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THE RECOVERY METHOD FOR PROPER ENVELOPE COMPLEX DISTORTIONAL AMPLITUDE MODULATED SIGNAL BASED ON HILBERT TRANSFORM

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Summary: In this paper the definition of Hilbert transform and analytical representation of the signal are presented. The method of determining the envelope of amplitude modulated signals using Hilbert transform is defined. The article discusses the benefits of this method with the use of radio transmission, which are affected by the interference from low frequency signals the source of which may be a network of electricity. This subject is supported by specific examples.

Keywords: Hilbert transform, analytic signal, amplitude modulated signal, the signal envelope

1. INTRODUCTION

In the transmission radio systems analogue and digital information have to be modulated. Demodulated signal is sent in the form of high frequency electromagnetic radio wave from a transmitter to a receiver where signal is demodulated and information is recovered. Assuming that the carrier signal is purely cosine after modulation it can be described as follows [1]:

$$s(t) = a(t) \cdot \cos\left(2 \cdot \pi \cdot f_c(t) \cdot t + \varphi(t)\right) \tag{1}$$

where:

$a(t)$ – amplitude of signal,
$f_c(t)$ – frequency of signal,
$\varphi(t)$ – phase of signal.

Depending on the identified parameters of the signal (1) changes over time, we can distinguish the following types of modulation: the **amplitude modulation** where the amplitude is modulated, the **frequency modulation**, if the frequency is modulated, and the **phase modulation** if the phase is modulated.

One of the basic and frequently used types of modulation is amplitude modulation, determined at a glance AM (Amplitude Modulation), which relies the changes of amplitude of the signal during the prescribed pattern (1) with predetermined values of frequency $f(t)$ and phases $\varphi(t)$ this signal. However, it must be added that the simplicity of the implementation of the amplitude modulation entails some disadvantages, such as the lack of resistance to interference of various kinds, among which the impact of low frequency signals stemming from the electricity network can be distinguished. Therefore, the discussed disturbances – distortions have an adverse effect on the modulated signal, causing the modulating signal of low frequency around 50 Hz – the frequency of electricity network in Poland. As the consequence of these distortions the faulty reception on the receiving side appears, which results in errors in transmission.

According to the definition and properties of the amplitude modulation, the modulated signal (1) is a narrowband signal, because it satisfies the assumption that the baseband of this signal is a small relative to the carrier frequency signal. Such a complex analysis can be used for the representation of narrowband signals. With respect to the modulated signal (1), after applying the formulas of trigonometric functions of angles and determining the amount of unchanged in time frequency $f_c(t) = f_c = const$ we get:

$$\begin{aligned} s(t) &= a(t)\cdot\cos\left(2\cdot\pi\cdot f_c\cdot t+\varphi(t)\right) = \\ &= a(t)\cdot\cos\left(\varphi(t)\right)\cdot\cos\left(2\cdot\pi\cdot f_c\cdot t\right) + \\ &- a(t)\cdot\sin\left(\varphi(t)\right)\cdot\sin\left(2\cdot\pi\cdot f_c\cdot t\right) = \\ &= x(t)\cdot\cos\left(2\cdot\pi\cdot f_c\cdot t\right) - y(t)\cdot\sin\left(2\cdot\pi\cdot f_c\cdot t\right) \end{aligned} \tag{2}$$

where:

$x(t)$ and $y(t)$ – the quadrature components of signal $s(t)$, which using the pattern (2) can be defined as $x(t)=a(t)\cdot\cos(\varphi(t))$ – the synphase component of signal $s(t)$, $y(t)=a(t)\cdot\sin(\varphi(t))$ – the quadrature component of signal $s(t)$. Using these components we can find the signal envelope:

$$z(t)=x(t)+j\cdot y(t)=a(t)\cdot\cos(\varphi(t))+j\cdot a(t)\cdot\sin(\varphi(t))=a(t)\cdot e^{j\cdot\varphi(t)} \tag{3}$$

While analyzing the modulated signal (1) it can be deduced that the complex carrier signal has the following form:

$$\underline{c}(t)=\cos\left(2\cdot\pi\cdot f_c\cdot t\right)+j\cdot\sin\left(2\cdot\pi\cdot f_c\cdot t\right)=e^{j\cdot 2\cdot\pi\cdot f_c\cdot t} \tag{4}$$

