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MODEL BADANIA POTWIERDZAJĄCEGO NIEZAWODNOŚĆ DLA DWUMIANOWYCH SYSTEMÓW O ROSNĄCEJ NIEZAWODNOŚCI

RELIABILITY DEMONSTRATION TEST MODEL FOR BINOMIAL SYSTEMS WITH RELIABILITY GROWTH

Wiele złożonych systemów inżynieryjnych przechodzi proces wzrostu niezawodności w kolejnych etapach badań. Większość istniejących modeli badań potwierdzających niezawodność nie uwzględnia jednak wzrostu niezawodności. W niniejszym artykule przedstawiono model badania niezawodności dla systemów dwumianowych o 2 etapach testowych, który bierze pod uwagę możliwy wzrost niezawodności. Określono wskaźniki prawdopodobieństwa dla różnych możliwych sytuacji oraz wyprowadzono statystyczne reguły decyzyjne. Podany przykład ilustruje względnie niższe ryzyko decyzyjne proponowanego modelu w stosunku do tego, jakie niosą klasyczne modele badań.

Słowa kluczowe: *System dwumianowy, badanie potwierdzające, dyskretny model wzrostu niezawodności.*

Many complex engineering systems experience reliability growth in successive test stages. In most existing reliability demonstration test models, reliability growth is not incorporated. In this paper, for binomial systems with 2 test stages, a reliability demonstration test model is presented that takes possible reliability growth into consideration. Likelihood ratios are defined for various possible situations, and statistical decision rules are derived. With a given example, the relative lower decision risk of the proposed model is illustrated by comparison with that of classical demonstration test models.

Keywords: *Binomial system, demonstration test, discrete reliability growth model.*

1. Introduction

The objective of reliability demonstration test is to confirm whether the newly developed system meet given reliability requirements. With the increasing complexity and testing cost of modern military systems, reliability demonstration test has to be done with limited number of samples and with changing technical configuration of system [8]. Consequently, it is necessary to develop reliability demonstration test models that can be used in such situations.

Some research works on reliability demonstration models have been reported in the literature [4]. However, most of the conventional demonstration models assume that the test data come under the same system state. In the practical test and evaluation process of a complex engineering system, once a failure occurs, the system must be fixed before starting the next test. As a result, the technical state of the system is different for the subsequent test stage. This in turn leads to possible reliability growth as tests progress [3]. In their reliability analysis of a solid rocket engine, An and Zhou pointed out that based on the test data in two successive test stages, many samples could have been saved if a reliability growth model had been used for estimation of the engine reliability [1]. Mazzuchi and Soyer used the ordered Dirichlet distribution in modeling the reliability growth process during product development [5,6]. Patterson and Dietrich introduced an ordered Dirichlet binomial attribute testing model[7]. They incorporated prior information in the model and used the posterior distribution for reliability assessment and prediction. However, their model cannot be directly used as a reliability demonstration model due to lack of consideration of decision risks to the producer and the consumer. Li and Yan introduced a test model based on test data of all stages [3]. They used the AMSAA growth model and provided test decision rules based on statistics of the AMSAA model. But their model is a continuous growth model and is not appropriate for binomial systems. In addition, the reliability growth is assumed to be consistent with the nonhomogeneous Poisson process (NHPP). Thus, to make full use of test information and reduce expensive test samples, there is a need to develop demonstration test models for binomial systems when the system configuration may change through the test stages.

In this paper, a reliability demonstration model for the binomial system (also called discrete reliability growth model for the binary-valued output case) is presented. It is assumed that there are only two possible test results from each test, either a failure or a success (pass). Reliability growth may be experienced by the system in successive stages, depending on the effectiveness of the corrective actions taken when a failure is detected. The proposed model can be used to validate the success probability at the end of the final test stage, based on all the available test results. In this paper, consideration is given to possible reliability growth during test stages, all possible situations that may change the system reliability are listed and the corresponding likelihood functions are given, then their likelihood ratios are defined. Using the principle of likelihood ratio test [2], we provide statistical decision rules along with formulas for calculating their risk levels. To illustrate the advantage of the proposed model over classical models, an example is presented.

