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## MODELLING PREVENTIVE MAINTENANCE BASED ON THE DELAY TIME CONCEPT IN THE CONTEXT OF A CASE STUDY

### MODELOWANIE KONSERWACJI ZAPOBIEGAWCZEJ W OPARCIU O POJĘCIE CZASU ZWŁOKI W KONTEKŚCIE STUDIUM PRZYPADKU

*Using the delay time concept and associated models, this paper presents a modelling study of optimising the preventive maintenance (PM) interval of a production plant within the context of a case study. To establish the relationship between the PM interval and expected downtime per unit time, we need the data of both failure times and the number of defects identified and removed at PM epochs. However, the available data to us was only the recorded times of failures. To overcome this problem, we obtained an estimated mean number of the defects identified at the PM epoch by the plant maintenance technicians. Based on these two types of data, we first establish a likelihood function of the observed times to failure and then a squared function of the difference between the number of defect identification estimated by the technician and the corresponding expected value from the model is mixed with the likelihood function to estimate the unknown model parameters. We test by simulation to show the validity of the above parameter estimation method. Once the parameters of the model are known, a PM model is proposed to optimize the expected downtime per unit time with respect to the PM interval. The modeling process is demonstrated by the case study presented.*

**Keywords:** delay time, Preventive Maintenance (PM), parameter estimation, modelling.

*Wykorzystując pojęcie czasu zwłoki oraz modele stowarzyszone, w artykule przedstawiono badania modelowe optymalizacji przerwy konserwacyjnej w zakładzie produkcyjnym w oparciu o studium przypadku. Aby ustalić związek pomiędzy przerwą konserwacyjną a oczekiwanym czasem przestoju na jednostkę czasu, potrzebne są dane dotyczące zarówno czasów uszkodzeń jak i liczby usterek wykrytych i usuniętych w okresach konserwacji zapobiegawczej. Niestety, w badanym przez nas przypadku jedynymi dostępnymi danymi były czasy uszkodzeń. Aby obejść ten problem, wykorzystaliśmy szacunkową średnią liczbę usterek wykrytych w okresie konserwacji zapobiegawczej przez obsługę techniczną zakładu. W oparciu o wspomniane dwa typy danych, ustaliliśmy, w pierwszej kolejności, funkcję wiarygodności dla obserwowanych czasów do uszkodzenia. Następnie, w celu określenia niewiadomych parametrów modelu, funkcję tę połączyliśmy z funkcją najmniejszych kwadratów dla różnicy pomiędzy liczbą wykrytych usterek oszacowaną przez pracownika obsługi technicznej a odpowiadającą jej oczekiwaną wartością wyprowadzoną z modelu. Wiarygodność powyższej metody oceny parametrów sprawdzono za pomocą symulacji. Znając wartości parametrów modelu, zaproponowano model konserwacji zapobiegawczej pozwalający na optymalizację oczekiwanego czasu przestoju na jednostkę czasu w odniesieniu do przerwy konserwacyjnej. Proces modelowania przedstawiono za pomocą studium przypadku.*

**Słowa kluczowe:** czas zwłoki, konserwacja zapobiegawcza, ocena parametrów, modelowanie.

#### 1. Introduction

The delay time concept proposed by Christer has been extensively applied to maintenance problems of plant inspection practice [9, 13, 16]. The period from the first point at which a defect can be identified at a PM inspection to the time when a repair is essential is called the delay time, denoted by  $h$ . The objective of most delay time based studies is to either minimize a cost function or a down time function subject to a preventive inspection interval [16].

A major task in modelling the above inspection practice based upon the delay time concept is the estimation of parameters

which describe (1)  $\lambda(u)$ , the rate of occurrence of defects at time  $u$ , (2)  $F(h)$ , the cumulative probability function of delay time  $h$ . (3) the probability of perfect defects identification at PM. In general, there are two established methods to estimate model parameters, namely the subjective method [2, 11, 12, 17] and the objective method, see Akbarov [11], Aven [3], Christer and Wang [6, 8], Jones *et al* [10], Wang [15]. The former is based on the subjective data obtained from maintenance engineers' experience. The latter is based on the observed data of recorded failure times and the number of defects identified at each PM epoch.