Making a composite complex carrier multiplication (4) and the envelope complex (3) we obtain:

$$\begin{aligned} \underline{c}(t)\cdot z(t) &= e^{j\cdot 2\cdot\pi\cdot f_c\cdot t}\cdot\left[a(t)\cdot\cos\left(\varphi(t)\right)+j\cdot a(t)\cdot\sin\left(\varphi(t)\right)\right] = \\ &= e^{j\cdot 2\cdot\pi\cdot f_c\cdot t}\cdot a(t)\cdot e^{j\cdot\varphi(t)} = a(t)\cdot e^{j\cdot\left(2\cdot\pi\cdot f_c\cdot t+\varphi(t)\right)} = \\ &= a(t)\cdot\left[\cos\left(2\cdot\pi\cdot f_c\cdot t+\varphi(t)\right)+j\cdot\sin\left(2\cdot\pi\cdot f_c\cdot t+\varphi(t)\right)\right] = \\ &= a(t)\cdot\cos\left(2\cdot\pi\cdot f_c\cdot t+\varphi(t)\right)+j\cdot a(t)\cdot\sin\left(2\cdot\pi\cdot f_c\cdot t+\varphi(t)\right) \end{aligned} \tag{5}$$

When comparing the formula (1) with the formula (5) a very important relationship can be noticed, namely, that the modulated signal $s(t)$ is the real part of the composite multiplying complex carrier signal $\underline{c}(t)$ by the complex envelope $z(t)$:

$$s(t) = \mathrm{Re}\left[\underline{c}(t) \cdot z(t)\right] \tag{6}$$

This analysis allows us to conclude that on the receiving side the received data can be decoded successfully without errors, when we have to get complex envelope of modulated signal. For this purpose, we use the Hilbert transform, which gives a quadrature component, which is the imaginary part of the complex envelope of the analyzed signal.

I would add that the application of Hilbert transform in signal processing can be found in the literature following sources: Zieliński P. T.: *Cyfrowe przetwarzanie sygnałów. Od teorii do praktyki*. WKŁ, Warszawa 2007, Wetula A.: *Zastosowanie transformaty Hilberta do wyznaczania obwiedni zespolonej napięć i prądów sieci elektroenergetycznej*. AGH Kraków, Oczeretko E., Borowska M., Laudański M.: *Przekształcenie Hilberta w przetwarzaniu sygnałów i obrazów biomedycznych*. Uniwersytet w Białymstoku. Analyzing sources of literature I can say that I have examined the problem is new, so devoted to him this publication.

2. DEFINITION AND PROPERTIES OF AMPLITUDE MODULATION

Assume that the carrier wave has the cosine character and is defined as follows:

$$c(t) = U_{c_m} \cdot \cos\left(2 \cdot \pi \cdot f_c \cdot t + \varphi_c\right) \tag{7}$$

where:

U_{c_m} – the amplitude of carrier wave;

f_c – the frequency of carrier wave;

φ_c – the phase of carrier wave.

The changes of carrier wave are made according to parameters of modulating wave, represented by the pattern:

$$m(t) = U_{m_m} \cdot \cos\left(2 \cdot \pi \cdot f_m \cdot t + \varphi_m\right) \tag{8}$$

where:

U_{m_m} – the amplitude of modulating wave;

f_m – the frequency of modulating wave;

φ_m – the phase of modulating wave.

Consequently, the instantaneous value of the amplitude modulated signal can be described as follows:

$$\begin{aligned} a(t) &= U_{c_m} + m(t) = U_{c_m} + U_{m_m} \cdot \cos\left(2 \cdot \pi \cdot f_m \cdot t + \varphi_m\right) = \\ &= U_{c_m} \cdot \left(1 + \frac{U_{m_m}}{U_{c_m}} \cdot \cos\left(2 \cdot \pi \cdot f_m \cdot t + \varphi_m\right)\right) \end{aligned} \tag{9}$$

Using amplitude modulation parameters, such as the sensitivity of the amplitude modulator and the ratio of modulation depth, the formula (9) can be expressed as follows:

$$\begin{aligned} a(t) &= U_{c_m} \cdot \left(1 + k_a \cdot U_{m_m} \cdot \cos\left(2 \cdot \pi \cdot f_m \cdot t + \varphi_m\right)\right) = \\ &= U_{c_m} \cdot \left(1 + m \cdot \cos\left(2 \cdot \pi \cdot f_m \cdot t + \varphi_m\right)\right) \end{aligned} \tag{10}$$

where:

$k_a = \dfrac{1}{U_{c_m}}$ – the sensitivity of the amplitude modulator;

$m = \dfrac{U_{m_m}}{U_{c_m}}$ – the ratio of modulation depth.

