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COMPUTER SIMULATION OF THE ELECTROMECHANICAL SYSTEMS USING CONVOLUTION INTEGRAL

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Summary. The method of the computer simulation of the electromechanical systems using the recurrent equations based on the convolution integral approximations is presented in the paper.

Keywords: electromechanical system, computer simulation, convolution integral, ordinary differential equation (ODE)

1. INTRODUCTION

The numeric methods for solving the ordinary differential equations (ODEs) are used for the computer simulations of the electromechanical systems widely. There are no problems to solve the systems of the ODEs with smooth or low varying solutions but problems appear when solutions are complicated with the very fast components of the process. The example to show these problems using well-known environment MATLAB with Simulink can be found in [1] – the different methods give the different results and automatic step control can't to improve this situation.

Different solutions obtained by means of different numeric methods intended for solving ODEs with the automatic step control strategy were described in [2] also as example for MATLAB ODEs suit (*only for second-order system of equations*!).

This phenomenon of ODEs solution using the numeric methods is the consequence of their basic principle – all numeric methods for solving the ODEs approximate the solution by the limited Taylor series that is suitable for continues smooth functions only. As the result the modern electromechanical system with power pulse-width modulation (PWM) can't be correct simulated using the traditional numeric methods because their signals are interrupted (discontinues). Such problem can be solved using analytic or integration-based methods some of which are based on the convolution integral [3].

2. FUNDAMENTALS

The proposed method based on the convolution integral approximation that produces simple but effective recurrent modelling equations [3], [4]. Some problems can be solved using presented approach:

- Numerical stability – the theorem of the strong numeric stability of this formulas was proved in [4];
- Simplicity of the obtained recurrent formulas – they are simple and comprehensible;
- Obtained equations are very effective – the operating step during the computer simulation is limited by the Shannon's sampling theorem.

The proposed method is based on the two principles:

1) The modelling object's transient behaviour is defined and described by the one of the common methods:
   - system of ODEs;
   - transfer function W(s);
   - structured model etc,

   and can be decomposed to the elementary dynamic blocks (*this method called as the parallel decomposition*).
2) The free input signal or excitation $x(t)$ of the modelled system is undefined symbolically that's must be approximated by the polynomial $x^*(t)$ using signal samplings. This principle is used in z-transform on the basis of zero- and first-order approximation for the sampled signals [5].

The output (response) of the dynamic system can be described by the convolution integral in the following form:

$$y(t) = y_0(t) + \int_0^t x(\tau) \cdot w(t-\tau)\, d\tau ,$$

where:

$y(t)$ – output system response;
$y_0(t)$ – system response for nonzero initial conditions;
$x(t)$ – system input or excitation;
$w(t)$ – impulse response of the system, corresponds to the inverse Laplace transform of the system transfer function $W(s)$.

The system response for nonzero initial conditions $y_0(t)$ can be written it was shown [3]:

$$y_0(t) = \mathrm{L}^{-1}\left(\frac{\sum_{j=1}^{n} \left(y_0^{(j-1)} \sum_{i=j}^{n} a_i s^{i-j} \right)}{A(s)} \right),$$

where:

L – Laplace transform;
n – system transfer function $W(s)$ order;
$y_0^{(j)}$ – j^{th} derivative of the $\left. y_0(t) \right|_{t=0}$;
$A(s)$ – polynomial denominator of the system transfer function $W(s)$;
a_i – polynomial coefficients of the denominator $A(s)$.

The final equation of the system response with the nonzero initial conditions is

$$y(t) = \mathrm{L}^{-1}\left(\frac{\sum\limits_{j=1}^{n} \left(y_0^{(j-1)} \sum\limits_{i=j}^{n} a_i s^{i-j} \right)}{A(s)} \right) + \int\limits_0^t x(\tau) \cdot w(t-\tau)\, d\tau .$$

The main problem of the convolution integral calculation is the missing of the analytic description of the input signal $x(t)$, because it is free of decision usually. For example, the input of the actuator or controller in the closed automatic system is unknown (undetermined) as the result there is no analytic description of the input $x(t)$ for convolution integral calculation. This problem can be solved by the polynomial approximation of the sampled signal $x^*(t)$.

