Zhong HAN
Jianmin GAO
Liudong XING
Fumin CHEN

OPTYMALNA ALOKACJA ZASOBÓW ZAPEWNIAJĄCA BEZPIECZEŃSTWO W ZŁOŻONYCH ROZPROSZONYCH SYSTEMACH ELEKTROMECHANICZNYCH

OPTIMAL RESOURCE ALLOCATION FOR SAFETY IN DISTRIBUTED COMPLEX ELECTROMECHANICAL SYSTEMS

Istniejące strategie optymalnej alokacji zasobów służące zapewnieniu bezpieczeństwa systemów skupiają się głównie na systemach szeregowo-równoległych lub na systemach, które można przekształcić w modele szeregowo-równoległe. Jednakże, w przypadku niektórych złożonych rozproszonych systemów elektromechanicznych, przetworzenie na model szeregowo-równoległy może być bardzo trudne lub wręcz niemożliwe. Dodatkowo, z powodu złożoności relacji sprzężeń w fizycznej strukturze tego rodzaju systemów, bezpieczeństwo niektórych jednostek systemowych jest niemierzalne. W niniejszym artykule przedstawiono nową metodę optymalnej alokacji zasobów gwarantującą maksymalne bezpieczeństwo złożonych rozproszonych systemów elektromechanicznych o strukturze innej niż szeregowo-równoległa. Metoda ta oparta jest na sieciach złożonych i wykorzystuje dynamiczne programowanie bazujące na zbiorach ścieżek. Jako miarę bezpieczeństwa systemu zastosowano pojęcie hierarchii bezpieczeństwa, zdefiniowane jako funkcja dwóch parametrów bezpieczeństwa: strat z tytułu awarii oraz prawdopodobieństwa awarii. Dla zilustrowania proponowanej metody i weryfikacji jej przydatności i możliwości zastosowania, przedstawiono przykład rzeczywistego systemu.

Słowa kluczowe: *straty z tytułu awarii, prawdopodobieństwo awarii, sieć złożona, system złożony, programowanie dynamiczne, optymalna alokacja zasobów, hierarchia bezpieczeństwa, bezpieczeństwo systemu.*

Existing optimal resource allocation for system safety mainly concentrates on series/parallel systems or systems that can be converted into series/parallel models. However, for some distributed complex electromechanical systems, it is very difficult or even impossible to refine them into a series/parallel model; in addition, the safety of some system units is immeasurable because of the coupling relationship complexity in the system composition structure. In this paper, a novel method based on complex networks and path set-based dynamic programming is proposed for the optimal resource allocation for maximal safety of distributed complex electromechanical systems with non-series-parallel structures. As a measurement of the system safety, safety importance is defined, which is a function of two safety feature parameters - accident loss and accident probability. A practical system is taken as an example to illustrate and verify the feasibility and applicability of the proposed method.

Keywords: *accident loss, accident probability, complex network, complex system, dynamic programming, optimal resource allocation, safety importance, system safety.*

1. Introduction

Due to the frequent accidents that have happened recently, safety has become a very critical problem for Distributed Complex Electromechanical Systems (DCES). Optimal resource allocation is an effective means to improve the system safety given limited resources. Considerable research efforts have been expended in the optimal resource allocation for system safety. Most of the work concentrates on systems with series structure, parallel structure, or combined series/parallel structure [1-2]. In some work, for example [18-20, 25, 28, 30], the weak points of the system are first identified and the vulnerability is evaluated. The optimal resource allocation is then carried out according to the level of vulnerability [32, 34]. In [8, 15, 17, 21, 26, 29, 33], safety control and resource allocation are performed in light of states or stages of life cycle of the system. In [4-5, 7, 11, 16, 23, 31, 36], the optimal configuration strategies are applied to solve the safety problems for complex systems, mostly discrete manufacturing systems and information systems. In these works, algorithms such as the genetic algorithm, ant colony algorithm, fuzzy random variables, cubic algorithm and particle algorithm were adopted.

However, those methods cannot be (at least directly) applied to solve the optimal resource allocation problem for the safety of the DCES because of the following reasons: 1) the series/parallel models are not sufficient to describe the DCES with complex network structure [24]; 2) most of the existing safety strategies are effective only when being applied to the simple coupling systems, and the evaluation of the system safety is not synthetically and systematically studied [30]. When being applied to the DCES with complex coupling, these strategies may cause erroneous, unbalanced, or inefficient resource allocation; and 3) the existing methods heavily depend on analytical models, which can be efficiently applied to information systems, discrete manufacturing systems and so on. But they typically cannot model a DCES with complex network structure accurately, thus the results from the resource allocation can be invalid [18, 31].

Therefore, in this paper, a novel method of the optimal resource allocation is presented for the safety of the DCES. The method integrates the complex network model, the newly-defined safety importance measure, and the dynamic programming to realize the optimal resource allocation.

The remainder of the paper is organized as follows. Section 2 summarizes the previous results from the literature on the analysis models and then introduces an extended model with the complex network structure. Section 3 defines several parameters for characterizing the safety of distributed complex electromechanical systems (DCES). Section 4 presents our methods for optimal resource allocation in DCES. Section 5 uses an example of a process system to illustrate the whole process of the optimal resource allocation. Finally, conclusions and some directions for future work are discussed.