Having the formula (1) the value of the instantaneous amplitude modulated signal, described by formulas (9) and (10), we can describe the amplitude modulated carrier signal:

$$s(t) = U_{c_m} \cdot \left(1 + m \cdot \cos\left(2 \cdot \pi \cdot f_m \cdot t + \varphi_m\right)\right) \cdot \cos\left(2 \cdot \pi \cdot f_c \cdot t + \varphi_c\right) \tag{11}$$

At this point it is worth mentioning that the envelope of amplitude modulated signal $s(t)$ will have the same shape as the modulating signal $m(t)$, if the following conditions will be fulfilled [2]:

- the amplitude of signal $k_a \cdot m(t) = k_a \cdot U_{m_m} \cdot \cos(2 \cdot \pi \cdot f_m \cdot t + \varphi_m)$ always has to be less then one, so $\forall t \in R : |k_a \cdot m(t)| < 1$. This condition guarantees, that the function $1 + k_a \cdot m(t) = 1 + k_a \cdot U_{m_m} \cdot \cos(2 \cdot \pi \cdot f_m \cdot t + \varphi_m)$ is always positive, therefore, the envelope of modulated signal is described by pattern (9). It could be considered a variant, when $\exists t \in R : |k_a \cdot m(t)| > 1$, the carrier wave is overmodulated, and consequently the reverse phase of the carrier wave at the points, where the function $1 + k_a \cdot m(t) = 1 + k_a \cdot U_{m_m} \cdot \cos(2 \cdot \pi \cdot f_m \cdot t + \varphi_m)$ is zero. However, when $\exists t \in R : |k_a \cdot m(t)| = 1$ then we have full modulation. An important suggestion is to protect the relation between the envelope of the AM wave, and the modulation wave all the time. That is why we have to select the appropriate value of the sensitivity of the amplitude modulator, or an appropriate ratio of modulation depth – typical of its value should be the range $m \in (0{,}2;0{,}8)$, in the audio systems this value should be in the range $m \in (0{,}3;0{,}5)$;
- the carrier frequency must be chosen in compliance with the transmission medium. When considering radio transmission, we can conclude that by reducing the frequency of the carrier signal frequency spectrum we can avoid the risk of overlapping

between the transmitted signals. The frequency range of radio wave is from 3 Hz to 3 THz, so the proper choice of carrier frequency signal is a signal of high frequency. The transmission of high frequency electromagnetic waves is simpler than low frequency electromagnetic wave. Thus, a carrier wave frequency f_c is much greater than that, which modulates the frequency f_m $(f_c >> f_m)$. The AM modulation bandwidth is defined as $W = 2 \cdot f_m$. This means that the amplitude modulated signal is a narrowband signal.

3. MATHEMATICAL MODEL AND SIMULATION OF THE INFLUENCE LOW FREQUENCY DISTORTION OF AMPLITUDE MODULATED SIGNAL

In this paper the influence of existing electromagnetic field induced by high voltage power lines is presented. For this purpose it is necessary to introduce the concept of distortion low frequency signal, which must be understood as a purely cosine signal, the frequency of which is approximately 50 Hz. The amplitude of this signal is low in relation to the amplitude modulated transmitted signal. In addition, it is worth mentioning that, as in the case of the signal, which modulates the carrier phase, it may be zero, that in case of distortion has not always phase have be zero, so we must take into consideration its change over time. This distortion will result not only in overmodulation amplitude of signal but also in the frequency of signal, still the frequency modulation is a very slow process.

The analysis will be incorporated into a clean signal modulation cosine, but more complex signals can be used too. An example of a complex signal may be an acoustic signal, with a very complicated process of defining the shape of the color and intensity of sound. This signal is the sum of multiple sinusoidal waveforms at fixed intervals, the frequency and amplitude of which change over time.