If the maintenance records of failures and the number of defects identified at PM are available and sufficient in quantity and quality, the delay time model parameters can be estimated by the objective method, generally the classical statistical method of maximum likelihood. If however, such a data set does not exist, or is insufficient in quantity and quality for the purpose of estimation, the alternative is to use expert judgment for obtaining those parameters [6].

In many cases, there are some objective data available, but those data are insufficient to estimate by merely the objective data, so more recent development in delay time modelling has established that these parameters can also be estimated using limited PM data and selective repair at PM [5, 7]. Wang and Jia [14] presented an empirical Bayesian based approach to estimate the delay time model parameters using both subjective and objective data. This approach starts with subjective data first, and then updates the estimates when objective data become available.

In this paper, because of the operating practice of PM and data constraints of the case we studied, we present an estimation procedure which is different from previous delay time models of complex plant. Here historic data exist for failure time points and PM times, but the interval of PMs is not equal, and no records exist for the number of the defects identified and removed at PM. However, we obtained latter a subjective estimate of the mean number of the defects identified and removed at PM from the factory technicians who maintained the plant. In this case a mixture of both objective data of failures and subjective PM data will be utilized in order to estimate model parameters. A mixed likelihood function with a least squared function (take the negative) is proposed and maximized to obtain the estimated values of the model parameters. Simulated data based upon imperfect inspections are generated to test whether the above mixed likelihood method can recover the underlying model parameters within a required accuracy. Finally an inspection model as a function of the PM interval is proposed and an optimal PM interval is obtained for the plant concerned. The modelling objective is to minimize the total downtime per unit time in terms of the PM interval.

The paper is organized as follows. Section 1 presents a basic introduction to the problem and a brief literature review. Section 2 outlines the modeling assumptions and notation, the modeling developments, and the test of this developed model using simulation. Section 3 proposes a downtime model. Section 4 presents a numerical example and section 5 concludes the paper.

## 2. The statistical model for model parameter estimation

### 2.1. Assumptions

Based upon the observation of the plant maintenance practice and referring to the published delay time papers [9, 13, 16],

the following modeling assumptions are proposed to characterize the operation of the plant over the period of data collection.

- 1) Defects arise according to a Homogeneous Poisson Process (HPP).
- 2) Defects are assumed to arise independently of each other.
- 3) The delay time  $h$  of a random defect is independent of its time origin and has a pdf,  $f(\bullet)$ , and a cdf,  $F(\bullet)$ , common to all defects.
- 4) Inspections carried out at a PM are assumed to be imperfect in the sense that a defect present will be identified with a known probability.
- 5) All identified defects are rectified by repairs or replacements during the PM period.
- 6) Failures are identified immediately, and repairs or replacements are made as soon as possible.

### 2.2. Notation and likelihood formulation

We shall adopt the following notation:

$\lambda$	The rate of occurrence of defects.
$v(t)$	The rate of occurrence of failures at time $t$ .
$r$	The probability of detecting a defect at PM, if it is present.
$h$	delay time of a random defect with pdf $f(\bullet)$ and cdf $F(\bullet)$ .
$T_i$	The time of the $i$ th PM from new.
$t_{(i-1)j}$	The time of the $j$ th failure occurring in $(T_{i-1}, T_i)$ , $j=1, 2, \dots, k_{i-1}$ , and $t_{(i-1)k_{i-1}}$ is the time of the last failure in $(T_{i-1}, T_i)$ .
$\Delta t$	A small time interval sufficiently small that only one failure event at most can arise within it.
$n_i$	The number of the defects identified at the $i$ th PM.
$EN_f(T_{i-1}, T_i)$	The expected number of the failures over the inspection interval $(T_{i-1}, T_i)$ .
$EN_p(T_i)$	The expected number of the defects identified and rectified at $T_i$ .