2. System modeling for distributed complex electromechanical systems

2.1. Related work

The purpose of the optimal resource allocation is to minimize investment and maximize the safety of the system. Strategies of the optimal resource allocation depend on the system structure analysis model. Existing work concentrates on systems with series structure, parallel structure, or combined series/parallel structure. In a series structure, all elements are connected one by one in a sequence as shown in fig. 1. In a parallel system, all elements are connected in juxtaposition, as shown in fig. 2. Some elements of a series structure can be parallel subsystems, forming a series-parallel system; similarly, some elements of a parallel structure can be series subsystems, forming a parallel-series system. For both series-parallel and parallel-series systems, they are also referred to as k-out-of-n systems. For example, the optimal sequential testing procedure was developed for k-out-of-n systems with equal testing costs and general costs for all components in [3, 12]; and it was then generalized for minimizing testing cost in [13]. Later on, heuristic sequential inspection procedures were introduced for minimizing inspection costs of k-out-of-n systems while decreasing the average malfunction probability [9], [10]. In [10], a discrete-valued function and a statistical classification procedure were adopted; in [31], probability distributions, in particular, component failure probabilities were adopted. Recently, in [1] more complicated system structures with both parallel and series subsystems similar to a networked system (fig. 3) were studied and closed-form results were derived.

Tab. 1. Symbols and explanation

Symbol	Explanation
G	graph
W	accident losses
w_i	the i^{th} specific accident loss in W
S	reliability system
s_i	the i^{th} specific element in S
P	accident probability
p_i	accident probability of element i
F	safety importance
F_j	safety importance of element i
C, B	investment cost
x_i	amount of investment in element i
X	specific investment cost
E	edge set
V, T, Γ_i	node set
$D(i, j)$	distance between nodes, i, j
v_i	node i
$F_k(x)$	safety importance increase of k elements after investment amount x
$f_i(j)$	safety importance increase of the k^{th} element after investment amount j

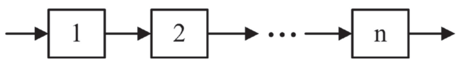

Fig. 1. Series model

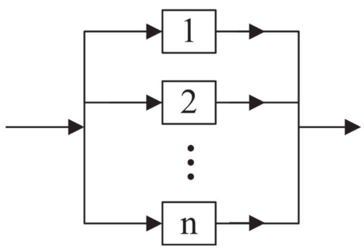

Fig. 2. Parallel model

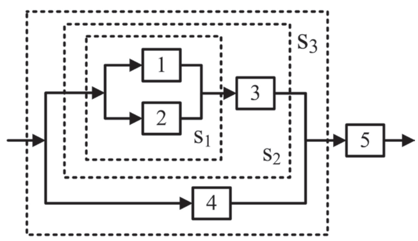

Fig. 3. An example of a series-parallel combined model

2.2. Extension to systems with complex network structures

As mentioned in Section 1, the series/parallel models are not sufficient to describe the DCES with complex coupling. In reality, there exist strong and weak couplings which form complex relationships between the numerous elements in the DCES. Hence, an object-oriented method is used to develop a complex network model for describing the complex relationships existing in the DCES.

Specifically, a complex system is physically divided into various independent objects O_i, which can be a subsystem, a piece of equipment, or a part. By independent we means that there are no common elements shared by any two objects, that is, $O_i \cap O_j = \varphi$. Then the logical multi-medium (such as material flow, energy flow, control flow, information flow) couplings/relationships between these objects are identified. The objects and their relationships constitute an object set and a relationship set, respectively, which are utilized to build a network model of the actual system. Each object is represented by a node, and each relationship is represented by an edge in a network model. Every node has its own inherent attributes (for example, voltage, temperature, pressure) and action modes (for example, transformation), and it can interact with other nodes according to the relationships between them. Every edge in the network model is associated with a parameter called coupling strength between the objects. According to the degree of the coupling strength, we have strong and weak connections. The coupling strength can be quantified as weight, which can be current, probability, capacity. The nodes, edges, and their associated parameters provide a tool for the safety analysis of DCES.

A network model of the DCES can be represented by a graph *G (V, E, R)*, where *V* represents a set of nodes (objects), *E* represents a set of edges (relationships), and *R* represents the relationship strengths. Note that the parameters associated with the nodes are not considered in this paper. fig. 4 illustrates an example of complex network models. Such models cannot be refined into a series structure, a parallel structure, or a combined series/parallel structure.

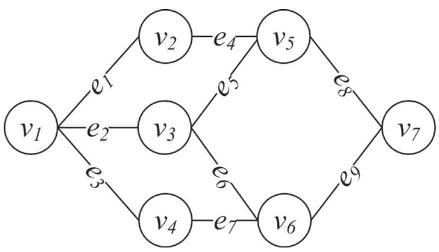

Fig. 4. Complex network model

3. Safety parameters for DCES

3.1. Safety factor analysis

A safe system means that there are no casualties, asset losses, or environment pollution when the system is in operation or fails. To guarantee the safety of a DCES, it is necessary to evaluate malfunctions and existing risks in an objective and comprehensive manner. In addition, based on the safety standard in industry, reasonable controlling methods and preventive measures against the malfunctions or risks must be determined according to outcomes from them.