Considering the assumptions above, for the purposes of simulation we can take the following mathematical descriptions and determine the parameters of individual signals:

- carrier signal:

$$c(t) = U_{c_m} \cdot \cos(2 \cdot \pi \cdot f_c \cdot t) \tag{12}$$

where:

$U_{c_m} = 200$ mV,

$f_c = 10$ kHz;

- modulating signal:

$$m(t) = U_{m_m} \cdot \cos(2 \cdot \pi \cdot f_m \cdot t) \tag{13}$$

where:

$U_{m_m} = 160$ mV,

$f_m = 1$ kHz;

- amplitude modulated signal:

$$\begin{aligned} s(t) &= \left(1 + k_a \cdot m(t)\right) \cdot c(t) = \\ &= U_{c_m} \cdot \left(1 + k_a \cdot U_{m_m} \cdot \cos\left(2 \cdot \pi \cdot f_m \cdot t\right)\right) \cdot \cos\left(2 \cdot \pi \cdot f_c \cdot t\right) \end{aligned} \tag{14}$$

where:

$$k_a = \frac{1}{U_{c_m}} = \frac{1}{200 \cdot 10^{-3}} = 5\ \mathrm{V}^{-1};$$

- low frequency distortion:

$$n(t) = U_{n_m} \cdot \cos\left(2 \cdot \pi \cdot f_n \cdot t + \varphi_n(t)\right) \tag{15}$$

where:

$U_{n_m} = 80\ \mathrm{mV}$,

$f_n = 50\ \mathrm{Hz}$,

$\varphi_n(t) = \frac{\pi}{4} \cdot \cos\left(2 \cdot \pi \cdot (1\ \mathrm{Hz}) \cdot t\right)$.

In accordance with the agreed objectives for the signals (12) and (13) can be obtained from a graph of function (14) describing the signal amplitude modulated with the envelope. It is a signal without low frequency distortion signal.

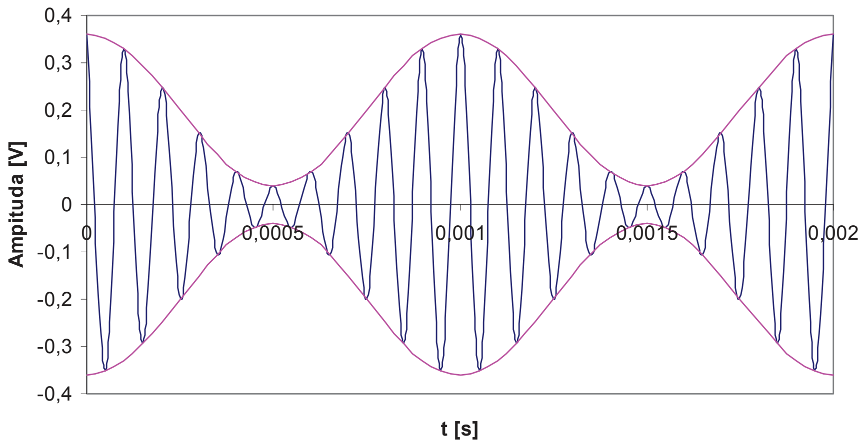

Fig. 1. Amplitude modulated signal $s(t)$ with envelope

A form of modulated carrier signal shown in Fig. 1, is without distortion and fully consistent with its mathematical description. In fact, transmitted signals are some obstacles that may make overmodulation signal $s(t)$. According to this assumption it can be stated that the modulated signal $s(t)$ will be the carrier signal, and the signal $n(t)$ representing the distortion will be the modulating one. Therefore, the mathematical representation of this situation will be as follows:

$$s_n(t) = \left(1 + k_{a_n}(t) \cdot n(t)\right) \cdot s(t) \tag{16}$$

where:

$k_{a_n}(t) = \dfrac{1}{U_{c_m} \cdot \left(1 + k_a \cdot U_{m_m} \cdot \cos(2 \cdot \pi \cdot f_m \cdot t)\right)}$ – instantaneous amplitude sensitivity, this parameter varies in time, because the amplitude of signal $s(t)$ varies in time.