Consider all observations in  $(T_{i-1}, T_i)$ , namely the number of the defects identified at  $T_i$ , and the failure times in  $(T_{i-1}, T_i)$ ,  $i=1, 2, \dots, n$ , and  $T_0=0$ , see Fig. 1. The likelihood function is the product of the probabilities of these observations arising. At  $T_p$  we need to formulate the probability of the number of the defects identified and rectified. Also, for each failure time in  $(T_{i-1}, T_i)$ , we need to formulate the probability of a failure arising at times  $t_{(i-1)j}$ ,  $j=1, 2, \dots, k_{i-1}$ , and of having no other failures between recorded consecutive failure times. Therefore, the likelihood function  $L$  is given by:

$$L = \prod_{i=1}^n \left\{ p(n_i \text{ defects identified at } T_i) \prod_{j=1}^{k_{i-1}} \left[ p(\text{a failure at time } t_{(i-1)j}) \cdot p(\text{no further failure between } t_{(i-1)(j-1)} \text{ and } t_{(i-1)j}) \right] \right\} \quad (1)$$

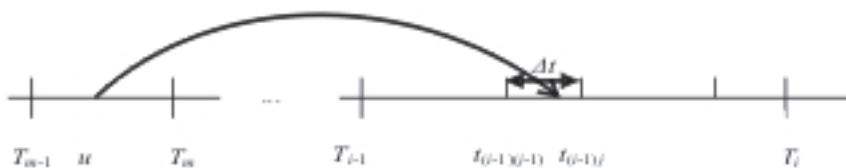


Fig. 1. The failure process in  $(T_{i-1}, T_i)$  due to a defect arising at  $u$  in  $(T_{m-1}, T_m)$

The log likelihood function is given by:

$$\ell = \sum_{i=1}^n \left\{ \log p(n_i, \text{defects identified at } T_i) + \sum_{j=1}^{k_{i-1}} \left[ \log p(\text{a failure at time } t_{(i-1)j}) + \log p(\text{no further failure between } t_{(i-1)(j-1)} \text{ and } t_{(i-1)j}) \right] \right\} \quad (2)$$

In equation (2), the term  $P(\text{no further failure between } t_{(i-1)j} \text{ and } t_{(i-1)(j+1)})$  is necessary because of the use of an HPP for the defect arrival so that the interval between failures has to be modeled. Equation (2) assumes that the necessary objective data are available from both PMs and failures.

To compute the above likelihood function, firstly, we consider the probability of a failure in  $(t, t+\Delta t)$ , namely  $P(t, t+\Delta t|u)$ , see Fig. 2, where,  $T_{n-1} < t \leq T_n$ ,  $T_{i-1} < u \leq T_i$

$$p(t, t+\Delta t|u) = \begin{cases} (1-r)^{n-i} (F(t+\Delta t-u) - F(t-u)) & T_{i-1} < u < T_i, i=1, \dots, n-1 \\ F(t+\Delta t-u) - F(t-u) & T_{n-1} < u < t \\ F(t+\Delta t-u) & t < u < t+\Delta t \\ 0 & u > t+\Delta t \end{cases} \quad (3)$$

So the rate of occurrence of failures,  $v(t)$ , is derived below:

$$\begin{aligned} v(t) &= \int_0^t \lambda \lim_{\Delta t \rightarrow 0} \frac{P(t, t+\Delta t)}{\Delta t} du \\ &= \lambda \left\{ \sum_{i=1}^{n-1} (1-r)^{n-i} \int_{T_{i-1}}^{T_i} f(t-u) du + \int_{T_{n-1}}^t f(t-u) du \right\} \\ &= \lambda \left\{ \sum_{i=1}^{n-1} (1-r)^{n-i} [(F(t-T_{i-1}) - F(t-T_i))] + F(t-T_{n-1}) \right\} \end{aligned} \quad (4)$$

where,  $T_{n-1} < t \leq T_n$ . For The derivation of Equation (4), see Christer and Wang [6]. So the expected number of failures over the inspection interval  $(T_{n-1}, T_n)$  is as follows:

$$\begin{aligned} EN_f(T_{n-1}, T_n) &= \int_{T_{n-1}}^{T_n} v(t) dt = \lambda \int_{T_{n-1}}^{T_n} \sum_{i=1}^{n-1} (1-r)^{n-i} \\ &[(F(t-T_{i-1}) - F(t-T_i))] dt + \lambda \int_{T_{n-1}}^{T_n} F(t-T_{n-1}) dt \end{aligned} \quad (5)$$