The system safety is relative and random because the occurrence of accidents is random and the consequence of the accidents can be catastrophic or negligible. Therefore, the occurrence of an accident can be regarded as a random event, which follows a probability distribution. In this section, based on the relative and random features of accidents, two parameters of *accident loss* and *accident probability* are introduced to measure the safety aspect of the DCES. Methods of obtaining those parameters are different for different application environments. Typically they can be obtained based on test data, expert data, literature data, simulation data or statistical data in history [5, 22].

3.2. Accident loss

Accident loss denoted by *W* of a system *S* (or an element) is used to describe the serious degree of consequence from the occurrence of an accident. The method to calculate the accident loss is based on value accounting [6], which converts all losses and effects brought by the occurrence of an accident into monetary loss (W_m) and non-monetary loss (W_n). That is,

$$W = W_m + W_n \tag{1}$$

The monetary loss means the loss that can be calculated in theory, and it can be further divided into personnel casualty (W_1), financial loss (W_2), and environmental pollution (W_3). The non-monetary losses can be further divided into loss of the stop production (W_4), yield reduction (W_5), work loss (W_6), and resource loss (W_7). Thus, Equation (1) can be elaborated as:

$$W = (W_1+W_2+W_3) + (W_4+W_5+W_6+W_7) = \sum_{k=1}^{7} W_k \tag{2}$$

In general, the accident loss *W* can involve *m* different kinds of losses, and it can represent as:

$$W = \sum_{k=1}^{m} W_k \tag{3}$$

Typically, *n* samples of accident loss for the same element will be collected and an average of those losses will be calculated and used in the optimal resource allocation of DCES. Let W_{ij} represent the value of W_i in the j^{th} sample/accident of the element. All the W_{ij} form an $m \times n$ matrix as shown in Equation (4):

$$M_W = (W_{ij})_{m \times n} \tag{4}$$

Then the expectation of the accident loss can be evaluated as:

$$\overline{W} = (n)^{-1} \sum_{i=1}^{n} \sum_{j=1}^{m} W_{ij} \quad (i = 1, 2 \ldots m;\ j = 1, 2 \ldots n) \tag{5}$$

3.3. Accident probability

Accident probability denoted by *P* of a system *S* (or an element) is used to describe the occurrence frequency or possibility of accident events. For DCES, the grey prediction theory is a suitable method for estimating the accident probability [27, 35]. Assuming that sequence statistics of the accident probability is the timing statistics with equal interval. The row vector in (6) represents the original sequence which is composed of probabilities of accidents occurring in *n* consecutive and equal intervals:

$$p^{(0)} = [p^{(0)}(1), p^{(0)}(2), \cdots, p^{(0)}(n)] \tag{6}$$

This initial grey vector is then treated with a generation operation given in Equation (7):

$$P^{(1)}(k) = \sum_{i=1}^{k} P^{(0)}(i) \qquad k = 1, 2 \cdots n \tag{7}$$

Equation (8) shows the resultant one-accumulative sequence:

$$p^{(1)} = [p^{(1)}(1), p^{(1)}(2), \cdots, p^{(1)}(n)] \tag{8}$$

The data generated by accumulation are fitted and approximated to obtain a continuous function $p^{(1)}(t)$ in the form of Equation (9). Equation (10) shows the discrete solution to Equation (9), for estimating the probability of accidents occurring in future times (n+1, n+2, ...). The parameters a and u can be obtained using the least square method.

$$\frac{dp^{(1)}(t)}{dt} + ap^{(1)}(t) = u \tag{9}$$

$$p^{(1)}(k+1) = (p^{(0)}(1) - \frac{u}{a})e^{-ak} + \frac{u}{a} \quad k = 1, 2 \ldots n \tag{10}$$

To verify the accuracy of Equation (10), a reverse subtractive process will be used to generate a subtractive sequence, which will be compared with the original sequence. If the difference between the two sequences exceeds a threshold value, the initial value of $p^{(0)}(1)$ will be adjusted and the whole process will be repeated.

3.4. Safety importance

Traditionally, considerable efforts were expended on the core elements of the system to improve their safety because the failure of those core elements causes larger loss than the failure of other minor elements does. As a result, the accident probability for the core elements is very low. On the other hand, little attention was paid to the minor elements; the accident probability for those minor elements can be so high that the total system loss resulting from the frequent accidents is high. In other words, minor accidents occurring with high-frequency but small-scale/loss can have similar impact on the system safety as the major accidents occurring on the core elements with low-frequency but large-scale/loss. Therefore, the traditional system safety parameter that focuses mainly on the accident loss $W(S)$ is not sufficient for evaluating the overall system safety.

In Equation (11) we define safety importance (denoted by F) for a system S as a function of accident loss (W) and accident probability (P) described in the last two subsections.

$$F(S) = W(S) \cdot P(S) \tag{11}$$

Such safety importance parameter allows us to objectively and reasonably evaluate the safety situation of real systems. And it is used as the evaluation metric in our resource allocation solution.

