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A SIMPLIFIED METHOD TO ASSESS FATIGUE LIFE OF SELECTED STRUCTURAL COMPONENTS OF AN AIRCRAFT FOR A VARIABLE LOAD SPECTRUM

UPROSZCZONA METODA OCENY TRWAŁOŚCI ZMĘCZENIOWEJ WYBRANYCH ELEMENTÓW KONSTRUKCJI STATKU POWIETRZNEGO DLA ZMIENNEGO WIDMA OBCIĄŻENIA*

The assessment of fatigue life of an aircraft's structural component operating under variable load spectrum causes many and various problems, hence the need for simplified methods that facilitate it. The presented study covers the question of rearranging an actual spectrum with variable values of cycles into a homogeneous spectrum with weighted cycles. A method for the evaluation of fatigue life of some selected aircraft's structural component with an initial crack has been presented using a rearranged spectrum. To model an increment in the crack length a difference equation has been applied which, after rearrangement, resulted in a partial differential equation of the Fokker-Planck type. A density function of the crack length is a particular solution to this equation. Using the density function of a crack length, fatigue life of the structural component has been determined for the crack that keeps growing up to the permissible value l_d lower than the critical value l_{kr}. What has been given consideration in this study is the case when the exponent of the Paris equation $m \neq 2$.

Keywords: *load cycle, weighted load cycle, reliability, durability, load spectrum.*

Ocena trwałości zmęczeniowej elementu konstrukcji pracującego pod wpływem zmiennego widma obciążenia przysparza wielu trudności. Stąd potrzeba poszukiwania uproszczonych metod umożliwiających tą ocenę. Przedstawiona praca obejmuje przekształcenie widma rzeczywistego o zmiennych wartościach cykli w widmo jednorodne o cyklach ważonych. Wykorzystując widmo przekształcone przedstawiono metodę oceny trwałości zmęczeniowej wybranego elementu konstrukcji statku powietrznego z początkowym pęknięciem. Do modelowania przyrostu długości pęknięcia wykorzystano równanie różnicowe z którego po przekształceniu otrzymano równanie różniczkowe cząstkowe typu Fokkera-Plancka. Rozwiązaniem szczególnym tego równania jest funkcja gęstości długości pęknięcia elementu. Wykorzystując następnie funkcję gęstości długości pęknięcia określono trwałość zmęczeniową elementu konstrukcji dla pęknięcia narastającego do wartości dopuszczalnej l_d mniejszej od wartości krytycznej l_{kr}. W pracy rozpatruje się przypadek, gdy wykładnik równania Parisa $m \neq 2$.

Słowa kluczowe: *cykl obciążenia, ważony cykl obciążenia, niezawodność, trwałość, widmo obciążenia.*

1. Introduction

The assessment of fatigue life of an aircraft structural component operating under variable load spectrum causes many and various problems, however it proves essential to flight safety. The present study is an effort to find a simplified method of fatigue life determination. This simplification consists in the rearrangement of an actual load spectrum into a homogeneous spectrum with weighted cycles. The rearrangement has been outlined in Section 2.

It has been assumed that an initial crack in the structural component is l_0. As affected by the load of a variable spectrum the crack grows up to some permissible length l_d (safe) shorter than the critical length l_{kr}. The crack growth rate, approached in a deterministic way, has been described with the Paris formula of the following form [1]:

$$\frac{dl}{dN} = C(\Delta K)^m \qquad (1)$$

where: ΔK – the range of changes in the stress intensity factor, C, m – material constants, N – a variable that denotes the number of structure-affecting load cycles.

(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie ***www.ein.org.pl***

2. How to find the cracking rate for the load in the form of a rearranged load spectrum

The following assumptions have been made for the need of rearranging the actual spectrum with variable load values into the spectrum with weighted cycles:

1) An aircraft structural component keeps operating under variable loads while performing its functions.
2) The load spectrum that affects the component in the course of a standard flight of the aircraft is known.
3) We assume that this spectrum of loads allows of the determination of:
   - the total number of load cycles N_c in the course of one flight,
   - there is the L number of thresholds of maximum load values $\sigma_1^{max}, \sigma_2^{max}, \ldots, \sigma_L^{max}$.
4) The number of repetitions of maximum threshold values in the assumed spectrum is as follows: σ_1^{max} occurs n_1 times, σ_2^{max} occurs n_2 times, …, σ_L^{max} occurs n_L times. The number of repetitions of specific threshold values of load in the course of one flight is: $N_c = \sum_{i=1}^{L} n_i$
5) The minimum value in thresholds is determined with the following relationship:

$$\sigma_{i,śr}^{min} = \frac{\sigma_{i,1}^{min} + \sigma_{i,2}^{min} + \cdots + \sigma_{i,n_i}^{min}}{n_i}$$, where $i = 1,2,\ldots,n_i$.