There are two ways to create the polynomial approximation $x^*(t)$:

1) the **explicit** rule that use current (index “i”) and previous samples of the data – only this principle can be applied in the real-time systems;
2) the **implicit** rule that uses the next (index “i+1”) and the current and previous (when are need) samples of the data – this principle may be apply for the computer simulation because implicit equations can be solved by the analytic or numeric method.

There are simple examples below illustrate the basic of the polynomial approximation: the coefficients of the first-order approximation polynomial $x^*(t) = a_1 \cdot t + a_0$ can be finding with two samples as shown below.

From explicit rule:

Coefficients using samples x_{i-1} and x_i on the discrete time t_{i-1} i t_i can be finding from the following set of equations:

$$\begin{cases} a_0 = x_i\,; \\ -a_1 \cdot h + a_0 = x_{i-1}\,; \end{cases} \Rightarrow \begin{cases} a_0 = x_i\,; \\ a_1 = \dfrac{x_i - x_{i-1}}{h}. \end{cases}$$

From implicit rule:

Coefficients using samples x_i and x_{i+1} on the discrete time t_i i t_{i+1} can be finding from the other system of equations:

$$\begin{cases} a_1 \cdot h + a_0 = x_{i+1}\,; \\ a_0 = x_i\,; \end{cases} \Rightarrow \begin{cases} a_0 = x_i\,; \\ a_1 = \dfrac{x_{i+1} - x_i}{h}. \end{cases}$$

There are polynomials of any order can be build using those rules but their high orders are not optimal for the signal reconstruction.

The analysis of the rational polynomial approximation order based on the classic control theory which operate by the Bode's diagrams of the investigating system. So, we can describe the polynomial as discrete filter and build discrete transfer function for it. The Bode analyse in such case is based on two rules:

- Equation $X_i = \frac{1}{h}\int_{t_i}^{t_{i+1}} x(t)\,dt$ corresponds to analytic integration which Laplace transform is $\frac{1}{s}$ and Bode's diagram build from the frequency-based form $\frac{1}{j\cdot\omega}$.
- Equation $X_i^* = \frac{1}{h}\int_{t_i}^{t_{i+1}} x^*(t)\,dt$ corresponds to integral of the approximation polynomial.

That's polynomial approximation errors using Bode analyse is the result of the research of the frequency errors between ideal integrator and its approximation by integration of the polynomial $x_{(n)}^*(t) = a_n t^n + ... + a_2 t^2 + a_1 t + a_0$:

$$\varepsilon_i = X_i - X_i^* = \frac{1}{h}\int_{t_i}^{t_{i+1}} x(t)\,dt - \frac{1}{h}\int_{t_i}^{t_{i+1}} x^*(t)\,dt = \frac{1}{h}\int_{t_i}^{t_{i+1}} \left(x(t) - x^*(t)\right)dt\,.$$

The high frequency for the Bode analysis depends on Shannon's sampling theorem and it is twice lower than the sampling frequency ω_0. The operational frequency ω for the most digital systems such as the computer models is lower than sampling frequency ω_0 ten times at least. The resulted Bode diagrams of the frequency errors of the polynomial approximations relative ideal integration are shown on Figure 1 [3]. The rational order of the polynomial approximation is not high than 3 but the 1st order implicit approximation produces better result because it is simple without phase errors.

