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With-Fracture Gurney Model to Estimate both Fragment and Blast Impulses

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Abstract: Work to extend Gurney's model for fragment velocity to predict blast impulse is ongoing by means of analytical calculations, based on gas and casing dynamics, with comparisons to available experimental data. The issue of early case fracture, with release of explosive gases thus retaining a greater degree of momentum, is also addressed. The method is based on that of G.I. Taylor and can include both case material compressive flow stress and explosive properties such as Chapman-Jouguet pressure. Comparisons between this new Taylor-based theory and blast data from studies of cased charges are shown. The potential effects on blast output of casing dynamic material properties appear considerable. Dynamic testing of case metals is needed to confirm the yield stresses implied by the blast data. It is expected that this method will be useful to the researcher in a number of roles.

Keywords: Gurney, Taylor, casing, fracture, explosive

Introduction

The munitions expert, whether asked to design munitions with certain blast and fragment attack capabilities, or asked to advise on the design of munitions storage or munitions-testing chambers and ranges, requires to know how much of the energy liberated by the explosive will be delivered in the form of blast pressure and impulse, and how much in the form of kinetic energy of casing fragments. The author, coming into this field in early 2007, as adviser to a new firing chamber project, has discovered a lack of sound analytical approaches to the determining the balance between casing and blast impulses and offers the following methods.

Calculations

Basic analytical equations

A simple analytical model for exploding cased munitions, derived by R.W. Gurney during WWII [1], is still very useful today and capable of further development. Given the simplifying assumptions of uniform internal gas density and linear variation of gas velocity from the centre of axis of the bomb out to the casing, the terminal casing velocity v_G was approximated by Gurney to be a function both of the explosive properties and of the ratio *M/C*, where *M* is the casing mass and *C* the charge mass:

$$v_G = \sqrt{2E}\left(\frac{M}{C} + \frac{n}{n+2}\right)^{-\frac{1}{2}}. \tag{1}$$

Here *E* is a value for energy per unit mass specific to each explosive and *n* is the dimensionality of the system, i.e. plane sandwich $n = 1$, cylinder $n = 2$ or sphere $n = 3$.

However, Gurney's focus was on the initial velocity of the casing fragments and he did not provide an explicit derivation for the momentum carried by the explosive gases. The author has recently derived and published extensions to Gurney's model [2], which significantly enlarge it to include the momentum of the explosive gases. This allows estimates of blast impulse, relative to those from bare charges. The first step was to calculate the scalar radial momentum, *p*, of the entire gas and casing system, relative to the explosive mass:

$$\frac{p}{C} = \sqrt{2E}\left(\frac{M}{C} + \frac{n}{n+1}\right)\left(\frac{M}{C} + \frac{n}{n+2}\right)^{-\frac{1}{2}}. \tag{2}$$

Within the first set of brackets, the second term, $n/n+1$, represents the momentum of the explosive gases. If this term is removed, equation (2) reduces to that of Gurney (1). However, if the term *M*/*C* for the casing is instead removed, one can obtain two values for the gases momentum, one for a bare charge ($M = 0$) and another for a cased charge. By taking the ratio of these two momenta, one can obtain:

$$\frac{C_{EB}}{C} = \left(\frac{n}{n+2}\right)^{\frac{1}{2}}\left(\frac{M}{C} + \frac{n}{n+2}\right)^{-\frac{1}{2}}. \tag{3}$$

Here C_{EB} represents the effective explosive mass for blast impulse. A complete derivation can be found in the author's recent paper [2]. Equation (3) allows one to set aside an earlier incorrect derivation by Fisher [3], which as can be seen in Figure 1 gave a fortuitously good fit for low M/C values, but does not accord with the fact that a very thick case can completely contain an explosion.