6) Table 1 gives maximum σ_i^{max} and minimum $\sigma_{i,śr}^{min}$ stress values in the cycles, and frequencies of their appearing in the spectrum P_i
7) Table 2 gives the statement of stress ratios $\hat{R}_i$ and empirical coefficients of influence on crack growth U_i where: $\hat{R}_i = \frac{\sigma_{i,śr}^{min}}{\sigma_i^{max}}$, $U_i = \alpha_1 + \alpha_2 \hat{R}_i + \alpha_3 \hat{R}_i^2$; α_1, α_2, α_3 – empirical coefficients [4, 5].
8) The range of stress variations has been shown in Table 3 $\Delta\sigma_i = \sigma_i^{max} - \sigma_{i,śr}^{min}$
9) Account has been taken of the effect of overload cycles upon the crack growth rate (table 4):
   $\Delta\sigma_{i,ef} = C_i^P \Delta\sigma_i$
   where: C_i^P - factor of crack growth retardation after overload cycles occurred [3].

In the case given consideration it has been also assumed that the rate of crack growth in the structural component, approached in a deterministic way, follows the Paris' law written down with formula (1). For the above specified assumptions, in this case for the i-th type of a load cycle (gained from the description of the spectrum of loading in a standard cycle), formula (1) takes the following form:

$$\frac{dl}{dN} = C U_i M_k^m \left(\Delta\sigma_{i,ef}\right)^m \pi^{\frac{m}{2}} l^{\frac{m}{2}} \quad (2)$$

where M_k specify influence of crack location and dimensions with relations to structural element dimensions on crack growth velocity [1].

With account taken of all types of load cycles, the relationship (2) takes the form:

$$\frac{dl}{dN} = C\pi^{\frac{m}{2}} \left(\sum_{i=1}^{L} P_i U_i \left(\Delta\sigma_{i,ef}\right)^m \right) M_k^m l^{\frac{m}{2}} \quad (3)$$

where: $i = 1,2,\ldots,L$.

Table 1. Maximum σ_i^{max} and minimum $\sigma_{i,śr}^{min}$ stress values in the cycles, and frequencies of their appearing in the spectrum P_i

σ_i^{max}	σ_1^{max}	σ_2^{max}	…	σ_i^{max}	…	σ_L^{max}
$\sigma_{i,śr}^{min}$	$\sigma_{1,śr}^{min}$	$\sigma_{2,śr}^{min}$	…	$\sigma_{i,śr}^{min}$	…	$\sigma_{L,śr}^{min}$
P_i	$P_1 = \frac{n_1}{N_c}$	$P_2 = \frac{n_2}{N_c}$	…	$P_i = \frac{n_i}{N_c}$	…	$P_L = \frac{n_L}{N_c}$

Table 2. Stress ratios $\hat{R}_i$ and empirical coefficients of influence on crack growth U_i

$cykle\ i$	1	2	…	i	…	L
$\hat{R}_i$	$\hat{R}_1$	$\hat{R}_2$	…	$\hat{R}_i$	…	$\hat{R}_L$
U_i	U_1	U_2	…	U_i	…	U_L

Table 3. Range of stress $\Delta\sigma_i$ and frequencies of their appearing in the spectrum P_i

$cycle\ types$	1	2	…	i	…	L
$\Delta\sigma_i$	$\Delta\sigma_1$	$\Delta\sigma_2$	…	$\Delta\sigma_i$	…	$\Delta\sigma_L$
P_i	P_1	P_2	…	P_i	…	P_L

Table 4. Range of effective stress $\Delta\sigma_{i,ef}$ which takes into consideration effect of overload cycles