4. Optimal allocation strategies

4.1. Optimization methods

According to the network characteristic of DCES, the optimization methods adopted should be feasible and dynamic, *i.e.*, can adapt to various disturbances and state changes from the system. Two common classes of methods that can meet the above requirement, and thus can be used for solving the optimal resource allocation problem for DCES safety: dynamic programming and genetic algorithm. Below is a brief comparison between those two methods:

- From *structure* point of view, dynamic programming is suitable for cases when the network model is known; the genetic algorithm is suitable for cases when the network model is unknown.
- From *accuracy* point of view, dynamic programming can obtain the exact solution; but the genetic algorithm can only obtain the approximate solution.
- From *resource (CPU time and memory) consumption* point of view, because the discrete points of the state variable have to be saved into the computer memory in the deducing process, dynamic programming has a great advantage when the quantity of data is not very large; the genetic algorithm has a great advantage when the quantity of data is very large.
- From *computational efficiency* point of view, dynamic programming is faster than the genetic algorithm.

Considering *pros* and *cons* of the two methods and the features of DCES, the dynamic programming method is selected as the optimization method used in this work. The principle of dynamic programming is briefed via an example of network model in fig. 5.

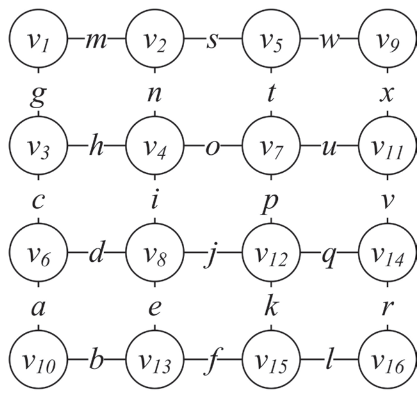

Fig. 5. An illustrated model for dynamic programming

Fig. 5 illustrates a network system with 16 nodes and 24 edges. The letter associated with each edge represents the distance between two adjacent nodes. There are totally 20 different paths from node v_1 to node v_{16} in the network. An optimization problem is to find the longest distance path from node v_1 to node v_{16}.

According to the principle of dynamic programming, the distance from node i to target node T on the longest path can be determined using the following formula:

$$w(i,T) = \max_{j \in \Gamma_i}\{w(i,j) + w(j,T)\} \tag{12}$$

Where, $\Gamma i = \{j |\ (i, j) \in E, j \in V\}$, and E and V represent the edge set and the node set respectively in fig. 5. Hence according to the iterative calculation in Equation (12), the optimal solution can be obtained step by step.

4.2. Optimal allocation modeling

In this subsection, we formulate the optimal resource allocation problem for the maximal safety of DCES. Assume a DCES system S consists of n elements: S_1, S_2... S_n. These elements may be equipment, software, or a work model. All allocation elements are viewed as the system's assets, and then the allocation processes can be regarded as reorganization of assets. Different types of assets have different attributes, and thus the same amount of investments for different assets/elements may lead to different gains in the overall system safety. Let x_i represent the allocated amount of investment for the i^{th} element S_i in terms of RMB. $f_i(x_i)$ is a function representing the increase in the safety importance when the amount of x_i is invested in element S_i, which is obtained based on Equation (11). Let B represent the total budget of the actual investment. The problem is how to allocate the system resources to increase the whole system safety the most while the total investment cost is within the budget limit. Such optimization problem can be formulated using the following equation:

$$\begin{cases} \max y = \sum_{i=1}^{n} f_i(x_i) \\ s.t. \quad \sum_{i=1}^{n} x_i \le B \\ x_i \ge 0, i = 1, 2, \cdots, n \end{cases} \tag{13}$$

This problem can be regarded as a multi-stage decision making problem, and thus can be solved using the dynamic programming technique to find the optimal solution. It is assumed that $F_k(x)$ is the maximal safety importance that is obtained by the total investment x on the first k elements in the allocation. Therefore, according to the optimal principle, the following recursive equation is obtained:

$$\begin{cases} F_1(x) = f_1(x) \\ F_k(x) = \max_{0 \le x_k \le x} \{ f_k(x_k) + F_{k-1}(x - x_k) \} \quad k = 2, 3, \cdots, n \\ 0 \le x \le B \end{cases} \tag{14}$$

In Equation (14), $F_n(B)$ is the optimal solution in question, i.e., the maximal increase in the safety importance of the system.

4.3. Optimal allocation process

Traditional approaches to optimal allocation typically adopt a test strategy which searches the element that is easy to malfunction and to be diagnosed and involves the least cost [1], [27]. More specifically, a test sequence is first determined according to the position of an element in the system structure. System elements are then tested according to the determined order for identifying the system state (operation or failures). Relevant cost and failure probability are produced for testing each element of the system. The problem is to determine the optimal inspection procedure for identifying the system state at minimal expected inspection cost. The results from the optimal inspection procedure are then used for the optimal allocation process. Those approaches are often applied to the series/parallel or combined series/parallel systems; they cannot be applied to the complex systems such as DCES with non-series-parallel structure. In this section, we introduce a novel optimization process for systems with complex network structures.