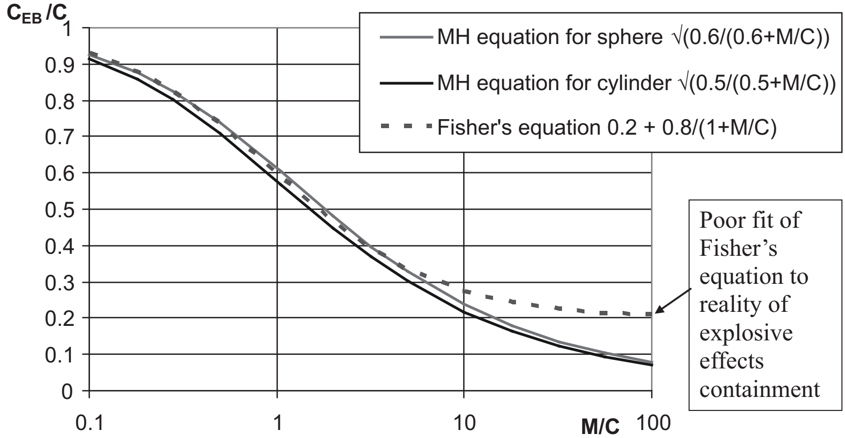

Figure 1. Variation of effective charge mass for blast with casing mass.

Radius dependency of cylinder wall velocity

Gurney's derivation is for terminal wall velocity, i.e. the velocity which the case material would reach if it continued to expand without fracture until the internal gas pressure is too low to impart significant further drive to the casing. However, early case fracture significantly affects the momentum and energy balance between explosive products and case fragments.

One therefore needs to know the state of the expanding case at intermediate expansion radii. Setting M in the first set of brackets in (2) to zero, the scalar momentum of the product gases is:

$$p_c = C\left(\frac{n}{n+1}\right)\sqrt{2E}\Big/\left(\frac{M}{C}+\frac{n}{n+2}\right)^{1/2}. \tag{4}$$

However, it is not possible to derive the kinetic energy per unit mass E_{kc} of the product gases directly from their momentum (or vice versa), since the product gases do not have a single velocity. In the Gurney model, their velocities range from zero on the cylinder axis linearly up to reach case velocity. The product gases kinetic energy per unit mass is given by:

$$E_{kc} = E\frac{n}{(n+2)}\bigg/\left(\frac{M}{C}+\frac{n}{n+2}\right) = E\bigg/\left(\frac{(n+2)M}{nC}+1\right) \tag{5}$$

(so when $M = 0$, $E_{kc} = E$).

Assuming the explosive products behave as an ideal gas of polytropic coefficient γ, the pressure P of gases internal to the case will depend on the expansion radius R as the casing expands from an initial state with radius R_0:

$$P(R) = P_0\left(\frac{R_0}{R}\right)^{2\gamma}. \tag{6}$$

In the above equation, P_0 is the value of the uniform gas pressure assumed by Gurney at the notional point in time where the case is still at static radius R_0 and the explosive is at uniform solid density ρ_0 (but has reacted to form a high pressure gas). P_0 is related to the Chapman-Jouget (C-J) pressure through the equation:

$$P_0 = P_{CJ}\left(\frac{\rho_0}{\rho_{CJ}}\right)^{\gamma}, \tag{7}$$

for the adiabatic expansion of the product gases from the C-J state back to the density they had in their original condensed state.

The acceleration of the case by the gas is proportional at each moment in time to the internal driving pressure, $P(R)$, multiplied by R since the instantaneous case mass per unit area is inversely proportional to R. From (6), and integrating with respect to R an expression for v^2, the radius dependency of case velocity v for explosively-driven expansion of thin-walled cylinders can be calculated to be:

$$\frac{v(R)}{v_G} = \sqrt{1-\left(\frac{R_0}{R}\right)^{2(\gamma-1)}}. \tag{8}$$

This equation can also be used to predict the variation of strain-rate with expansion radius, as shown in Figure 2.

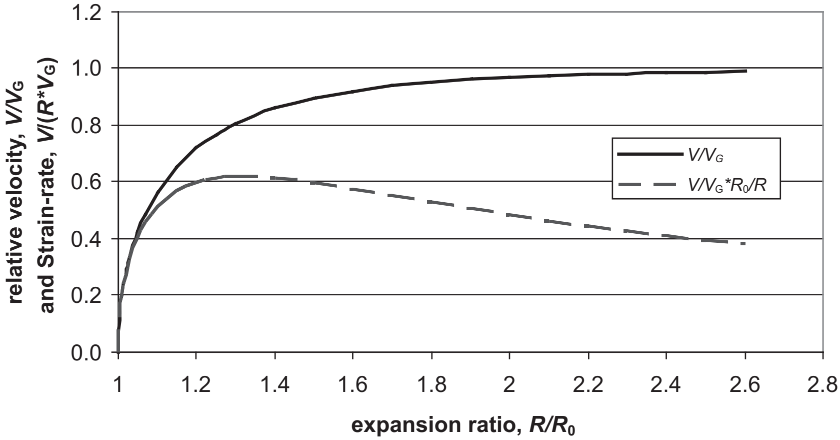

Figure 2. Relative cylindrical case velocity and strain-rate vs. expansion ratio.