$cycle\ types$	1	2	…	i	…	L
$coefficients$	C_1^P	C_2^P	…	C_i^P	…	C_L^P
$\Delta\sigma_{i,ef}$	$\Delta\sigma_{1,ef}$	$\Delta\sigma_{2,ef}$	…	$\Delta\sigma_{i,ef}$	…	$\Delta\sigma_{L,ef}$

Formula (3) can be expressed as a function of time, or more precisely, of the aircraft's flying time. Therefore, we assume that:

$$N = \lambda t \tag{4}$$

where: λ – the rate of load cycles, N – the number of load cycles, t – flying time of an aircraft.

In our case $\lambda = 1/\Delta t$, where Δt denotes duration of the fatigue loading cycle that affects the structural component. We can assume the following formula for Δt:

$$\Delta t = \frac{T}{N_c} \tag{5}$$

where: T – the duration of the aircraft's standard flight to determine load spectrum, N_c – the number of load cycles during the standard flight.

After these rearrangements, formula (3) takes the following form:

$$\frac{dl}{dt} = \lambda C \pi^{\frac{m}{2}} \left(\sum_{i=1}^{L} P_i\, U_i \left(\Delta\sigma_{i,ef} \right)^m \right) M_k^m l^{\frac{m}{2}} \tag{6}$$

Formula (6) describes the crack growth rate for a homogeneous load spectrum with weighted cycles.

3. Finding the density function of the crack length

Let $U_{l,t}$ denote probability that for the aircraft's flying time t the structural component's crack length is l. The difference equation for the above listed assumptions takes the following form [2, 6]:

$$U_{l,t+\Delta t} = (1 - \lambda \Delta t) U_{l,t} + \lambda \Delta t U_{l-\Delta l,t} \tag{7}$$

where: Δl – the crack length increment during one equivalent load cycle.

The value of the crack-length increment calculated on the basis of (6) will be:

$$\Delta l = \lambda C \pi^{\frac{m}{2}} \left(\sum_{i=1}^{L} P_i\, U_i \left(\Delta\sigma_{i,ef} \right)^m \right) M_k^m l^{\frac{m}{2}} \Delta t \tag{8}$$

Equation (7) expressed in terms of function notation takes the following form:

$$U(l, t + \Delta t) = (1 - \lambda \Delta t) U(l, t) + \lambda \Delta t U(l - \Delta l, t) \tag{9}$$

where: $U(l,t)$ – the density function of the crack length after the aircraft's flying time t expressed in terms of flight hours has elapsed; $(1 - \lambda \Delta t)$ – the probability that no equivalent load cycle occurs in time Δt; $\lambda \Delta t$ - the probability that an equivalent load cycle occurs in time Δt.

Equation (9) can be rearranged into a partial differential equation using the following approximations:

$$\left. \begin{aligned} U(l, t + \Delta t) &\cong U(l,t) + \frac{\partial U(l,t)}{\partial t} \Delta t \\ U(l - \Delta l, t) &\cong U(l,t) - \frac{\partial U(l,t)}{\partial l} \Delta l + \frac{1}{2} \frac{\partial^2 U(l,t)}{\partial l^2} (\Delta l)^2 \end{aligned} \right\} \tag{10}$$

Having substituted equation (10) into equation (9) the following is arrived at:

$$\frac{\partial U(l,t)}{\partial t} = -\lambda \frac{\partial U(l,t)}{\partial l} \Delta l + \frac{1}{2} \lambda (\Delta l)^2 \frac{\partial^2 U(l,t)}{\partial l^2} \tag{11}$$

where:

$$\Delta l = \lambda C \pi^{\frac{m}{2}} \left(\sum_{i=1}^{L} P_i\, U_i \left(\Delta\sigma_{i,ef} \right)^m \right) M_k^m l^{\frac{m}{2}} \Delta t$$

Since $\lambda \Delta t = 1$, the above written equation takes the form:

$$\Delta l = C \pi^{\frac{m}{2}} \left(\sum_{i=1}^{L} P_i\, U_i \left(\Delta\sigma_{i,ef} \right)^m \right) M_k^m l^{\frac{m}{2}} \tag{12}$$