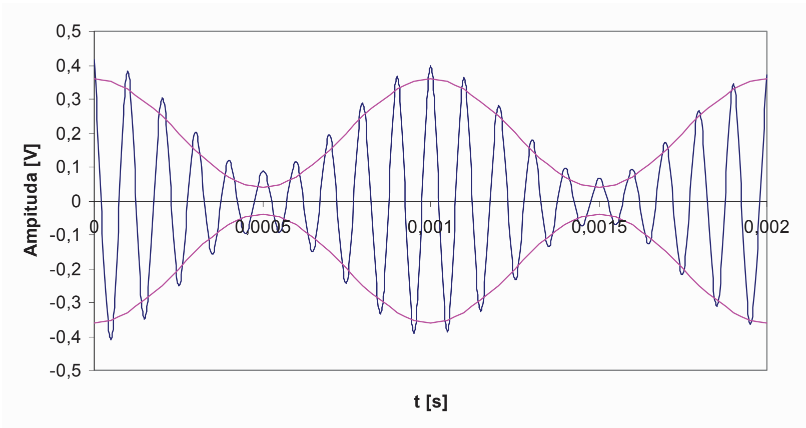

Fig. 2. Amplitude modulation signal with distortion $n(t)$ and with evolve of amplitude modulated signal $s(t)$

In Figure 2, we can see that the modulated signal does not contain the envelope. This confirms the correctness of the assumption that low frequency distortion has a significant impact on the transmission of amplitude modulated signals. It is necessary to develop appropriate methods for filtration in order to determine the correct carrier signal without distortions, which can be easily decoded.

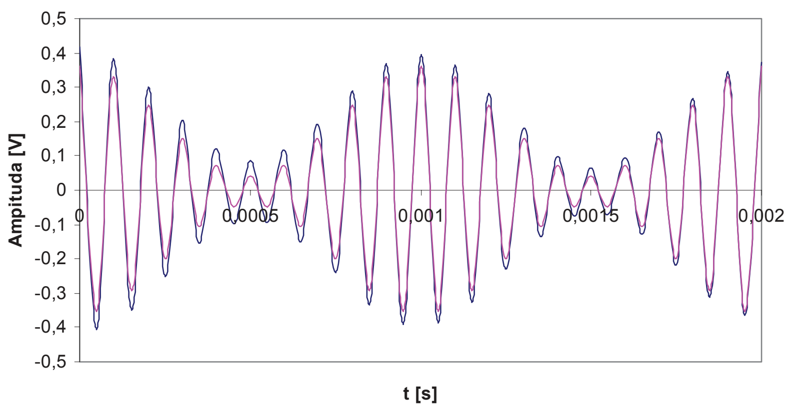

Fig. 3. Amplitude modulated signal with distortion $n(t)$ and amplitude modulated signal without distortion $s(t)$

4. DEFINITION, PROPERTIES AND APPLICATION OF HILBERT TRANSFORM

Hilbert transformation is defined by the operation of integration, which represents the signal studied in a different function. If the signal $x(t)$ is real, then if there is the Hilbert transform, it can be summarized as follows [1]:

$$\mathrm{H}\{x(t)\} = y(t) = x(t) * h(t) = x(t) * \frac{1}{\pi \cdot t} = \frac{1}{\pi} \cdot \int_{-\infty}^{+\infty} \frac{x(\tau)}{t-\tau} d\tau \tag{17}$$

where:

$h(t) = \frac{1}{\pi \cdot t}$ – impulse response filter, which gives moving phase about $\frac{\pi}{2}$ in the whole frequency range;

$*$ – convolution operation.

Knowing the Hilbert transform of a signal $x(t)$ can be determined composite analytical directly related to it. Its form is as follows:

$$z(t) = x(t) + j \cdot y(t) = x(t) + j \cdot \mathrm{H}\{x(t)\} = x(t) + j \cdot x(t) * h(t) \tag{18}$$

Using the Euler's identity to the pattern (18) we obtain a generalized form of the instantaneous amplitude and instantaneous phase of the composite signal $z(t)$. They are as follows:

$$a(t) = |z(t)| = \sqrt{x^2(t) + y^2(t)} = \sqrt{x^2(t) + \left(\mathrm{H}\{x(t)\}\right)^2} = \sqrt{x^2(t) + \left(x(t) * h(t)\right)^2} \tag{19}$$

$$\varphi(t) = arctg\left(\frac{\mathrm{Im}\, z(t)}{\mathrm{Re}\, z(t)}\right) = arctg\left(\frac{y(t)}{x(t)}\right) = arctg\left(\frac{\mathrm{H}\{x(t)\}}{x(t)}\right) = arctg\left(\frac{x(t) * h(t)}{x(t)}\right) \tag{20}$$

where:

$a(t)$ – a generalized form of the instantaneous amplitude;

$\varphi(t)$ – a generalized form of the instantaneous phase.