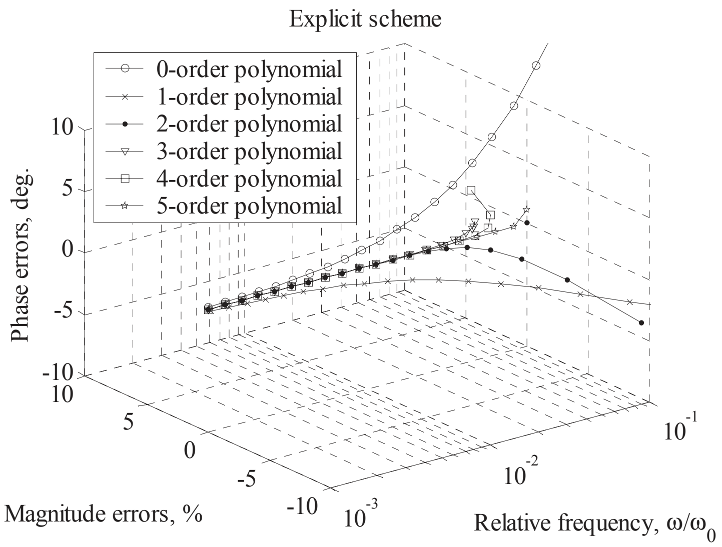
The whole system impulse response $w(t)$ has the Laplace transformation $W(s)$ that can be decomposed on the poles and zeros using Heaviside theorem. So, the complete convolution integral can be represented as the sum of the convolution integrals of the k decomposed elementary parts:

$$y(t) = y_0(t) + \int_0^t x(\tau)\cdot w(t-\tau)\,d\tau = \sum_{k=1}^{N} y_{0k}(t) + \sum_{k=1}^{N}\int_0^t x(\tau)\cdot w_k(t-\tau)\,d\tau\,.$$

The real poles are the universal representation of the set of poles because zero pole (integrator) is the particle occasion of the real pole for example and corresponds to ordinary differential equation. The pair of the complex poles can be represented as the connection of the real and zero poles [5] that can be describe by the two first order differential equations:

$$T^2 y'' + 2\xi T y' + y = x \qquad \Rightarrow \qquad \begin{cases} y' = z\,; \\ \dfrac{T}{2\xi} z' + z = \dfrac{x-y}{2T\xi}\,. \end{cases}$$

This way reduces accuracy slightly (*that can be compensated by small step size reducing*) but simplifies obtained recurrent equations very much [5].



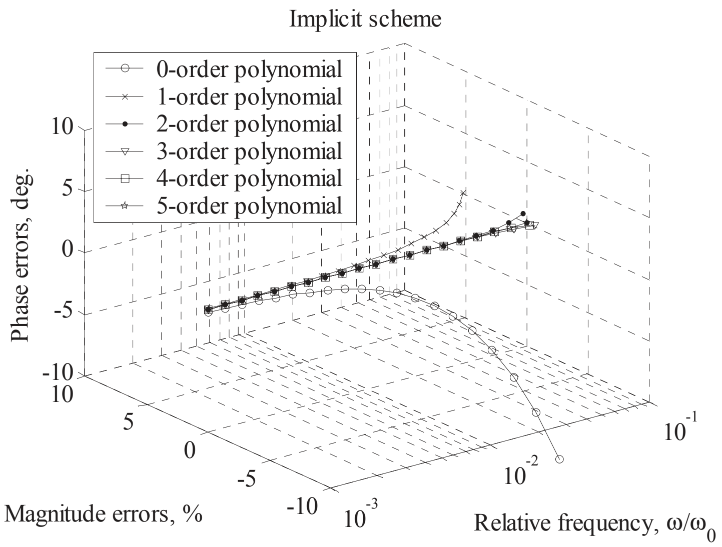


Fig. 1. Bode's plots of the polynomial approximation of the sampled signal.

The impulse response of the block with the real pole is the fading exponent $w_1(t) = \frac{1}{T} e^{-\frac{h}{T}}$ where h is the time step and T corresponds to the real pole. The first order **explicit** polynomial approximation used for signal approximation $x^*(t)$ produces response of first-order pole for nonzero initial conditions and free input as the result:

$$y_1(t) = y_{01}(t) + \int_0^t x^*(\tau)\cdot w_1(t-\tau)\,d\tau = \mathrm{L}^{-1}\left(\frac{y_0\cdot T\cdot s}{T\cdot s+1}\right) + \int_0^t (a_1\cdot\tau + a_0)\cdot\frac{1}{T}e^{-\frac{t-\tau}{T}}\,d\tau =$$

$$= y(0)\cdot e^{-\frac{t}{T}} + \frac{1}{T}\int_0^t \left(\frac{x_i - x_{i-1}}{h}\tau + x_i\right)e^{-\frac{t-\tau}{T}}\,d\tau.$$