2. The proposed reliability demonstration test model

2.1. Test scheme

Consider a system reliability demonstration test that has two stages. In each stage, a number of identical system tests are repeatedly conducted, and the test results are observed. After each test stage, redesign and correction are done to improve reliability. Suppose that the technical configuration of the system under test is fixed within each stage, and thus the reliability of the system will remain unchanged within each stage. However, the system reliability either grows or remains unchanged between successive test stages (due to possible design improvement or modification). This test scheme is much like the so-called delayed-fix scheme in traditional reliability growth test.

In the first stage, suppose that the number of tests conducted is n_1 and the number of successes is s_1, so the test result is denoted as (n_1, s_1). Similarly, the test result of the second stage is denoted as (n_2, s_2).

Assume that there are two possible reliability levels for the system under test. One is called the high reliability level p_1, and the other is called the low reliability level p_0, where $p_1 > p_0$. In engineering practice, p_0 is also called the tolerable reliability level, while p_1 is called the acceptable reliability level.

Let $p(j)$ denote the reliability level of the system at stage j, $j = 1,2,\cdots$. Because of possible reliability growth from stage 1 to stage 2, we have $p(2) \geq p(1)$.

2.2. Test decision

Assume that $x=\{(n_1, s_1), (n_2, s_2)\}$ is the test results of the two stages. The numbers of failures in each stage are $f_1=n_1-s_1$, $f_2=n_2-s_2$ respectively.

To verify the system reliability level using the statistical decision model, the following hypotheses are given:

$$H_0: p(2) = p_1 \quad H_1: p(2) = p_0 \tag{1}$$

where H_0 and H_1 are called null hypothesis and alternative hypothesis, respectively.

Given the above, the likelihood ratio test (LRT) statistic[2] can be defined as

$$\lambda(x) = \frac{L\left(p(2) = p_0 | x\right)}{L\left(p(2) = p_1 | x\right)} \tag{2}$$

For the reliability levels of the system at the two stages, depending on the effectiveness of corrective actions at the end of stage 1, there are 3 and only 3 possible mutually exclusive cases as follows:

Case 1: This is the case that the system reliability originally does not meet the requirement, and the corrective measures taken at the end of the first stage are effective in improving the reliability, so the system reliability level changes from p_0 at the first stage to p_1 at the second stage (Fig.1(a)).

$$p(1) = p_0 \quad p(2) = p_1 \tag{3}$$

Case 2: This is the case that the system reliability originally does not meet the requirement, and the corrective measures taken at the end of the first stages are not effective in removing faults, so the system reliability level is not improved significantly, and still remains at p_0 at the second stage (Fig. 1(b)).

$$p(1) = p_0 \quad p(2) = p_0 \tag{4}$$

Case 3: This is the case that the system reliability originally does meet the requirement, measures taken at the end of the first stage actually are not necessary for the system to pass the demonstration test. Therefore, the system reliability is not changed and remains at p_1 at both stages (Fig. 1(c)).

$$p(1) = p_1 \quad p(2) = p_1 \tag{5}$$

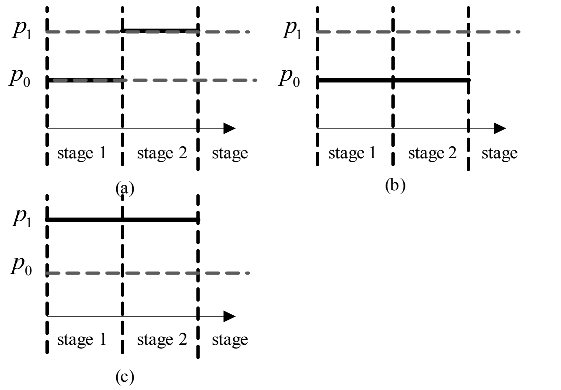

Fig.1. Cases of system reliability change

By enumeration of all the possible cases outlined above, the corresponding likelihood functions can be expressed as