Briefly speaking, the proposed optimization process involves identifying the path sets between the source node and the destination node in the network model, evaluating the safety importance of each path sets, and allocating resources to each path using dynamic programming while considering the balance of safety among different paths. Next we elaborate the optimization process in a seven-step process.

1) In a complex system network, there can be multiple distinct paths between the source node and the destination node, and different subsets of the system elements are involved on different paths. All the elements on the same path form a path set, denoted by V_i, i = 1, 2, ..., m where m represents the total number of path sets. And define $V = \bigcup_{i=1}^{m} V_i$ (i = 1, 2, ..., m). Also define SN to be the sequence number with the initial value of 1.
2) The safety importance for each path set is then computed as the sum of the safety importance of elements constituting the path set. Assuming that a path set V_i contains n_i system elements. Let F_i represent the safety importance of V_i, and f_{ij} represent the safety importance of a component in V_i. Then we have:

$$F_i = \sum_{j=1}^{n_i} f_{ij} \qquad i = 1, 2 \cdots m \tag{15}$$

3) The path set with the maximal safety importance is identified among all path sets in $\boldsymbol{V}$ and is assigned the sequence number SN.
4) The intersection operation is carried out between the present maximal path set V_{max} and all other path sets V_j identified in Step 1. The result is saved in a temporary set variable T:

$$T = V_{max} \cap V_j \tag{16}$$

5) Each non-maximal path set V_j is updated by subtracting the temporary set T, and its safety importance is updated correspondingly:

$$V_j = V_j - T \tag{17}$$

$$F_j = F_j - F_T \tag{18}$$

6) Let $V = V - \{V_{max}\}$ and SN=SN+1. If V is not NULL, then go to Step 3.
7) According to the balance rule of safety importance, the investment cost C_i in each path set V_i is computed as the follow:

$$C_i = B * F_i / F \tag{19}$$

Where, F is the total safety importance of the system, F_i is the updated safety importance of each path set after Step 6.

After the total investment cost for each path set C_i is decided, the dynamic programming approach explained in Section 4.2 is used to allocate the resource to elements within each set.

5. An example

5.1. Example descriptions

A fuel process system is used as an example to illustrate the method of the optimal resource allocation for system safety presented in previous sections. This system uses pipes to transport oil and vapor to the reactor and a burning reaction happens to provide energy. The schematic diagram of the system structure is shown in fig. 6. Fig. 7 illustrates the corresponding network model, which is composed of 9 nodes and 10 couplings relationships. Each node represents a different facility that can perform functions such as oil supply, vapor supply, heat exchange, and waste oil disposal.

The safety problems of the process systems are mainly caused by the aging, corrosion, abrasion, and fatigue of the equipment. In this example, accident losses are mainly determined by the cost of the equipment. The computation of the accident loss is focused on the equipment replacement, maintenance, and repair. Accident probabilities are mostly obtained based on documentations and/or experts' experiences. Table 2 shows the accident loss and accident probability for each element of the example process system, which are obtained through the statistic method and data analysis on a large amount of history data.

According to equation (11) and the data in table 2, the safety importance of the nine elements in the process system can be obtained, and their values are given in table 3.

According to the seven-step procedure for optimal resource allocation in Section 4.3, there are four path sets for the example system and at the end of Step 6, those four path sets are updated as:

$$V_1 = \{v_2, v_5, v_7, v_9\};\ V_2 = \{v_1, v_4, v_8\};\ V_3 = \{v_3\};\ V_4 = \{v_6\}.$$

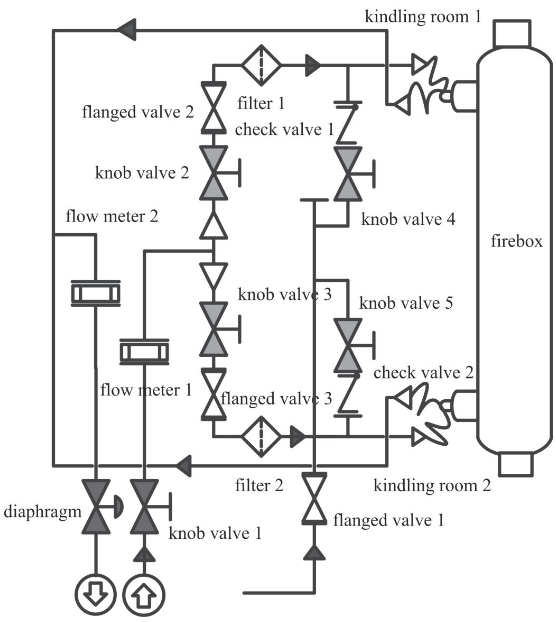

Fig. 6. Schematic diagram of a process system

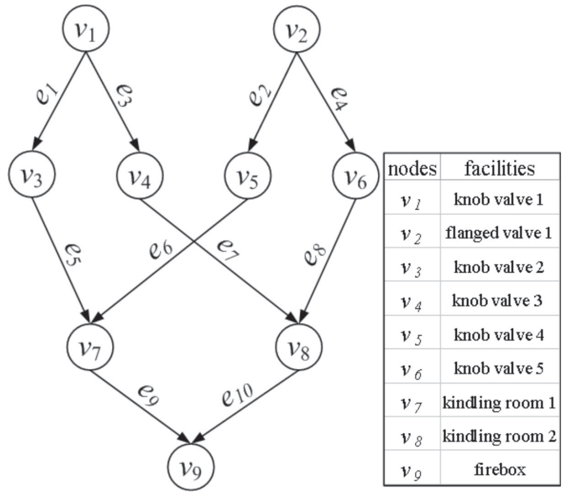

Fig. 7. Network model of the process system

Tab. 2. Elements, accident losses (*10000 RMB), and accident probabilities (‰)