Using equation (4), we obtain the probability of a failure arising in time interval  $(t_{(i-1)j}, t_{(i-1)j}+\Delta t)$ , for sufficiently small  $\Delta t$ ,

$$P(\text{a failure in } (t_{(i-1)j}, t_{(i-1)j}+\Delta t)) = v(t_{(i-1)j}) \Delta t \quad (6)$$

Since the failure process is NHPP, it is straightforward that:

$$p(\text{no failure in } (t_{(i-1)(j-1)}, t_{(i-1)j})) = e^{-\int_{t_{(i-1)(j-1)}}^{t_{(i-1)j}} v(t) dt} \quad (7)$$

If failure durations are negligible, the logged probability of no further failure between recorded failures within  $(T_{i-1}, T_i)$  is simply given by

$$\begin{aligned} &\sum \log p(\text{no further failure in } (t_{(i-1)(j-1)}, t_{(i-1)j})) = \\ &= \sum_{j=1}^{k_{i-1}} \left( -\int_{t_{(i-1)(j-1)}}^{t_{(i-1)j}} v(t) dt \right) - \int_{t_{(i-1)k_{i-1}}}^{T_i} v(t) dt = \int_{T_{i-1}}^{T_i} v(t) dt \end{aligned} \quad (8)$$

From equation (3), we obtain the expected number of the defects found at  $T_n$ , namely  $EN_p(T_n)$ , given by Christer *et al.* [6].

$$EN_p(T_n) = \lambda \sum_{i=1}^{n-1} (1-r)^{n-i} r \int_{T_{i-1}}^{T_i} [1 - F(T_n - u)] du + \lambda r \int_{T_{n-1}}^{T_n} [1 - F(T_n - u)] du \quad (9)$$

Because the number of defects identified at PMs follows a Poisson distribution with the mean defined by equation (9), [6], the probability of  $n_n$  defects identified at  $T_n$  is

$$p(n_n \text{ defects identified at } T_n) = \frac{(EN_p(T_n))^{n_n} e^{-EN_p(T_n)}}{n_n!} \quad (10)$$

Dividing equation (6) by  $\Delta t$  and taking the log of equation (10). The log likelihood function for the problem described becomes:

$$\log L = \sum_{i=1}^n \left\{ (n_i \log EN_p(T_i) - EN_p(T_i) - \log n_i!) + \sum_{j=1}^{k_{i-1}} \log v(t_{(i-1)j}) - \int_{T_{i-1}}^{T_i} v(t) dt \right\} \quad (11)$$

In this case, PM inspection data are not available, so the first part of the right hand side of equation (11) cannot be computed, but the estimated mean number of the defects identified and rectified at PMs are provided by the maintenance technicians. So we used the likelihood of the failure events (the second part of the right hand side of equation (11)) and a least square function and the function,  $Z$ , to be maximized is given by:

$$Z = \sum_{i=1}^n \left\{ \left[ \sum_{j=1}^{k_{i-1}} \log v(t_{(i-1)j}) - \int_{T_{i-1}}^{T_i} v(t) dt \right] - [EN_p(T_i) - ES_p(T)]^2 \right\} \quad (12)$$

where  $ES_p(T)$  denotes the subjective estimate of the mean number of the defects identified and rectified given  $T$  where  $T$  is the average PM interval length. This equation has not been used before in delay time based models. Maximizing equation (12), we may obtain the estimated parameters of the model, namely  $\lambda, r$  and those in  $f(h)$  from actual failure records and subjective PM data.

### 2.3. The assessment of the model

#### 2.3.1. The simulation test

We have run a simulation experiment to test the validity and feasibility of equation (12). The failure processes with imperfect inspection of  $r=0.2, 0.5$  and  $0.8$  are simulated respectively.

