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OCENA WZROSTU NIEZAWODNOŚCI W BEZZAŁOGOWYM STATKU LATAJĄCYM PODCZAS KOLEJNYCH FAZ BADANIA W LOCIE

RELIABILITY GROWTH ESTIMATION FOR UNMANNED AERIAL VECHICLE DURING FLIGHT-TESTING PHASES

Samoloty muszą być testowane w locie podczas procesu ich opracowywania i dla zapewnienia niezawodności powinny przejść, podczas faz badania w locie, proces wzrostu niezawodności obejmujący kolejne etapy: testowania, poszukiwania ukrytego uszkodzenia, udoskonalania i ponownego testowania. Jednakże z powodu złożonej budowy samolotów i wysokich kosztów badań w locie, badania wzrostu niezawodności z reguły przeprowadza się na małych próbkach. Trudno jest zatem ocenić wzrost niezawodności w kolejnych fazach badań w locie. W niniejszej pracy do estymacji wzrostu niezawodności zastosowano metodę bayesowską dla dwumianowego wzrostu niezawodności opartą na rozkładzie a priori Dirichleta oraz obliczono parametry rozkładu a posteriori wykorzystując metodę symulacji Markov-Chain Monte Carlo. Metodę zastosowano w kolejnych fazach badań w locie bezzałogowego statku latającego (Unmanned Aerial Vehicle), a użyty przykład pokazuje, iż metoda oparta na rozkładzie a priori Dirichleta może skrócić czas badań w locie. Parametry rozkładu a priori łatwo jest potwierdzić na podstawie uprzednio znanych informacji. Proponowana metoda nadaje się do oceny badań wzrostu niezawodności podczas kolejnych etapów badań w locie.

Słowa kluczowe: *Niezawodność, wzrost niezawodności, badanie wzrostu niezawodności, metoda bayesowska.*

It is necessary for airplanes to be flight-tested during the development process, and they should pass the testing/failure-finding/improvement/re-testing reliability growth process during the flight-testing phases to ensure its reliability. However, due to airplane complexity and the high costs of flight-testing, the reliability growth testing is usually done with small samples. It is thus difficult to estimate the reliability growth during the flight-testing phases. In this paper, Bayesian method for binomial reliability growth based on the Dirichlet prior distribution is applied to reliability growth estimation, and the parameters of the posterior distribution are calculated by using the simulation method of Markov-Chain Monte Carlo. The method is applied to the Unmanned Aerial Vehicle test flight phases, and the example shows that the method based on the Dirichlet prior distribution can save the flight-testing time. It is easy to confirm the parameters of the prior distribution by using the prior information. The proposed method is suitable for reliability growth testing estimation during flight-testing stages.

Keywords: *Reliability, reliability growth, reliability growth testing, Bayesian method.*

Nomenclature

UAV	Unmanned Aircraft Vehicle
RGM	Reliability Growth Model
MCMC	Markov-Chain Monte Carlo
R_i	Reliability of the *i*th stage of testing
$\alpha i, \beta$	Dirichlet distribution location and scale parameters
S_i	The number of successes during the *i*th stage of testing
f_i	The number of failures during the *i*th stage of testing
m	Total number of testing phases
R_s	The mission reliability index for the whole system
R_{op}	The reliability index of the operating personnel
R_{UAV}	The reliability index of an airplane

1. Introduction

During the process of design, manufacture, and testing, the reliability and quality of a product are improved through uncovering faults, analyzing the cause, and improving the design, which is the process of reliability growth [2-5]. This process has become an essential part of reliability improvement projects. For aircraft design, the flight-testing phases are the reliability growth process, and the attribute data (pass/failure data) is usually obtained during this process. For reliability growth estimation based on attribute data, some reliability growth models have been reported in literature. The ordering restriction model is presented by Barlow, Proschan & Scheuer [12]. Based on the model, Smith [9] brought forward Bayesian Reliability Growth Model for attribute data in 1977 which applied Bayesian method to deal with binomial reliability growth estimation. Based on the Dirichlet prior distribution, Bayesian method for binomial reliability growth had been put forward by Mazzuchi and Soyer [7]. Erkanli, Mazzuchi and Soyer [8] discussed the calculation involved in the Bayesian reliability growth model by using MCMC (Markov Chain Monte Carlo) method.