Let

$$C \pi^{\frac{m}{2}} M_k^m = C_m \tag{13}$$

$$\Delta l = C_m \left(\sum_{i=1}^{L} P_i\, U_i \left(\Delta\sigma_{i,ef} \right)^m \right) l^{\frac{m}{2}} \tag{14}$$

Substitution of relationship (14) into equation (11) gives:

$$\begin{aligned} \frac{\partial U(l,t)}{\partial t} = &-\lambda \frac{\partial U(l,t)}{\partial l} C_m \left(\sum_{i=1}^{L} P_i\, U_i \left(\Delta\sigma_{i,ef} \right)^m \right) l^{\frac{m}{2}} + \\ &+ \frac{1}{2} \lambda (C_m \left(\sum_{i=1}^{L} P_i\, U_i \left(\Delta\sigma_{i,ef} \right)^m \right) l^{\frac{m}{2}})^2 \frac{\partial^2 U(l,t)}{\partial l^2} \end{aligned} \tag{15}$$

The result of equation (6) should be substituted for the crack length l in equation (15). What we get is:

$$\begin{aligned} &\frac{dl}{dt} = \lambda C \pi^{\frac{m}{2}} M_k^m \left(\sum_{i=1}^{L} P_i\, U_i \left(\Delta\sigma_{i,ef} \right)^m \right) l^{\frac{m}{2}} \\ &\int_{l_0}^{l} \frac{dx}{x^{\frac{m}{2}}} = \int_0^t \lambda C_m \left(\sum_{i=1}^{L} P_i\, U_i \left(\Delta\sigma_{i,ef} \right)^m \right) dt \\ &\frac{2}{2-m} l^{\frac{2-m}{2}} - \frac{2}{2-m} l_0^{\frac{2-m}{2}} = \lambda C_m \left(\sum_{i=1}^{L} P_i\, U_i \left(\Delta\sigma_{i,ef} \right)^m \right) t \\ &\frac{2}{2-m} l^{\frac{2-m}{2}} = \frac{2}{2-m} l_0^{\frac{2-m}{2}} + \lambda C_m \left(\sum_{i=1}^{L} P_i\, U_i \left(\Delta\sigma_{i,ef} \right)^m \right) t \\ &l = \left[l_0^{\frac{2-m}{2}} + \frac{2-m}{2} \lambda C_m \left(\sum_{i=1}^{L} P_i\, U_i \left(\Delta\sigma_{i,ef} \right)^m \right) t \right]^{\frac{2}{2-m}} \end{aligned} \tag{16}$$

With account taken of (16), coefficients of equation (15) can be written down as:

$$\begin{aligned} \alpha(t) = &\lambda C_m \left(\sum_{i=1}^{L} P_i\, U_i \left(\Delta\sigma_{i,ef} \right)^m \right) \cdot \\ &\cdot \left[l_0^{\frac{2-m}{2}} + \frac{2-m}{2} \lambda C_m \left(\sum_{i=1}^{L} P_i\, U_i \left(\Delta\sigma_{i,ef} \right)^m \right) t \right]^{\frac{m}{2-m}} \end{aligned} \tag{17}$$

$$\begin{aligned} \beta(t) = &\lambda \left[C_m \left(\sum_{i=1}^{L} P_i\, U_i \left(\Delta\sigma_{i,ef} \right)^m \right) \cdot \right. \\ &\left. \cdot \left[l_0^{\frac{2-m}{2}} + \frac{2-m}{2} \lambda C_m \left(\sum_{i=1}^{L} P_i\, U_i \left(\Delta\sigma_{i,ef} \right)^m \right) t \right]^{\frac{m}{2-m}} \right]^2 \end{aligned} \tag{18}$$

Equation (15) with coefficients (17) and (18) takes the following form for m ≠ 2:

$$\frac{\partial U(l,t)}{\partial t} = -\alpha(t)\frac{\partial U(l,t)}{\partial l} + \frac{1}{2}\beta(t)\frac{\partial^2 U(l,t)}{\partial l^2} \tag{19}$$