Elements	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9
Accident losses	2.8	3.2	2.8	2.8	2.8	2.8	2.05	2.0	6.0
Accident probabilities	0.03	0.03	0.026	0.021	0.031	0.024	0.04	0.035	0.015

Tab. 3. Element safety importance

Elements	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9
Safety importance	0.84	0.96	0.72	0.60	0.86	0.68	0.82	0.70	0.90

Tab. 4. Relationship among elements, amount of investment and safety importance

$f_i(j)$ \ j / i	1	2	3	4
1	4	5	6	7
2	0	2	4	7
3	1	2	4	9
4	5	6	6	8

The safety importance of each set is obtained as follows:

F_1=3.54; F_2=2.14; F_3=0.72; F_4=0.68.

Thus, the total safety importance of the entire system is *F=7.08*. Suppose that the total investment for the system safety is 80,000 RMB, the resource allocation of each set can be computed in terms of formula (19), and the results are given as follows:

C_1=40,000; C_2=24,000; C_3=8,000; C_4=8,000.

Now, take set V_1 as an example to illustrate the optimal allocation process for the elements within the set. Set V_1 includes 4 elements v_2, v_5, v_7, v_9, which are renumbered as v_1, v_2, v_3, v_4, respectively. The total investment cost for these elements of set V_1 for safety is C_1, which equals 40,000 RMB as computed previously. The safety increase that results from the investment obeys certain rules, and the value can typically be found using statistic analysis and grey estimation. Assuming that the investment cost is *j* units (10,000 RMB/unit), then the increase of the safety importance in element *i* is denoted by $f_i(j)$ $(i, j = 1, 2, 3, 4)$. The values of $f_i(j)$ for set V_1 is given in table 4; and $f_i(0) = 0$. At this stage, the task is to rationally allocate the investment budget C_1 = 40,000 RMB to the four elements of set V_1 for maximizing the safety importance of the entire set.

Applying Equation (13) to set V_1 with *B*=4, *n*=4, we obtain:

$$\begin{cases} \max y = \sum_{i=1}^{4} f_i(x_i) \\ s.t. \ \sum_{i=1}^{4} x_i = 4 \\ x_i \geq 0, i = 1,2,3,4 \end{cases}$$

$F_k(x)$ indicates the maximum increase of the safety importance caused by investing *x* on the first *k* elements in set V_i. $x_k(n)$ shows the amount of investment required by the k^{th} element in the *n*th step when the overall effect of the investments is optimal. Applying the recursive Equation (14) to set V_1, we obtain the following equations:

$$\begin{aligned} F_1(x) &= f_1(x) \\ F_2(x) &= \max_{0 \le x_2 \le x} \{f_2(x_2) + F_1(x - x_2)\} \\ F_3(x) &= \max_{0 \le x_3 \le x} \{f_3(x_3) + F_2(x - x_3)\} \\ F_4(x) &= \max_{0 \le x_4 \le x} \{f_4(x_4) + F_3(x - x_4)\} \end{aligned}$$

Consider a specific example, when *x=1*, then $F_2(1)$ can be calculated as:

$$\begin{aligned} F_2(1) &= \max_{0 \le x_2 \le x} \{f_2(x_2) + F_1(x - x_2)\} \\ &= \max\{f_2(0) + F_1(1), f_2(1) + F_1(0)\} \\ &= \max\{f_2(0) + f_1(1), f_2(1) + f_1(0)\} \\ &= \max\{0 + 4, 0 + 0\} = 4 \end{aligned}$$

The above calculation shows that when $x_2=0$, $F_2(1)=4$. x_2 is the investment on the second element, $x_1 = x - x_2$ is the investment on the first element for increasing the system safety. $F_2(1)=4$ means that the increase in safety importance is 4 when 10,000 RMB is allocated to the first element and 0 RMB is allocated to the second element. Similarly, values of other $F_i(j)$ and their corresponding parameters $x_k(n)$ can be derived and they are shown in table 5.

