ =2
0.7	0.9884	0.9882	0.9879	0.9876	α_3=0.01, μ =0.33
0.9	0..9902	0.9900	0.9898	0.9896	α_4=0.015, σ =0.02
1.1	0.9913	0.9912	0.9910	0.9908	λ_1=0.001, λ_2=0.002

Tab. 6. Effect of failure and repair rate of the sub-system Extruder (B) on Availability when both sub-systems failed simultaneously.

ψ \ α_2	0.007	0.009	0.011	0.013	Constant values
2	0.9853	0.9843	0.9832	0.9823	α_1=0.0057, ϕ=0.5
4	0.9869	0.9865	0.9860	0.9855	α_3=0.01, μ =0.33
6	0..9875	0.9872	0.9869	0.9865	α_4=0.015, σ =2
8	0.9878	0.9876	0.9873	0.9871	λ_1=0.001, λ_2=0.002

Tab. 7. Effect of failure and repair rate of the Sub-System Die (C) on Availability when both Sub-Systems failed simultaneously.

μ \ α_3	0.01	0.02	0.03	0.04	Constant values
0.33	0.9853	0.9853	0.9853	0.9853	α_1=0.0057, ϕ=0.5
0.53	0.9853	0.9853	0.9853	0.9853	α_2=0.007, ψ =2
0.73	0..9854	0.9854	0.9854	0.9855	α_4=0.015, σ =2
0.93	0.9854	0.9854	0.9854	0.9855	λ_1=0.001, λ_2=0.002

Tab. 8. Effect of failure and repair rate of the Sub-System Cutter (D) on Availability when both Sub-Systems failed simultaneously.

σ \ α_4	0.015	0.030	0.045	0.060	Constant values
2	0.9853	0.9853	0.9852	0.9852	α_1=0.0057, ϕ=0.5
4	0.9853	0.9853	0.9853	0.9852s	α_2=0.007, ψ =0.04
6	0..9853	0.9852	0.9852	0.9852	α_3=0.33,μ =0.02
8	0.9854	0.9853	0.9853	0.9852	λ_1=0.001, λ_2=0.002

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