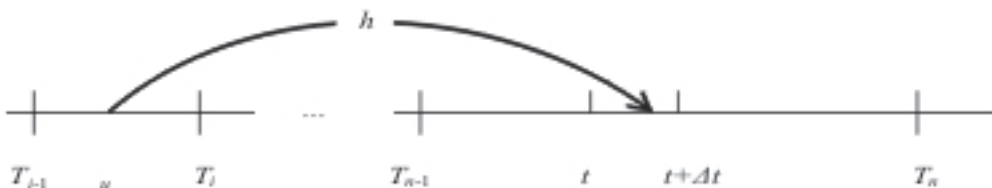


Fig. 2 The failure process of a defect arising in  $(T_{i-1}, T_i)$

We then averaged the observed number of the defects identified at PM to be used as an assumed estimate from the technicians. Table 1 shows that fitted parameter values, in which the rate of occurrence of faults is  $\lambda=1.1528$  and the scale parameter of the exponential delay time  $\alpha=0.0288$ , and the inspection interval is 7 days. The full data likelihood is also run to compare with that from equation (11). From table 1 it can be seen that the estimates from using equation (12) are not far from the true parameter values and validate our approach. Though they are not as good as the estimate using the full data, but the method is a good approximate way for model parameter estimates.

**2.3.2. Choice of possible candidates for model**

Before fitting a model to the data, the functional form of the delay time distribution must be specified. The best choice of the distribution from a family of distributions for  $h$  is chosen, using the criterion of minimum Akaike information criterion (AIC) [4]. Possible candidates for  $F(\cdot)$  are 1) Exponential distribution  $F(x)=1-e^{-\alpha x}$ ; 2) Mix delta-exponential distribution  $F(x)=1-(1-p)e^{-\alpha x}$ ; and 3) Weibull distribution  $F(x)=1-e^{-(\alpha x)^{\beta}}$ . Exponential distribution is usually selected first. If there is any defect with zero delay time, model (2) is preferred, where  $p$  is the proportion of defects with zero delay time and  $\alpha$  is the scale parameter of the exponential distribution. This mixed distribution can be used for Weibull as well.

**3. Downtime model**

The relationship between the PM frequency and the total downtime is established as shown below, [16]:

$$ED(T) = \frac{d_f \cdot EN_f(T) + d_p}{T} \tag{13}$$

where:  $ED(T)$  - the total expected downtime per unit time over an infinite horizon with PM interval  $T$ ,  $d_f$  - the average downtime per failure,  $EN_f(T)$  - the expected number of failures over PM interval  $T$ ,  $d_p$  - the average downtime per PM, where since we assume that the plant is already operated very long to be in a steady state so,  $EN_f(T_{n-1}, T_n) = EN_f(T)$  for sufficient large  $n$ .

Table 1 Estimation result for an exponential delay-time distribution via various  $r$  values

$r$	PM cycle	Sample data	Use failure data and actual PM data			Use failure data and mean PM data		
			$\hat{\lambda}$	$\hat{\alpha}$	$\hat{T}$	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{T}$
$r=0.2$	10	58	1.1530	0.0245	0.1340	0.9780	0.0555	0.3000
	50	372	1.1040	0.0330	0.2040	1.0480	0.0560	0.3000
	100	792	1.1600	0.0270	0.1920	1.2090	0.0410	0.3000
$r=0.5$	10	69	1.1110	0.0235	0.4020	1.0830	0.0415	0.6000
	50	381	1.1040	0.0340	0.4860	1.1040	0.0455	0.6000
	100	798	1.1460	0.0350	0.6000	1.2370	0.0320	0.6000
$r=0.8$	10	74	1.0970	0.0295	0.7019	1.0620	0.0440	0.8999
	50	391	1.1180	0.0425	0.8999	1.0690	0.0450	0.8999
	100	805	1.1530	0.0355	0.8959	1.0480	0.0400	0.8999

True parameter values are  $\lambda=1.1528$ ,  $\alpha=0.0288$ , and  $d$  the PM period is 7 days.

**4. Case study**

This case study involves an important machine in Harbin Turbine Co Ltd. The machine is called the NC Gantry-type Milling Machine which is an advanced numerical controlled machine which is key plant item within the company with over 80% of products being processed on it at some stages of their production. This machine is operated 22 hours a day (three shifts), 7 days a week, excluding public holidays. At the time of the study, in order to reduce the downtime, preventive maintenance was performed 4 times per year, namely on Spring Festival, 1st May, 1st October and New Year. The company's objective is to reduce the downtime caused both by failures and PM activities, and thereby increase the availability of the plant. The key issue of concern is: How long the PM interval should be the best for the machine?