Based on these considerations, we can define the following Hilbert transform properties:

- it is carried out in the time domain, this is the same area where the test signal is presented;
- the shape of the imaginary component $y(t)$ of the analytical signal $z(t)$ is uniquely determined by the actual shape of the real component $x(t)$ and vice versa;
- based on the analytical signal $z(t)$ the instantaneous amplitude $a(t)$ and instantaneous phase $\varphi(t)$ can be defined;
- the analytical signal can be called the complex envelop, the knowledge of which allows us to reconstruct easily the signal with the modulated signal. This property is apparent from the pattern (5).

5. DETECTION OF MODULATED SIGNALS, AND DISTORTIONAL SIGNALS

As assumed in this paper low frequency distortion has a significant influence on the modulated signal, describes this relationship (16), as illustrated on Figure 2 and Figure 3 Therefore, it is necessary to remove the disorder of the signal described by formula (15) which is described by formula (16) to obtain a valid modulated signal (14). To do this, we perform the following operations on a received signal:

- the appointment of Hilbert transform and the complex envelope of the received signal $s_n(t)$ is described by formula (16):

$$\begin{aligned}
&\mathrm{H}\{s_n(t)\} = y(t) = s_n(t) * h(t) = \left(1 + k_{a_n}(t)\cdot n(t)\right)\cdot s(t) * \frac{1}{\pi\cdot t} = \\
&= \frac{1}{\pi}\cdot\int_{-\infty}^{+\infty}\frac{\left(1 + k_{a_n}(\tau)\cdot n(\tau)\right)\cdot s(\tau)}{t-\tau}d\tau = \frac{1}{\pi}\cdot\int_{-\infty}^{+\infty}\frac{s(\tau) + k_{a_n}(\tau)\cdot n(\tau)\cdot s(\tau)}{t-\tau}d\tau = \\
&= \frac{1}{\pi}\cdot\int_{-\infty}^{+\infty}\frac{s(\tau)}{t-\tau}d\tau + \frac{1}{\pi}\cdot\int_{-\infty}^{+\infty}\frac{k_{a_n}(\tau)\cdot n(\tau)\cdot s(\tau)}{t-\tau}d\tau = \frac{U_{c_m}}{\pi}\cdot\int_{-\infty}^{+\infty}\frac{\cos(2\cdot\pi\cdot f_c\cdot\tau)}{t-\tau}d\tau + \\
&+ \frac{k_a\cdot U_{c_m}\cdot U_{m_m}}{\pi}\cdot\int_{-\infty}^{+\infty}\frac{\cos(2\cdot\pi\cdot f_m\cdot\tau)\cdot\cos(2\cdot\pi\cdot f_c\cdot\tau)}{t-\tau}d\tau + \\
&+ \frac{U_{n_m}}{\pi}\cdot\int_{-\infty}^{+\infty}\frac{\cos(2\cdot\pi\cdot f_n\cdot\tau)\cdot\cos(\varphi_n(\tau))\cdot\cos(2\cdot\pi\cdot f_c\cdot\tau)}{t-\tau}d\tau - \\
&- \frac{U_{n_m}}{\pi}\cdot\int_{-\infty}^{+\infty}\frac{\sin(2\cdot\pi\cdot f_n\cdot\tau)\cdot\sin(\varphi_n(\tau))\cdot\cos(2\cdot\pi\cdot f_c\cdot\tau)}{t-\tau}d\tau
\end{aligned} \tag{21}$$

It can be noted that the Hilbert transform described by (21) is the sum of four Hilbert transformations to be examined individually [3]. In order to determine the pattern we are interested in describing the Hilbert transform distortional received modulated signal. Hilbert transform to the designated individual components contained in the formula (21) will be used instead of as Fourier transform (22) and (23) and the method of projections (24) and (25).

$$\frac{U_{c_m}}{\pi}\cdot\int_{-\infty}^{+\infty}\frac{\cos(2\cdot\pi\cdot f_c\cdot\tau)}{t-\tau}d\tau = \frac{U_{c_m}}{\pi}\cdot\sin(2\cdot\pi\cdot f_c\cdot t) \tag{22}$$

$$\begin{aligned}
&\frac{k_a\cdot U_{c_m}\cdot U_{m_m}}{\pi}\cdot\int_{-\infty}^{+\infty}\frac{\cos(2\cdot\pi\cdot f_m\cdot\tau)\cdot\cos(2\cdot\pi\cdot f_c\cdot\tau)}{t-\tau}d\tau = \\
&= \frac{k_a\cdot U_{c_m}\cdot U_{m_m}}{\pi}\cdot\sin(2\cdot\pi\cdot f_m\cdot t)\cdot\sin(2\cdot\pi\cdot f_c\cdot t)
\end{aligned} \tag{23}$$