A particular solution of equation (19) takes the form [2, 6]:

$$U(l,t) = \frac{1}{\sqrt{2\pi A(t)}} e^{-\frac{(l-B(t))^2}{2A(t)}} \tag{20}$$

where:

$$B(t) = \int_0^t \alpha(t)dt \tag{21}$$

$$A(t) = \int_0^t \beta(t)dt \tag{22}$$

Now, the integral (21) is calculated:

$$B(t) = \lambda C_m \left(\sum_{i=1}^{L} P_i\, U_i\big(\Delta\sigma_{i,ef}\big)^m\right) \cdot$$

$$\cdot \int_0^t \left[l_0^{\frac{2-m}{2}} + \frac{2-m}{2}\lambda C_m \left(\sum_{i=1}^{L} P_i\, U_i\big(\Delta\sigma_{i,ef}\big)^m\right) t \right]^{\frac{m}{2-m}} dt =$$

$$= \lambda C_m \left(\sum_{i=1}^{L} P_i\, U_i\big(\Delta\sigma_{i,ef}\big)^m\right) \frac{1}{\left(\frac{m}{2-m}+1\right)} \cdot$$

$$\cdot \left[l_0^{\frac{2-m}{2}} + \frac{2-m}{2}\lambda C_m \left(\sum_{i=1}^{L} P_i\, U_i\big(\Delta\sigma_{i,ef}\big)^m\right) t \right]^{\frac{m}{2-m}+1} \cdot$$

$$\cdot \left. \frac{1}{\frac{2-m}{2}\lambda C_m \left(\sum_{i=1}^{L} P_i\, U_i\big(\Delta\sigma_{i,ef}\big)^m\right)} \right|_0^t =$$

$$= \left[l_0^{\frac{2-m}{2}} + \frac{2-m}{2}\lambda C_m \left(\sum_{i=1}^{L} P_i\, U_i\big(\Delta\sigma_{i,ef}\big)^m\right) t \right]^{\frac{2}{2-m}} - l_0$$

i.e. $B(t)$ is an average of the crack length for the flying time t of the aircraft. The computational formula takes, therefore, the following form:

$$B(t) = \left[l_0^{\frac{2-m}{2}} + \frac{2-m}{2}\lambda C_m \left(\sum_{i=1}^{L} P_i\, U_i\big(\Delta\sigma_{i,ef}\big)^m\right) t \right]^{\frac{2}{2-m}} - l_0 \tag{23}$$

Calculation of the integral (22) follows:

$$A(t) = \lambda {C_m}^2 \left(\sum_{i=1}^{L} P_i\, U_i\big(\Delta\sigma_{i,ef}\big)^m\right)^2 \cdot$$

$$\cdot \int_0^t \left[l_0^{\frac{2-m}{2}} + \frac{2-m}{2}\lambda C_m \left(\sum_{i=1}^{L} P_i\, U_i\big(\Delta\sigma_{i,ef}\big)^m\right) t \right]^{\frac{2m}{2-m}} dt =$$

$$= \lambda {C_m}^2 \left(\sum_{i=1}^{L} P_i\, U_i\big(\Delta\sigma_{i,ef}\big)^m\right)^2 \frac{1}{\left(\frac{2m}{2-m}+1\right)} \cdot$$

$$\cdot \left[l_0^{\frac{2-m}{2}} + \frac{2-m}{2}\lambda C_m \left(\sum_{i=1}^{L} P_i\, U_i\big(\Delta\sigma_{i,ef}\big)^m\right) t \right]^{\frac{2m}{2-m}+1} \cdot$$

$$\cdot \left. \frac{1}{\frac{2-m}{2}\lambda C_m \left(\sum_{i=1}^{L} P_i\, U_i\big(\Delta\sigma_{i,ef}\big)^m\right)} \right|_0^t =$$

$$= \lambda {C_m}^2 \left(\sum_{i=1}^{L} P_i\, U_i\big(\Delta\sigma_{i,ef}\big)^m\right)^2 \frac{1}{\left(\frac{m+2}{m-2}\right)} \cdot$$

$$\cdot \left[l_0^{\frac{2-m}{2}} + \frac{2-m}{2}\lambda C_m \left(\sum_{i=1}^{L} P_i\, U_i\big(\Delta\sigma_{i,ef}\big)^m\right) t \right]^{\frac{m+2}{m-2}} \cdot$$