**4.1. Data collection and Analysis of failure data**

Through collecting records for this milling machine over a period of two years, we obtain some valuable information including the time of failures, causes of failures, or the failure mode, the length of the downtime for each failure and repair actions to the failures.

Based on the failure data, the following analysis is carried out, 1) Frequency analysis of failure modes; 2) Analysis of the causes of failures.

The number of failures occurred in different subsystem of NC Gantry-type Milling Machine is shown in table 2. From table 2, the number of failures occurred over past two years total to 77.

The frequency of failure modes for different main components with each subsystem of the machine is shown in Figures 3, 4 and 5 respectively.

The main failure modes are shown in table 3. It can be seen that the downtime due to the main shaft electric motor in the electric system, totaling 817 hours, accounts for 27 percent total downtime. Next is the brake controller of girder, its downtime reaches 611 hours and accounts for 20 percent total downtime. Others failure modes influencing the availability include the cooler system and attachments for the cutting tool.

Table 2. Failures number of different subsystems of NC Gantry-type Milling Machine and its percentage

Subsystems	Failures number	Percentage
Mechanical system	12	15.6
Hydraulic system	23	29.9
Electric system	42	55.5
Total	77	100

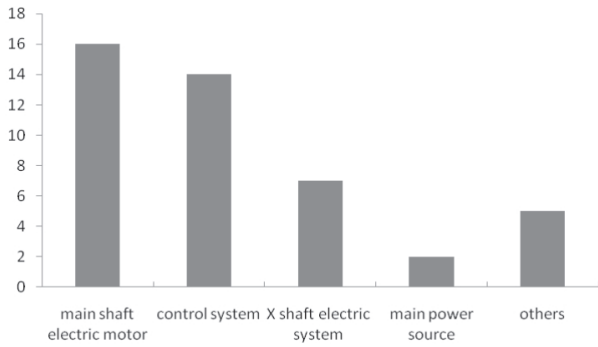


Fig. 3 Frequency analysis of electric system failures over two years

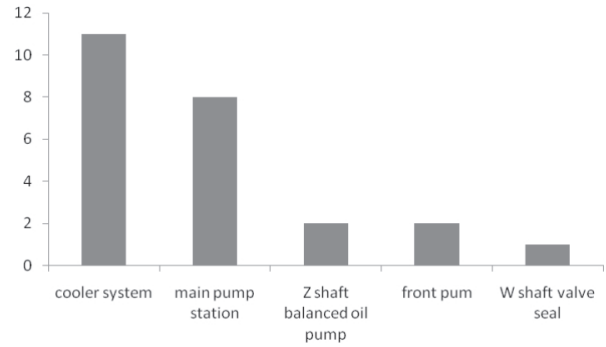


Fig.4 Frequency analysis of hydraulic system failures over two years

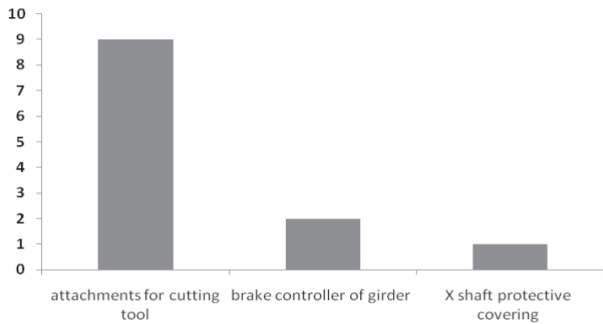


Fig. 5. Frequency analysis of mechanical system failures over two years

Table 3. Main failure modes influencing NC Gantry-type Milling Machine's downtime

Failure mode	Downtime (hr)	Percent %
Main shaft electric motor	817	27
Brake controller of girder	611	20
Cooler system	220	7
Attachments for cutting tool	215	7
Main pump station	150	5
Control system	149	5
Others	881	29
Total downtime	3043	100

Discussion with the company maintenance technicians revealed that the main reasons for the failures are as follows: 1) Inadequate maintenance lead to the frequently failure occurrence of the main shaft electric motor, the cooler system and the main pump station as well. 2) Poor design for the attachments for the cutting tool resulted in its frequent failure occurrence. 3) Too much time waiting for the repair parts lead to the long downtime of the failure of the brake controller of girder. Some advices are proposed as follows: 1) Enhancing the preventive maintenance activity, e.g. determining an optimal PM interval is expected to reduce the failure number occurred for the main shaft electric motor, the cooler system and the main pump station. 2) Redesigning the attachments for the cutting tools, will help to reduce the failure downtime resulting from the bad system design. 3) Enhancing the supply chain management and avoiding the long waiting for repair parts, will increase greatly the availability of the system. In this paper we pay attention to first item.