An identical dependency on v upon R to (8) has been derived by Koch, Arnold & Estermann [4], based on the conservation equations. Their analysis also derives the same value of v_G as obtained by Gurney himself and places the value of constant A at 1. It should be well noted that any derivation that includes a calculation of the case acceleration *must* include a term for the gases that follow behind the casing, or it will *not* agree either with Gurney's equation, which can be re-arranged as follows:

$$\frac{2EC}{v_G^2} = M + \frac{nC}{n+2} \; . \tag{9}$$

The acceleration equation that *should* be used for a cylindrical system, where $n/(n+2) = ½$, is therefore:

$$\frac{dv}{dt} = \frac{2\pi R}{M + ½C} P(R) \; . \tag{10}$$

Given this basic hydrodynamics, the author's enlarged Gurney model can now be expanded to include case material properties at high rates of strain, either yield stress or fracture strain, and explosive properties such as C-J pressure and Gurney energy.

Taylor yield stress approach

This approach is based on that of G.I. Taylor [5], again derived during WWII. He postulated that the fracture of the casing is suppressed until the internal pressure of the explosive gases drops below the yield stress of the case material. The case material then ceases to flow under pressure and instead fractures.

Based on equations similar to those in the part: Radius Dependency of Cylinder Wall Velocity, A.B. Crowley [6] has recently used Taylor's approach to derive a relationship between casing yield stress and internal gas pressure. She establishes a case kinetic energy reduction factor *f*, where:

$$f = 1 - A\left(\frac{\sigma_y}{P_0}\right)^{2/3}. \tag{11}$$

Crowley used the yield stress σ_y to represent the strength of the case material, but the compressive flow stress (CFS) may be more appropriate. For the purposes of this discussion, it shall be assumed that yield stress and CFS are more or less equal.

However, there are issues with Crowley's analysis. Firstly, she compared her fracture-limited case kinetic energies to heats of detonation, whereas it is more straightforward to compare them with the ideal Gurney kinetic energies, which in this analysis have been based on velocities one-third of the relevant detonation velocities tabulated in [7]. This leads to a slightly different definition of *f* as the ratio of actual to ideal case kinetic energy.

Secondly, Crowley assumes a value of 0.35GPa for steel σ_y, when a dynamic value >1GPa is arguably much more appropriate. Using a value of about 1.5GPa for steel σ_y leads to an experimental value of A~1, as shown in Figure 3. Error bars have been placed on the plotted (1-*f*) values, based on the conservative assumption of a ±2.5% random error both in the cylinder test data and in the detonation velocities tabulated by Dobratz & Crawford [7].

These corrections bring the experimental value for constant *A* into line with the theoretical value of 1. The case kinetic energy reduction factor *f* can now be added to equation (3) to provide a material sensitive expression for cased charge blast effectiveness:

$$\frac{C_{EB}}{C} = 1 - \left[1 - \left(\frac{n}{n+2}\right)^{1/2}\left(\frac{M}{C} + \frac{n}{n+2}\right)^{-1/2}\right]\sqrt{f}. \tag{12}$$

This expression reduces to 1, if $M/C = 0$, and to Equation (3), if $f = 1$. If the case kinetic energy reduction factor f, which from Equation (11) includes both casing and charge material properties, be known, then the blast output from a charge whose casing fractures before reaching its ideal Gurney velocity can be now predicted using Equation (12).