$$\cdot \left. \frac{1}{\frac{2-m}{2}\lambda C_m \left(\sum_{i=1}^{L} P_i\, U_i\big(\Delta\sigma_{i,ef}\big)^m\right)} \right|_0^t =$$

$$= C_m \frac{2}{m+2} \left(\sum_{i=1}^{L} P_i\, U_i\big(\Delta\sigma_{i,ef}\big)^m\right) \cdot$$

$$\cdot \left[l_0^{\frac{2-m}{2}} + \frac{2-m}{2}\lambda C_m \left(\sum_{i=1}^{L} P_i\, U_i\big(\Delta\sigma_{i,ef}\big)^m\right) t \right]^{\frac{m+2}{m-2}} +$$

$$-C_m \frac{2}{m+2} \left(\sum_{i=1}^{L} P_i\, U_i\big(\Delta\sigma_{i,ef}\big)^m\right) \left(l_0^{\frac{2-m}{2}}\right)^{\frac{m+2}{m-2}} =$$

$$= C_m \frac{2}{m+2} \left(\sum_{i=1}^{L} P_i\, U_i\big(\Delta\sigma_{i,ef}\big)^m\right) \cdot$$

$$\left[\left[l_0^{\frac{2-m}{2}} + \frac{2-m}{2}\lambda C_m \left(\sum_{i=1}^{L} P_i\, U_i\big(\Delta\sigma_{i,ef}\big)^m\right) t \right]^{\frac{m+2}{m-2}} - l_0^{\frac{m+2}{2}} \right]$$

i.e. $A(t)$ is a variance of the crack growth for the flying time t. The computational formula takes the form:

$$A(t) = C_m \frac{2}{m+2} \left(\sum_{i=1}^{L} P_i\, U_i\big(\Delta\sigma_{i,ef}\big)^m\right) \cdot$$

$$\cdot \left[\left[l_0^{\frac{2-m}{2}} + \frac{2-m}{2}\lambda C_m \left(\sum_{i=1}^{L} P_i\, U_i\big(\Delta\sigma_{i,ef}\big)^m\right) t \right]^{\frac{m+2}{m-2}} - l_0^{\frac{m+2}{2}} \right] \tag{24}$$

4. Determination of fatigue life of the selected structural component of the aircraft

For the density function of the crack length versus the flying time of the aircraft the structural component's reliability can be found from the relationship [2, 6]:

$$R(t)_{l_d} = \int_{-\infty}^{l_d} U(l,t)dl \tag{25}$$

where the form of the density function of the crack length $U(l,t)$ has been determined by the relationship (20). The permissible crack length l_d can be found using the stress intensity factor of the following form:

$$K = M_k \sigma \sqrt{\pi l} \tag{26}$$

When the crack length and the stress reach their critical values, l_{kr} and σ_{kr} respectively, the factor determined by the relationship (26) also becomes a critical value K_c and is then called fracture toughness of the material:

$$K_c = M_k \sigma_{kr} \sqrt{\pi l_{kr}} \tag{27}$$

This relationship together with the safety factor allow of finding the permissible crack length:

$$l_d = \frac{K_c^2}{k M_k^2 \sigma_{kr}^2 \pi} \tag{28}$$

where: k – safety factor.

Normalization of the integral in equation (25) results in:

$$R(t)_{l_d} = \int_{-\infty}^{\frac{l_d - B(t)}{\sqrt{A(t)}}} U(z,t)dz \tag{29}$$

where: $B(t)$ and $A(t)$ are determined with relationships (23) and (24), respectively.

With the reliability level found we take values of the upper limit of the integral (29) from the normal distribution table. Hence the relationship:

$$Q_{l_d} = \frac{l_d - B(t)}{\sqrt{A(t)}} \tag{30}$$

Where: Q_{l_d} – value of the upper limit of the integral (29), for which value of the integral equals $R(t)_{l_d}$.

From relationship (30) we can find value of the flying time such that makes the assumed reliability level reached.

5. Final remarks and a computational example

To illustrate the above described method, a computational example has been presented. The example covers the rate of growth of an average-length fatigue crack in a structural component made from the steel of specified material properties, subjected to an actual load spectrum. Computations have been performed for the spectrum of variable- amplitude loads, which represents an actual component-loading spectrum and has been rearranged in the way discussed in Section 2 [2]. Table 5 below shows quantities that describe the rearranged loading spectrum used in our study.