#### 4.2. The calculation of interval of PM

Now we focus on the determination of the optimal interval of PM for the whole system since a PM is usually scheduled for the whole system. The available data is as follows: the time of each failure happened, the length of downtime per failure. However, we have not had the number of the defects identified and identified at PM. According the experience of the chief technician who has been responsible for the maintenance of this machine for years, the estimate of the number of defects identified at PM is about 3-5, so we take its mean value, namely 4.

Using equation (12), the fitted values of parameters are shown in Table 4. From Table 4, the mixed exponential distribution is selected as having the lowest AIC value.

Using the mixed exponential delay time, from equation (5), we have [5]:

$$EN_f(T) = \lambda T - \frac{\lambda r (e^{\alpha T} - 1)(1 - p)}{\alpha (e^{\alpha T} - 1 + r)} \quad (14)$$

Table 4. Models and fitted values of parameters from the real data

Models	$F(x)=1-e^{-\alpha x}$	$F(x)=1-(1-p)e^{-\alpha x}$	$F(x)=1-e^{-(\alpha x)^p}$
$\hat{\lambda}$	0.1283	0.1233	0.1294
$\hat{\alpha}$ (scale parameter)	0.0321	0.0301	0.0341
$\hat{\beta}$ (shape parameter)	-	-	0.8844
$\hat{p}$	-	0.10	-
$\hat{r}$	0.8521	0.8411	0.8023
Maximum log-likelihood	-73.3779	-72.2862	-73.1773
AIC	152.7558	152.5724	154.3546

$p$  is the proportion of zero delay time.  $\lambda$  is the rate of occurrence of faults.  
 $AIC = -2 * \log \text{maxlikelihood} + 2 * (\text{number of parameters})$



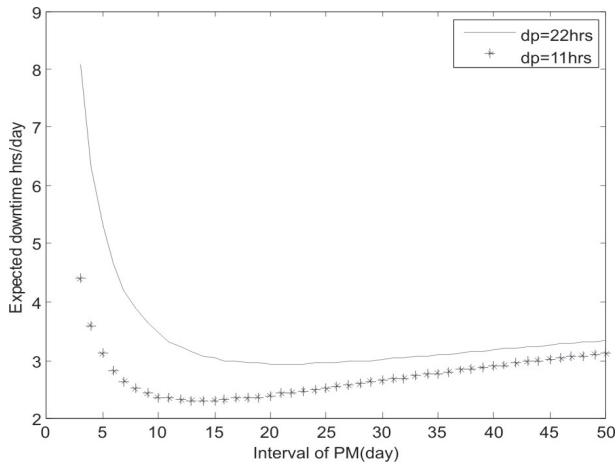


Fig. 6. Expected downtime (hour) per day against PM cycle length (day)

From tables 2 and 3, we obtain  $d_f=3043/77=39.5195$  hours, and  $d_p=22$  hours from the PM schedule. Substituting equation (14) into equation (13), we obtain the model output shown in Figure 3. From Figure 3, it can be seen that the optimal PM interval should be around 19 days. Since the expected downtime per unit time, when  $T=14$  days,  $T=30$  days, is increased less 5% than that when  $T=19$  days, the suitable PM interval range is from 2 weeks to a month.

If the interval of PM is changed to 19 days, the expected downtime per day for this machine is 2.9340 hours, and the observed average downtime when  $T=3$  months is 3043/

$(2*340)=4.4750$  hours per day. So with the optimal PM interval, the expected downtime of this machine will be reduced to  $(4.4750-2.9340)$  hour/day  $\times$  340 days/year = 524 hours/year. Since the average loss for this machine is 500 RMB/hour, so the decision made by the above optimal model will help the company to save at least 262,000 RMB per year. When  $T=90$  days, the output of the model is 3.8907 hours per day, so it is not far from 4.4750 hours per day from the data.