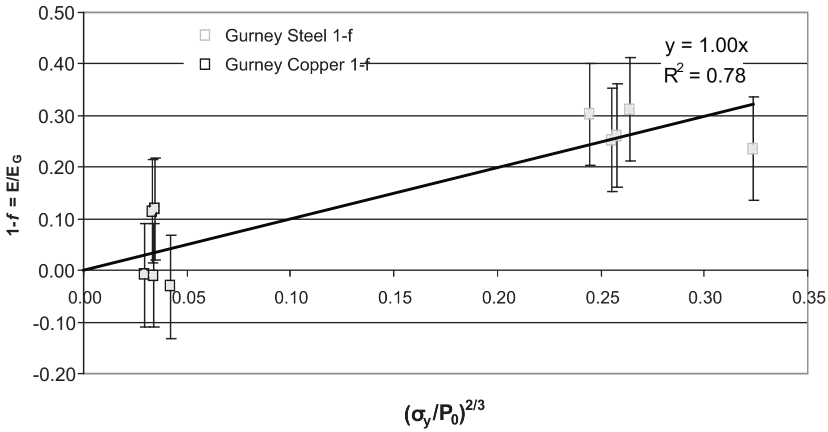

Figure 3. Cylinder test data analysis for constant, A (= y in equation for linear fit). Assumed steel σ_y = 1.47 GPa.

Experimental Comparisons with Taylor-Based Method

Blast data from cased charges

Before introducing comparisons with the blast equivalence equation (12) that includes the case kinetic energy reduction factor f, it should be remembered that this approach is based on Taylor's prediction that fracture strain is independent of case thickness (and hence of strain-rate). This may not always be true, because Taylor's method is one of fracture suppression, following which fracture may not occur immediately.

However, it will be shown that the Taylor-based model is consistent with two sets of experimental data on blast from cylindrical cased charges, both recent data from BAE Systems data [8] and also some 1973 AWE data [9]. In both sets of experiments, blast pressure gauges were ranged at a certain set of distances from both bare and cased charges of the same formulation. Attention was also paid to the consistency of casing material and manufacture.

Measured blast pressures were time integrated to provide data for blast impulses. The blast impulses from bare and cased charges, as fractions of the blast from a bare charge of the same formulation, i.e. C_{EB}/C, can be plotted against the expression $(\frac{1}{2}/(\frac{1}{2} + M/C))^{\frac{1}{2}}$ in equation (12) where $n = 2$ for a cylinder. The plots should show a linear relationship with an intercept on the C_{EB}/C axis of $1-\sqrt{f}$, as confirmed in Figures 4 and 5.

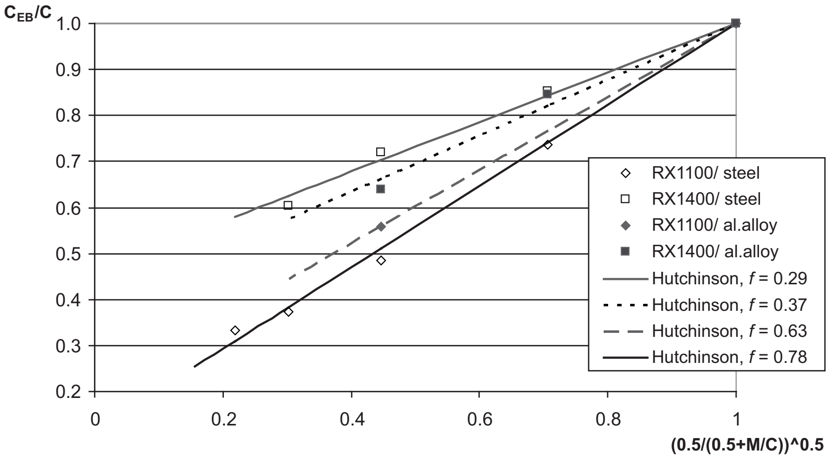

Figure 4. BAE Systems blast data compared with new expression for blast from cased cylindrical charges and linear fits to early case fracture.