Let us replace t by h and calculate integral on time distance $t_i \le t < t_{i+1}$ we obtain the explicit modelling recurrent formula after the simple algebra conversation:

$$y_{i+1} = y_i e^{\frac{-h}{T}} + \left(1 - e^{\frac{-h}{T}}\right)\cdot x_i + \left(1 - \frac{T}{h}\left(1 - e^{\frac{-h}{T}}\right)\right)\cdot(x_i - x_{i-1})\,.$$

Using the first order **implicit** polynomial approximation (the best choice for author view – see [3]) used for signal approximation $x^*(t)$ produces the other form of the response:

$$y_1(t) = y_{01}(t) + \int_0^t x^*(\tau)\cdot w_1(t-\tau)\,d\tau = \mathrm{L}^{-1}\left(\frac{y_0\cdot T\cdot s}{T\cdot s+1}\right) + \int_0^t (a_1\cdot\tau + a_0)\cdot\frac{1}{T}e^{-\frac{t-\tau}{T}}\,d\tau =$$

$$= y(0)\cdot e^{-\frac{t}{T}} + \frac{1}{T}\int_0^t \left(\frac{x_{i+1} - x_i}{h}\tau + x_i\right)e^{-\frac{t-\tau}{T}}\,d\tau.$$

As the result we obtain the implicit modelling recurrent formula:

$$y_{i+1} = y_i e^{\frac{-h}{T}} + x_{i+1} - x_i e^{\frac{-h}{T}} - \frac{T}{h}(x_{i+1} - x_i)(1 - e^{\frac{-h}{T}})\,.$$

Those formulas types (explicit and implicit) are using for computer simulation based on traditional prediction-correction rules [2]

3. EXAMPLES

The proposed method was tested on the complicated electromechanical object as electric drive system that describe by the 10th order system of the nonlinear ODEs – this is the 2-motor 3-mass electric drive system of the swing drive of the power mining shovel

Fig. 2. Nonlinearities of the ODE's system are:

- the saturation of the controllers;
- the static and dynamic nonlinearities of the thyristor exciter;
- magnetic saturation of the DC generator;
- air gaps in the mechanical part of the swing drive.

Simulation results for this model are shown in the Fig. 3 (electric drive voltage and current) and Fig. 4 (the shaft torques).

Computer simulation time on the time interval 6 s for the proposed formula with the fixed time step 10 ms (*emulate discrete properties of the DC generator thyristor* *exciter*) was 0.13 s using MATLAB environment on Intel Celeron-1400 processor and over 25 s and more for the standard MATLAB functions for ODEs – ode23, ode45 and ode113. The MATLAB functions for the stiff ODEs didn't work properly because gaps were presented in the mechanical joints.

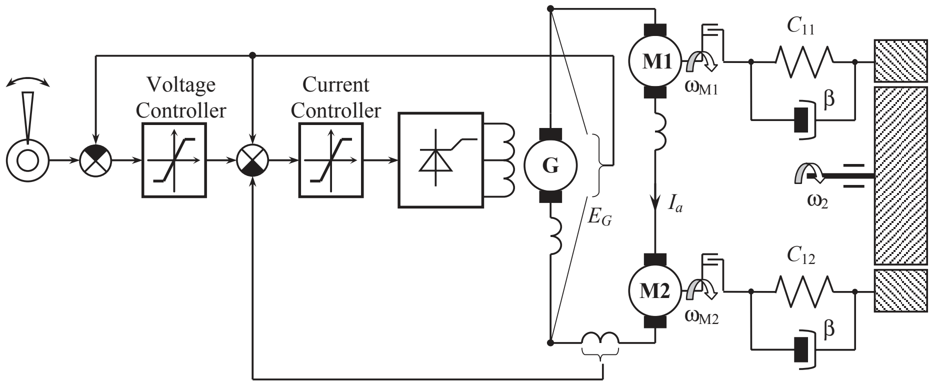


Fig. 2. Two-motors 3-mass electric drive system for the swing drive of the power mining shovel.