$$L_{00} = L\left(p(1) = p_0, p(2) = p_0 | x\right) = C_{n_1}^{s_1} p_0^{s_1} q_0^{f_1} C_{n_2}^{s_2} p_0^{s_2} q_0^{f_2} \tag{6}$$

$$L_{01} = L\left(p(1) = p_0, p(2) = p_0 | x\right) = C_{n_1}^{s_1} p_0^{s_1} q_0^{f_1} C_{n_2}^{s_2} p_1^{s_2} q_1^{f_2} \tag{7}$$

$$L_{11} = L\left(p(1) = p_0, p(2) = p_0 | x\right) = C_{n_1}^{s_1} p_1^{s_1} q_1^{f_1} C_{n_2}^{s_2} p_1^{s_2} q_1^{f_2} \tag{8}$$

where $q_0 = 1-p_0$, $q_1 = 1-p_1$.

Define the likelihood ratios as

$$\lambda_{0001} = \frac{L\left(p(1) = p_0, p(2) = p_0 | x\right)}{L\left(p(1) = p_0, p(2) = p_1 | x\right)} = \frac{C_{n_1}^{s_1} p_0^{s_1} q_0^{f_1} C_{n_2}^{s_2} p_0^{s_2} q_0^{f_2}}{C_{n_1}^{s_1} p_0^{s_1} q_0^{f_1} C_{n_2}^{s_2} p_1^{s_2} q_1^{f_2}} \tag{9}$$

$$\lambda_{0001} = \frac{L\left(p(1) = p_0, p(2) = p_0 | x\right)}{L\left(p(1) = p_0, p(2) = p_1 | x\right)} = \frac{C_{n_1}^{s_1} p_0^{s_1} q_0^{f_1} C_{n_2}^{s_2} p_0^{s_2} q_0^{f_2}}{C_{n_1}^{s_1} p_0^{s_1} q_0^{f_1} C_{n_2}^{s_2} p_1^{s_2} q_1^{f_2}} \tag{10}$$

Based on the idea of likelihood ratio test, the following form of decision rules are suggested

- H_0 is rejected if

$$\lambda(x) = \max\{\lambda_{0001}, \lambda_{0011}\} > c$$

- H_0 is not rejected if

$$\lambda(x) = \max\{\lambda_{0001}, \lambda_{0011}\} \leq c$$

where c is a decision parameter and takes a constant value.

By the rules defined above, the Producer Risk α is calculated as

$$\begin{aligned} \alpha &= \sum_x P\{\lambda(x) > c | p(2) = p_1\} \\ &= \sum_{\lambda(x)>c} P\{x | p(2) = p_1\} \\ &= \sum_{\lambda(x)>c} \frac{1}{2} C_{n_1}^{s_1} C_{n_2}^{s_2} \left[p_0^{s_1} q_0^{f_1} p_1^{s_2} q_1^{f_2} + p_1^{s_1+s_2} q_1^{f_1+f_2} \right] \end{aligned} \tag{11}$$

In establishing the final equality given above, we have implicitly assumed that the two cases leading to $p(2) = p_1$ are equally likely. Of course, different probabilities can be considered as a model parameter when necessary.

The Consumer (Subscriber) Risk is calculated as

$$\begin{aligned}\beta &= \sum_x P\left\{\lambda(x) \le c \,\middle|\, p(2) = p_0\right\} \\ &= \sum_{\lambda(x)\le c} P\left\{x \,\middle|\, p(2) = p_0\right\} \\ &= \sum_{\lambda(x)\le c} C_{n_1}^{s_1} C_{n_2}^{s_2}\left[p_0^{s_1+s_2} q_0^{f_1+f_2}\right]\end{aligned} \tag{12}$$