Tab. 5. Investment and safety importance

x	1	2	3	4
$F_1(x)$	4	5	6	7
$x_k(1)$	1	2	3	4
$F_2(x)$	4	5	6	8
$x_k(2)$	0	0	0,2	3
$F_3(x)$	4	5	6	9
$x_k(3)$	0	0,1	0,1,2	4
$F_4(x)$	5	9	10	11
$x_k(4)$	1	1	1,2	1

5.2. Results discussion

In table 5, $F_4(4)=11$ means that the maximal increase in the safety importance 11 can be obtained by investigating 40,000 RMB on the four elements in set V_1. To trace back the path that leads to the maximal safety increase, we check $x_4(4)$ in table 5. $x_4(4)=1$ means that the fourth element requires 10,000 RMB investment so that the maximal safety importance increase can be obtained. Then 30,000 RMB out of 40,000 RMB remains for the first three elements. Then we go back to check $F_3(3)$, which is 6 meaning that the maximal increase in the safety importance is 6 when 30,000 RMB is invested in the first three elements. Similarly we check $x_3(3) = 0, 1, 2$ meaning that the maximal value of 6 is obtained through three ways: no investment in the third element; or 10,000 RMB investments in the third element, or 20,000 RMB investment in this element. For illustration purpose, we discuss further about the latter two ways as follows.

In the second way, 10,000 RMB are invested in the third element and thus (30,000 - 10,000) = 20,000 RMB will be invested in the first and second elements. From table 4, we have $F_2(2)=5$. Correspondingly, $x_2(2)=0$ meaning that the investment on the second element is zero for maximal increase in safety importance. Thus, the investment on the first element is 20,000 RMB. Therefore,

$$x_1 = 2, x_2 = 0, x_3 = 1, x_4 = 1$$

In the third way, 20,000 RMB is invested in the third element, and thus 10,000 RMB remains to be invested in the first two elements. In table 4, $F_2(1) = 4$. Correspondingly, $x_1(2)=0$ meaning that the investment on the first element is zero in the second step. Then, we check $F_1(1)=4$. Correspondingly, $x_1(1)=1$ meaning that the investment on the first element is 10,000 RMB. Therefore,

$$x_1 = 1, x_2 = 0, x_3 = 2, x_4 = 1$$

Similarly, the optimal resource allocation for other sets V_2, V_3, V_4 can be obtained by applying the dynamic programming procedure. Note that as shown though the example above, there can be multiple optimal solutions for each set.

6. Conclusions and future works

In this paper, we proposed a novel method based on complex networks and path set-based dynamic programming to solve the optimal resource allocation problem for distributed complex electromechanical systems with non-series-parallel structures. Simple series, parallel, or combined series/parallel systems are special cases of the proposed method. A new concept of safety measure called safety importance was proposed and maximized during the optimal resource allocation process. Any safety is relative; absolute safety does not exist. However, the system risk could be continuously identified and the resources could be properly and efficiently allocated so that the accident probability and accident loss could be reduced. This method can effectively and rationally allocate the resources to the key points of the complex network at any stage in the system safety engineering. In the design stage, application of the proposed optimal resource allocation can save money; in the operation stage, the method can eliminate/reduce the potential risks in the system; in the maintenance stage, the method can offer optimal maintenance strategies and facilitate quick repair. In the future work, we will investigate other optimization approaches such as the genetic algorithm [7] for solving the optimal resource allocation problem for complex systems.

****

The work is supported by the National High Technology Research and Development Program of China (863 program) (Grant No.: 2006AA04Z441 and 2007AA04Z432), by the State Key Laboratory for Manufacturing Systems at Xi'an Jiaotong University. We need to express our thanks to Wei He Chemical LTD of Shaanxi of China for providing a practical environment and analytical data for us.