The table 5 includes: values of ranges of changes in stress in cycle $\Delta\sigma_i$ for assumed load factors i and frequencies of their occurrence P_i, and factors with both load cycle asymmetry and how it affects the crack growth taken into account.

For some specified material of the pattern component, the following values of material constants have been used in the computations:

$m = 3{,}5$

$C = 3{,}2 \cdot 10^{-12}$

In our example the following values have been used for the computations: the initial crack length in the component assumed to be l_0 = 10mm, and permissible crack length found from the relationship (28) l_d = 25mm. It has been also assumed that the crack growth retardation factor after overload cycles $C_i^P = 1$, whereas the factor with the load cycle asymmetry and how it affects the crack growth taken into account has been defined by the empirically formulated equation $U_i = 0{,}55 + 0{,}33\hat{R}_i + 0{,}12\hat{R}_i^{\,2}$. In numerical calculations account has been also taken of the change in the M_k coefficient in the course of the crack growth. Then, the rearranged equation (23) for the average crack length has been used to make it depend on the number of loading cycles N, on the basis of equation (4).

$$B(N) = \left[l_0^{\frac{2-m}{2}} + \frac{2-m}{2} C \pi^{\frac{m}{2}} M_k^m \cdot \left(\sum_{i=1}^{L} P_i U_i \left(\Delta\sigma_{i,ef} \right)^m \right) N \right]^{\frac{2}{2-m}} - l_0 \tag{31}$$

Using the above written relationship, the increment in the average crack length against the number of loading cycles N over the range from the initial crack length l_0 to the permissible crack length l_d = 25mm has been found. Figure 1 shows the change in the average crack length against the number of loading cycles.

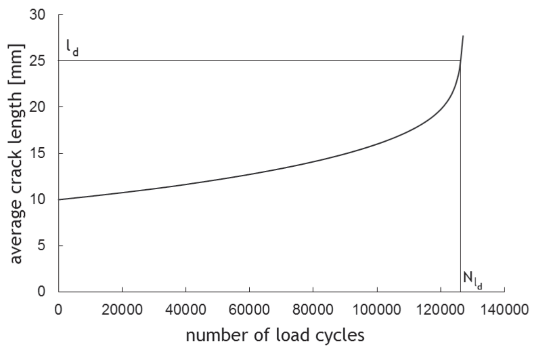

Fig. 1. Increment in the average crack length against the number of loading cycles

Table 5. Quantities which describe the rearranged loading spectrum

Load factor i	1	2	3	4	5	6	7
Number of cycles	1	5	4	10	30	50	140
σ_i^{max} [MPa]	186	159	141	129	112	93	72
$\sigma_{i,sr}^{min}$ [MPa]	-28	-13	8	17	23	27	27
Factor $\hat{R}_i$	-0,1505	-0,0818	0,0567	0,1317	0,2053	0,2903	0,375
Range of stress $\Delta\sigma_{i,ef}$[MPa]	214	172	133	112	89	66	45
Empirical function U_i	0,5030	0,5238	0,5691	0,5955	0,6228	0,6559	0,6906
Share of load factor in the spectrum P_i (frequency of occurrence)	0,0042	0,0208	0,0167	0,0417	0,125	0,2083	0,5833

On the basis of computations of growth of an average-length fatigue crack *B*(*N*) one can find that the permissible crack length l_d = 25mm will be reached after $N_{l_d} = 124\ 110$ loading cycles. To find fatigue life of the structural component given consideration, with probabilistic approach adopted, one should also take account of the crack length scatter *A*(*N*) defined with equation (24). Then, for the already found density function of the crack length against the number of loading cycles the structural component's reliability can be determined:

$$R(N)_{l_d} = \int_{-\infty}^{l_d} U(l, N_c)dl \tag{32}$$

A great advantage of the presented method is that it takes account of physical phenomena accompanying the variable loading spectrum. Values of material constants used in this method, and of other types of factors as well, all of them indispensable for the computations, are to be found experimentally, whereas some of them (e.g. *C,m* in Paris equation) can be estimated using service data on the crack growth. The method of moments or the function of likelihood prove applicable.

6. References

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