If improving the skills of maintenance technicians and strengthening the management of maintenance activity, the inspection time could be reduced to  $d_p=11$  hours, then the optimal inspection interval is 12 days from equation (13), and the expected downtime per day is 2.2981 hour per day, so the expected gain will be the  $(4.4750-2.2981)$  hour/day  $\times$  340 days/year  $\times$  500yuan/hour = 370,000 RMB.

### 5. Conclusion

In this paper, we propose a model to determine the optimal PM interval. The model is based upon the delay-time concept. A mixed likelihood and least squares method based upon actual failures and the subjectively estimated PM data has been used to obtain the estimated values of the model parameters. A PM inspection model has then been used to find the optimal PM inspection interval which minimizes the total expected downtime per day caused by failures and PMs. The model shows that if the machine can be checked up every 19 days, the expected downtime is minimized. Of course, some important factors such as production schedule, maintenance manpower, and spare parts should also be considered together before making the final decision of the PM inspection interval.

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### 6. References

1. Akbarov A, Wang W, Christer AH. Problem identification in the frame of maintenance modelling: a case study. *Int J Prod Res* 2008; 46(4): 1031-46.
2. Apeland S, Scarf P.A. A fully subjective approach to modeling inspection maintenance. *European Journal of Operational Research* 2003; 148: 410-425.
3. Aven T, Castro IT. A delay time model with safety constraint. *Reliab Eng & Syst Saf* 2009; 94:261-267.
4. Bozdogan H. Model selection and Akaike's Information Criterion (AIC): The general theory and its analytical extensions. *Psychometrika* 1987; 52(3): 345-370.
5. Christer A H, Lee C, Wang W. A data deficiency based parameter estimating problem and case study in delay-time PM modelling. *Int J. Production Economics* 2000; 67: 63-76.
6. Christer A H, Wang W, Baker R.D., Sharp J. Modelling maintenance practice of production plant using the delay time concept. *IMA Journal of Mathematics Applied in Business & Industry* 1995; 6: 67-83.
7. Christer A H, Wang W, Cho I K, Sharp J. The delay-time modelling of preventive of preventive maintenance of plant given limited PM data and selective repair at PM. *IMA Journal of Mathematics Applied in Medicine and Biology* 1998; 15: 355-379.
8. Christer A H, Wang W. A delay-time-based maintenance model of a multi-component system. *IMA Journal of Mathematics Applied in Business & Industry* 1995; 6: 205-222.
9. Christer A H. Developments in delay time analysis for modelling plant maintenance. *Journal of the Operational Research Society* 1999; 50: 1120-1137.
10. Jones B, Jenkinson I, Wang J. Methodology of using delay-time analysis for a manufacturing industry. *Reliability Engineering and System Safety* 2009; 94: 111-124.
11. Jones B, Jenkinson I, Yang Z, Wang J. The use of Bayesian net work modeling for maintenance planning in a manufacturing industry. *Reliability Engineering and System Safety* 2010; 95: 267-277.

12. Pillay A, Wang J, Wall A. Optimal inspection period for fishing vessel equipment: a cost and down time model using delay time analysis. *Mar Technol SNAME N* 2001; 38(2): 122–9.
13. Wang W, Carr M. Scheduling asset maintenance and technology insertions, to appear in complex engineering service systems. In: NgI, Parry G, Wild P, McFarlane D, editors. *Concepts and research*. Amsterdam: Springer; 2010.
14. Wang W, Jia J. A Bayesian approach in delay time maintenance model parameters estimation using both subjective and objective data. *Quality Maintenance and Reliability Int* 2007; 23: 95-105.
15. Wang W. An inspection model for a process with two types of inspections and repairs. *Reliab Eng & Syst Saf* 2009; 94: 526–533.
16. Wang W. Delay time modelling. In: Murthy DNP, Kobbacy AKS, editors. *Complex system maintenance handbook*. Amsterdam: Springer 2008; 345–70.
17. Wang W. Subjective estimation of the delay time distribution in maintenance modeling. *European Journal of Operational Research* 1997; 99: 516-529.

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