$$\frac{U_{n_m}}{\pi}\cdot\int_{-\infty}^{+\infty}\frac{\cos(2\cdot\pi\cdot f_n\cdot\tau)\cdot\cos(\varphi_n(\tau))\cdot\cos(2\cdot\pi\cdot f_c\cdot\tau)}{t-\tau}d\tau\approx$$
$$\begin{aligned}&\approx\frac{U_{n_m}}{4\cdot\pi}\cdot\sin\left[2\cdot\pi\cdot\tau\cdot(f_n+f_c)+\varphi_n(\tau)\right]+\\&+\frac{U_{n_m}}{4\cdot\pi}\cdot\sin\left[2\cdot\pi\cdot\tau\cdot(f_n+f_c)-\varphi_n(\tau)\right]+\\&+\frac{U_{n_m}}{4\cdot\pi}\cdot\sin\left[2\cdot\pi\cdot\tau\cdot(f_n-f_c)+\varphi_n(\tau)\right]+\\&+\frac{U_{n_m}}{4\cdot\pi}\cdot\sin\left[2\cdot\pi\cdot\tau\cdot(f_n-f_c)-\varphi_n(\tau)\right]\end{aligned}\tag{24}$$

$$\frac{U_{n_m}}{\pi}\cdot\int_{-\infty}^{+\infty}\frac{\sin(2\cdot\pi\cdot f_n\cdot\tau)\cdot\sin(\varphi_n(\tau))\cdot\cos(2\cdot\pi\cdot f_c\cdot\tau)}{t-\tau}d\tau\approx$$
$$\begin{aligned}&\approx\frac{U_{n_m}}{4\cdot\pi}\cdot\sin\left[2\cdot\pi\cdot\tau\cdot(f_n+f_c)-\varphi_n(\tau)\right]+\\&-\frac{U_{n_m}}{4\cdot\pi}\cdot\sin\left[2\cdot\pi\cdot\tau\cdot(f_n+f_c)+\varphi_n(\tau)\right]+\\&+\frac{U_{n_m}}{4\cdot\pi}\cdot\sin\left[2\cdot\pi\cdot\tau\cdot(f_n-f_c)-\varphi_n(\tau)\right]+\\&-\frac{U_{n_m}}{4\cdot\pi}\cdot\sin\left[2\cdot\pi\cdot\tau\cdot(f_n-f_c)+\varphi_n(\tau)\right]\end{aligned}\tag{25}$$

Correctness of the solutions envisaged (24) and (25) was confirmed in a numerical way. Aggregation of results presented in patterns (22), (23), (24) and (25) can identify the Hilbert transform the received signal on the basis of its complex envelope. Based on the formula:

$$s_n(t)=\mathrm{Re}\left[\underline{s}(t)\cdot z_n(t)\right]\tag{26}$$

where:

$\underline{s}(t)$	– complex carrier signal, which is a modulated signal without distortion;
$z_n(t)=s_n(t)+j\cdot y(t)$	– complex envelope;
$s_n(t)$	– signal received;

you can specify the carrier signal that interests us $s(t)$. The algorithm of proposed method is shown on Figure 4.

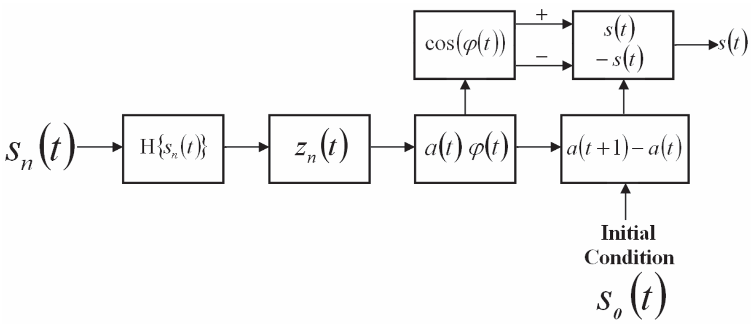

Fig. 4. The algorithm of proposed method

6. CONCLUSIONS

- the proposed method significantly improves the possibility of amplitude modulation, and thereby increases the area of application;
- it is indicated how the analysis of the received signal containing the distortion can easily eliminate them. Thanks to the demodulation we can use simple demodulation systems, for example, a peak detector;
- the correctness of the proposed algorithm shows the results in Table 1.