****

7. References

1. Azaiez M N, Bier V M. Optimal resource allocation for security in reliability systems. Eur J Oper Res 2007; 181(2): 773-786.
2. Bier V M, Nagaraj A, Abhichandani V. Protection of simple series and parallel systems with components of different values. Reliab Eng Syst Saf 2005; 87(3): 315-323.
3. Ben-Dov Y. Optimal testing procedures for special structures of coherent systems. Management Science 1981; 27(12): 1410–1420.
4. Chebouba A, Yalaoui F, Smati A, Amodeo L, Younsi K, Tairi A. Optimization of natural gas pipeline transportation using ant colony optimization. Comput Oper Res 2009; 36(6): 1916-1923.
5. Chen X, Zhou K, Aravena J. Probabilistic robustness analysis-risks, complexity, and algorithms. SIAM J Contr Optim 2008; 47(5): 2693-2723.
6. Chowdhury A A, Mielnik T C, Lawton L E, Sullivan M J, Katz A, Koval D O. System Reliability Worth Assessment Using the Customer Survey Approach. IEEE Tran Indus App 2009; 45(1): 317-322.
7. Coit D W, Smith A E. Reliability Optimization of Series-Parallel Systems Using a Genetic Algorithm. IEEE Trans Reliab 1996; 45(2): 254-266.
8. Colombo S, Demichela M. The systematic integration of human factors into safety analyses: An integrated engineering approach. Reliab Eng Syst Saf 2008; 93(12): 1911-1921.
9. Cox L, Chiu S, Sun X. Least-cost failure diagnosis in uncertain reliability systems. Reliability Engineering and System Safety 1996; 54(2-3): 203–216.
10. Cox L, Qiu Y, Kuehner W. Heuristic least-cost computation of discrete classification functions with uncertain argument values. Annals of Operations Research 1989; 2(1): 1-29.
11. HaiyangY, Chengbin C, Ėric C, Farouk Y. Reliability optimization of a redundant system with failure dependencies. Reliab Eng Syst Saf 2007; 92(12): 1627-1634.
12. Halpern J. Fault-testing of a k-out-of-n system. Operations Research 1974; 22(6): 1267-1271.
13. Halpern J. The sequential covering problem under uncertainty. INFOR 1977; 15: 76-93.
14. Hsu C I, Wen Y H. Application of Grey theory and multiobjective programming towards airline network design. European Journal of Operational Research 2000; 127(1): 44-68.
15. Kołowrocki K, Kwiatuszewska-Sarnecka B. Reliability and risk analysis of large systems with ageing components. Reliab Eng Syst Saf 2008; 93(12): 1821-1829.
16. Kunin I, Chernykh G, Kunin B. Optimal chaos control and discretization algorithms. Int J Eng Sci 2006; 44(1-2): 59-66.
17. Levitin G, Amari S V. Multi-state systems with multi-fault coverage. Reliab Eng Syst Saf 2008; 93(11): 1730-1739.
18. Levitin G, Lisnianski A. Optimizing survivability of vulnerable series–parallel multi-state systems. Reliab Eng Syst Saf 2003; 7(9): 319–331.
19. Levitin G. Optimal multilevel protection in series–parallel systems. Reliab Eng Syst Saf 2003; 81(1): 93-102.
20. Lisnianski A, Levitin G, Ben-Haim H. Structure optimization of multi-state system with time redundancy. Reliab Eng Syst Saf 2000; 67(2): 103-112.
21. Li W, Zou M J. Optimal design of multi-state weighted k-out-of-n systems based on component design. Reliab Eng Syst Saf 2008; 93(11): 1673-1681.
22. Ljiljana B, Peter H, Mirka M. An Optimization Problem in Statistical Databases. SIAM J Discrete Math 2000; 13(3): 346-353.
23. Nagai H. Optimal strategies for risk-sensitive portfolio optimization problems for general factor models. SIAM J Contr Optim 2003; 41(6): 1779-1800.
24. Overkamp A, Van Schuppen JH. Maximal solutions in decentralized supervisory control. SIAM J Contr Optim 2000; 39(2): 492-511.
25. Petersson J. On continuity of the design-to-state mappings for trusses with variable topology. Int J Eng Sci 2001; 39(10): 1119-1141.

26. Ramirez-Marquez J E, Coit D W. Optimization of system reliability in the presence of common cause failures. Reliab Eng Syst Saf 2007; 92(10): 1421-1434.
27. Rus G, Palma R, Pérez-Aparicio J L. Optimal measurement setup for damage detection in piezoelectric plates. Int J Eng Sci; 2009; 47(4): 554-572.
28. Sarhan, Ammar M. Reliability equivalence factors of a general series–parallel system. Reliab Eng Syst Saf 2009; 94(2): 229-236.
29. Shevchuk P, Galapats B, Shevchuk V. Mathematical modelling of ageing and lifetime prediction of lacquer-paint coatings in sea water. Int J Eng Sci 2000; 38(17): 1869-1894.
30. Tang J. Mechanical system reliability analysis using a combination of graph theory and Boolean function. Reliab Eng Syst Saf 2001; 72(1): 21-30.
31. Tavakkoli-Moghaddam R, Safari J, Sassani F. Reliability optimization of series-parallel systems with a choice of redundancy strategies using a genetic algorithm. Reliab Eng Syst Saf 2008; 93(4): 550-556.
32. Xing L, Amari S V. Effective Component Importance Analysis for the Maintenance of Systems with Common-Cause Failures. Int J Reliab Qual Saf Eng 2007; 14(5): 459-478.
33. Xing L, Dai Y. A New Decision Diagram Based Method for Efficient Analysis on Multi-State Systems. IEEE Trans. Dependable and Secure Computing 2009; 6(3): 161-174.
34. Xing L. Reliability Evaluation of Phased-Mission Systems with Imperfect Fault Coverage and Common-Cause Failures. IEEE Trans Reliab 2007; 56(1): 58-68.
35. Wang YH, Dang YG, Li YQ, et al. An approach to increase prediction precision of GM(1,1) model based on optimization of the initial condition. Expert Systems with Application 2010; 37(8): 5640-5644.
36. Zeng Z, Veeravalli B. On the Design of Distributed Object Placement and Load Balancing Strategies in Large-Scale Networked Multimedia Storage Systems. IEEE Tran Knowl Da Eng 2008; 20(3): 369-382.

Dr. Zhong HAN
State Key Laboratory for Manufacturing Systems Engineering
Xi'an Jiaotong University
Xi'an, 710049, P.R.China
Department of Electrical and Computer Engineering
University of Massachusetts Dartmouth
e-mail: hanyaozhong@sina.com
MA 02747, USA

Prof. Jianmin GAO
Asso. Prof. Fumin CHEN
State Key Laboratory for Manufacturing Systems Engineering
Xi'an Jiaotong University
Xi'an, 710049, P.R.China
e-mail; gjm@mail.xjtu.edu.cn;.

Dr. Liudong XING
Department of Electrical and Computer Engineering
University of Massachusetts Dartmouth
MA 02747, USA
e-mail: lxing@umassd.edu