3. Existing reliability demonstration test models

In existing reliability demonstration test models for binomial systems, it is assumed that the system have the same reliability level in all test stages. That is, reliability growth is not considered. Therefore, in this case, actually the following statistical hypotheses are assumed:

$$H_0{:}p = p_1 \qquad H_1{:}p = p_0 \tag{13}$$

Let $x = (n_1 + n_2, s_1 + s_2) = (n, s), f = n - s$, then the test ratio statistic becomes

$$\lambda(x) = \frac{L(p = p_0|x)}{L(p = p_1|x)} = \frac{C_n^s p_0^s q_0^f}{C_n^s p_1^s q_1^f} = \frac{p_0^s q_0^f}{p_1^s q_1^f} \tag{14}$$

The test rules are:

- H_0 is rejected if $\lambda(x)>c$;
- H_0 is not rejected if $\lambda(x)\le c$.

Here, c is a decision constant. With such statistical decision rules, the expected risks of both sides are given respectively as follows.

$$\begin{aligned}\alpha &= \sum_x P\left\{\lambda(x) > c \,\middle|\, p(2) = p_1\right\} \\ &= \sum_{\lambda(x)>c} P\left\{x \,\middle|\, p(2) = p_1\right\} \\ &= \sum_{\lambda(x)>c} \frac{1}{2} C_{n_1}^{s_1} C_{n_2}^{s_2}\left[p_0^{s_1} q_0^{f_1} p_1^{s_2} q_1^{f_2} + p_1^{s_1+s_2} q_1^{f_1+f_2}\right]\end{aligned} \tag{15}$$

$$\begin{aligned}\beta &= \sum_x P\left\{\lambda(x) \le c \,\middle|\, p(2) = p_0\right\} \\ &= \sum_{\lambda(x)\le c} P\left\{x \,\middle|\, p(2) = p_0\right\} = \sum_{\lambda(x)\le c} C_n^s p_0^s q_0^f\end{aligned} \tag{16}$$

In the following section, we will use an example to compare the proposed model considering reliability growth with models that do not consider reliability grow between stages.

4. Example

To verify the effectiveness of our proposed approach, suppose that in the test and evaluation of a new tactical missile, the target hitting reliability is p, and $p_1 = 0.9$, $p_0 = 0.6$. The number of tests in each stage is 5.

In Fig. 2, the risk levels of the two models described in sections 2 and 3 are compared at different critical values (logarithm values). As we can seen from Fig. 2, if we require equal risk levels for both the Producer and the Consumer sides, then using our model, which considers possible reliability growth, the results are c=0.4059, α=β=0.1864. However, for the conventional test model without consideration of reliability growth, the results are c=1.3286, α=β=0.2288. It can be seen that the two risk levels of the conventional model are higher than those with our model when the number of tests is the same. Thus, if reliability growth exists between successive stages, the classical demonstration likelihood ratio test model will have high risk of making wrong decisions.

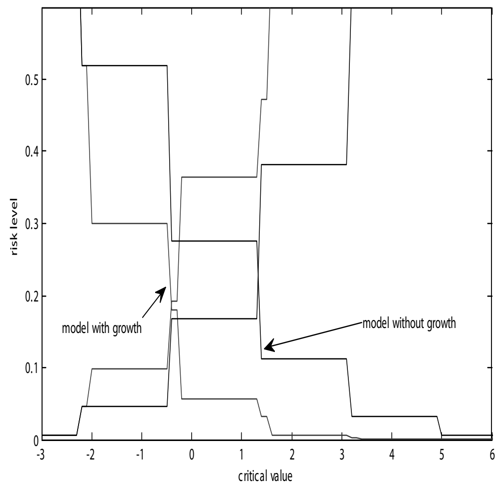

Fig.2. Comparison of risk levels

5. Conclusions

We have presented a new reliability demonstration test model for binomial systems that incorporates possible reliability growth between the two test stages. Based on all available test results of test stages, using likelihood ratio test approach, our proposed model can give more accurate decision results for test and evaluation of new systems with varying technical configuration among successive test stages.

In our future research work, this model will be considered to be further extended for systems with reliability growth in more than two stages, and with other types of reliability distributions.

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6. References

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