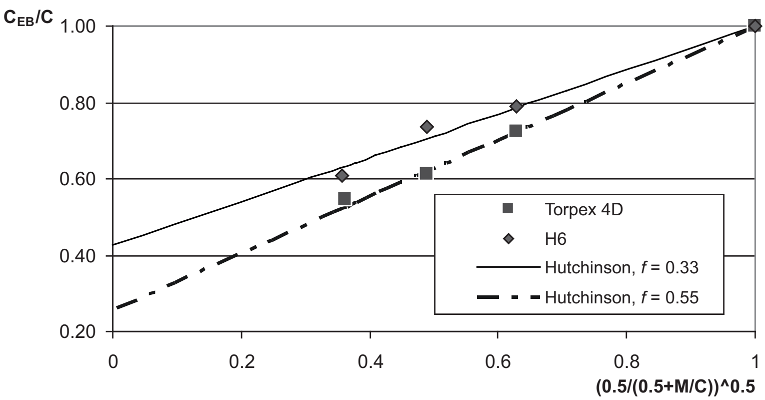

Figure 5. AWRE Foulness Systems blast data compared with new expression for blast from cased cylindrical charges and linear fits to early case fracture.

Because the dynamic yield stress for the casing materials, under the particular conditions to which they are subjected by each explosive formulation, is not known, values of f have been chosen arbitrarily to provide the best linear fits.

However, the analysis does indicate that the Taylor-based approach is sound, at least for the casing metals used under the conditions applied.

Apparently, only one CFS value needs to be determined for each case metal and explosive combination over the range of strain-rates in this study. A small variation in failure strain with strain-rate may still exist but be lost within the limited accuracy of the data used. Also, the failure strains and CFS values needed to fit this data may be very different to those for the same metals which have not been shocked and which are tested under quasi-static conditions.

<u>BAE systems blast data</u>

The BAE Systems experimenters used two casing metals, aluminium alloy 6062 and steel EN24, and two of BAE's own RDX-based explosive formulations, Rowanex 1100 and their aluminium-loaded Rowanex 1400.

In Figure 4, linear regression fits from equation (12) to RX1100 data are shown as solid lines, fits to RX1400 data as dotted lines. Values of f are best fits to the blast data. Looking first at the data for aluminium alloy cases (solid points), there is an increase in *relative* blast impulse for cased charges of the less powerful RX1400 explosive, confirming the Taylor hypothesis that when less powerful explosives are used, the internal gas pressure drops below the casing yield stress at a smaller case expansion radius. This allows more blast impulse, as a fraction of that from a bare charge, to escape, while reducing the drive to the case material.

The data for steel cases is very interesting. When driven by RX1400, it appears to be less ductile than the aluminium. However, when exposed to the more powerful RX1100 (open points), the data is consistent with the steel expanding a lot further before releasing the explosive gases. This observation underlines the importance of knowing case material properties over a range of dynamic stress conditions before blast impulse can be predicted.

<u>AWRE Foulness blast data</u>

A 1973 AWE internal report by Bishop and James [9] describes experiments to compare blast outputs from cased charges including the well-known formulations Torpex 4D and H6. The case metal was steel BS 970 EN2E (SAE 1010 equivalent).

Again, the values of f used in Figure 5 have been chosen arbitrarily to provide the best linear fits, but as with the BAE Systems data, when the less powerful H6 explosive is used, the internal gas pressure drops below the casing yield stress at a smaller case expansion and allows a larger fraction of the bare charge blast impulse to escape. The Taylor-based method again provides a good approximation to reality.

Discussion and Conclusions

Gurney's model for dynamic bomb case expansion has been expanded in two ways. Firstly, the momentum of the explosive gases has been included and the Fisher equation has been replaced. Secondly, a method in which the casing is allowed to fracture, before it is fully expanded, at a yield stress equal to the internal pressure of explosive products (Taylor's hypothesis), has been shown. Fits of equation (12) to experimental data are promising.

Calculations based on Gurney theory and energy conservation confirm that the value $A = 1$ used to determine factor f is correct. While the exact P_0 values for the Gurney model require confirmation, the major requirement is for further data on case metal yield stresses and failure strains, from experiments conducted under the appropriate conditions of initial shock and subsequent high strain-rate extension, are needed to complete this potentially quantitative and predictive model.

The effects of case and explosive material properties regarding both performance and safety of munitions can be considerable. Blast impulse depends not only on charge mass and composition, not only on relative casing mass, but also on case material response to the explosive drive in terms of either early or late fracture. Therefore, blast output can depend very strongly on the case material response to the specific explosive drive, and can be difficult to predict with any accuracy. The methods reported here can be used to minimise the need for dedicated blast trials, particularly if the relevant properties of the explosive and casing metal are known, and to give greater confidence in predictions of the effects of exploding munitions.

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