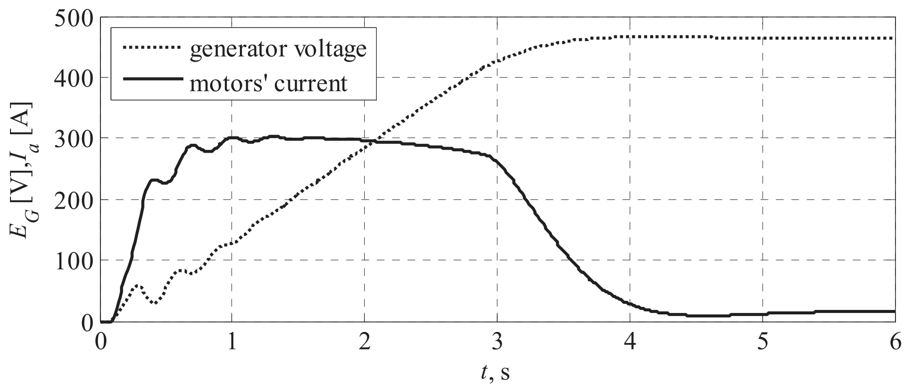


Fig. 3. Generator voltage and motors' current for electric drive model.

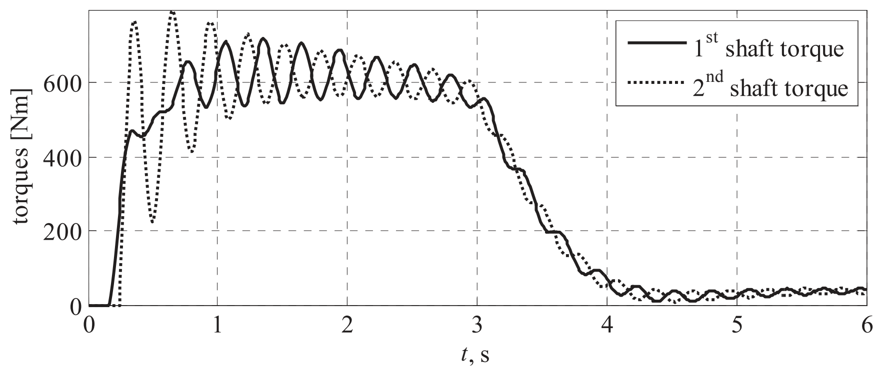


Fig. 4. Shafts' torques for swing drive of the power mining shovel.

The root-mean-square (RMS) relative errors of the proposed formula weren't higher than 1.65% for the torques, no more 0.86% for the motors' current and were less than 0.27% for the angular velocities comparing to any MATLAB numeric methods for ODEs with 10^{-6} tolerance.

4. CONCLUSIONS

The proposed approach is suitable for various problems solution in the field of electrical engineering. Main advantages of proposed method are:

- obtained modelling equations are numerically stable and don't dependent on the step size;
- the proposed approach produce quit simple but effective equations that are suitable for simulation of linear and nonlinear dynamic objects;
- the proposed method is suitable for the real-time computer simulation and produces the effective recurrent modelling formulas with the good tolerance.

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SYMULACJA KOMPUTEROWA UKŁADU ELEKTROMECHANICZNEGO Z WYKORZYSTANIEM CAŁKI SPLOTU

Streszczenie

W artykule przedstawiono metodę symulacji komputerowej układu elektromechanicznego z wykorzystaniem równań rekurencyjnych opartych na aproksymacji całki splotu.

Słowa kluczowe: układ elektromechaniczny, symulacja komputerowa, całka splotu, układ równań różniczkowych