Table 1. Comparative table of the results of theoretical and obtained based on the proposed method

t [s]	$s(t)$ [V] theoretical values	$\cos(\varphi(t))$ calculated values	$a(t)$ [V] calculated values	$s(t)$ [V] calculated values
0.00000	0.36000	0.99907	0.41657	0.36000
0.00005	–0.35217	–0.99900	0.40825	–0.35168
0.00010	0.32944	0.99883	0.38465	0.32809
0.00015	–0.29405	–0.99854	0.34840	–0.29183
0.00020	0.24944	0.99801	0.30295	0.24638
0.00025	–0.20000	–0.99706	0.25270	–0.19613
0.00030	0.15056	0.99529	0.20250	0.14594
0.00035	–0.10595	–0.99198	0.15723	–0.10067
0.00040	0.07056	0.98614	0.12127	0.06470
0.00045	–0.04783	–0.97817	0.09800	–0.04144
0.00050	0.04000	0.97310	0.08943	0.03286
0.00055	–0.04783	–0.97623	0.09612	–0.03955
0.00060	0.07056	0.98378	0.11743	0.06086
0.00065	–0.10595	–0.99007	0.15138	–0.09481
0.00070	0.15056	0.99388	0.19462	0.13805
0.00075	–0.20000	–0.99599	0.24277	–0.18621
0.00080	0.24944	0.99716	0.29099	0.23443
0.00085	–0.29405	–0.99781	0.33442	–0.27785
0.00090	0.32944	0.99817	0.36866	0.31209
0.00095	–0.35217	–0.99834	0.39025	–0.33369

Table 1 continuous

0.00100	0.36000	0.99837	0.39697	0.34040
0.00105	–0.35217	–0.99826	0.38804	–0.33147
0.00110	0.32944	0.99800	0.36424	0.30767
0.00115	–0.29405	–0.99749	0.32779	–0.27122
0.00120	0.24944	0.99656	0.28219	0.22562
0.00125	–0.20000	–0.99484	0.23181	–0.17525
0.00130	0.15056	0.99146	0.18156	0.12499
0.00135	–0.10595	–0.98462	0.13633	–0.07976
0.00140	0.07056	0.97120	0.10055	0.04400
0.00145	–0.04783	–0.95056	0.07758	–0.02101
0.00150	0.04000	0.93658	0.06911	0.01254
0.00155	–0.04783	–0.94646	0.07539	–0.01883
0.00160	0.07056	0.96705	0.09604	0.03948
0.00165	–0.10595	–0.98183	0.12942	–0.07285
0.00170	0.15056	0.98969	0.17223	0.11566
0.00175	–0.20000	–0.99365	0.22008	–0.16351
0.00180	0.24944	0.99569	0.26805	0.21148
0.00185	–0.29405	–0.99679	0.31127	–0.25471
0.00190	0.32944	0.99737	0.34534	0.28878
0.00195	–0.35217	–0.99766	0.36679	–0.31022
0.00200	0.36000	0.99773	0.37337	0.31680

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METODA ODZYSKIWANIA WŁAŚCIWEJ OBWIEDNI ZESPOLONEJ ZABURZONEGO SYGNAŁU ZMODULOWANEGO AMPLITUDOWO W OPARCIU O PRZEKSZTAŁCENIE HILBERTA

Streszczenie

W pracy przedstawiono definicję przekształcenia Hilberta oraz związane z nim zagadnienia analitycznej reprezentacji sygnału oraz splotu sygnałów. Określono metodę wyznaczania obwiedni sygnałów zmodulowanych amplitudowo z wykorzystaniem przekształcenia Hilberta. Omówiono korzyści wynikające ze stosowania niniejszej metody przy stosowaniu transmisji radiowej, na którą wpływ mają zakłócenia spowodowane sygnałami niskoczęstotliwościowymi, których źródłem może być sieć elektroenergetyczna. Rozpatrywane zagadnienia teoretyczne poparto konkretnymi przykładami.

Słowa kluczowe: przekształcenie Hilberta, sygnał analityczny, sygnał zmodulowany amplitudowo, obwiednia sygnału