In this paper, the case study on the reliability growth management process for the UAV in the Northwestern Polytechnical University is presented. Considering the operational reliability requirement, the UAV mission reliability index is allocated and calculated and the failure criterion is presented for distinguishing failure category A and failure category B. The prior information distribution parameters of the model are obtained from experts, and then field testing data is collected. The Bayesian model based on Dirichlet prior distribution is applied to estimating the reliability growth of attribute data for UAV flight-testing. The parameters of the posterior distribution are calculated by using the simulation method of Markov-Chain Monte Carlo.

2. Reliability growth estimation based the dirichlet prior distribution

Assume that there are m flight-testing phases before an airplane is put into quantity production. During the ith stage of testing, S_i denotes the number of successes, f_i denotes the number of failures. $R_i (i = 1,2,\cdots,m)$ denotes the reliability for the ith stage of testing, and R_{m+1} denotes the field reliability. During the m testing phases, the airplane should be redesigned after every failure is discovered and the reliability of the product is improved by the stages of testing. So it is reasonable to assume that

$$0 < R_1 < R_2 < \cdots < R_{m+1} < 1 \tag{1}$$

$R = (R_1, R_2, \cdots R_{m+1})$ obeys the ordered Dirichlet[4] distribution and its density is defined by the following multivariate distribution:

$$\begin{aligned} P(R) &\sim D(\alpha_1,\alpha_2,\cdots\alpha_{m+2}) \\ &= \frac{\Gamma(\beta)}{\prod_{j=1}^{m+2}\Gamma(\beta\alpha_j)}\prod_{j=1}^{m+2}(R_j - R_{j-1})^{\beta\alpha_j - 1} \end{aligned} \tag{2}$$

where, $R_0 \equiv 0$, $R_{m+2} \equiv 1$, β is the scale parameter of the prior distribution, $\alpha_i > 0$, and $\sum_{i=1}^{m+2}\alpha_i = 1$.

The main characteristic of the Dirichlet distribution is that the relevant conditional distribution and the marginal distribution are Beta distributions.

The marginal distributions is

$$P(R_i) \sim beta(\beta\alpha_i^*, \beta(1-\alpha_i^*)) \tag{3}$$

where $\alpha_i^* = \sum_{j=1}^{i}\alpha_j$.

According to this characteristic, we have: $E(R_i) = a_i^*$, $Var(R_i) = (a_i^* \times (1-a_i^*))/(\beta+1)$. β is the confidence parameter. The variance of R is inverse proportional to the value of β.

$$P(R_j - R_i) \sim beta(\beta(\alpha_j^* - \alpha_i^*), \beta(1+\alpha_i^* - \alpha_j^*)) \tag{4}$$

where $i<j$. We then have

$$E(R_i - R_{i-1}) = \alpha_i \tag{5}$$

Equation (5) means that the parameters α_i is the expected reliability growth value from the (i-1)th to the ith stages of testing.

If $\beta = m + 2$, $\alpha_i = \dfrac{1}{m+2}$, the prior distribution of Dirichlet can be seen as the non-informative prior distribution.

According to the characteristic of Dirichlet distribution, the parameters α_i is the reliability growth expectation value from the (i-1)th to the ith stages of testing. If the prior information is easy to be quantified by experts, it's easy to obtain the parameters of the prior Dirichlet distribution by using the prior information. Based on the testing data, the likelihood function can be obtained by

$$L(s_i, f_i; R_1, \cdots, R_m) = c\prod_{i=1}^{m} R_i^{s_i}(1-R_i)^{f_i} \tag{6}$$

According to Bayesian theory, the posterior distribution intensity function can be presented by

$$\begin{aligned} \pi(R) \propto{}& \frac{\Gamma(\beta)}{\prod_{j=1}^{m+2}\Gamma(\beta\alpha_j)}\prod_{j=1}^{m+2}(R_j - R_{j-1})^{\beta\alpha_j - 1} \\ &\times c\prod_{j=1}^{m} R_j^{s_j}(1-R_j)^{f_j} \end{aligned} \tag{7}$$

Then, the reliability of each phase can be calculated by using the MCMC method [6]. The MCMC methods are a class of algorithms for sampling from probability distributions based on constructing a Markov chain that has the desired distribution as its equilibrium distribution. The state of the chain after a large number of steps is then used as a sample from the desired distribution.

The Gibbs sampler [1, 6, 10] is a kind of widely used MCMC method, and it enables the drawing of samples from the posterior distribution without actually computing the exact distributional form. This is achieved by successive drawings from the full conditional distributions $\pi(R_j \mid R_0, \cdots R_{j-1}, R_{j+1,} \cdots R_{m+1})$.

The sampling process starts with a vector of arbitrary starting values $R^0 = (R_1^0, R_2^0, \cdots, R_{m+1}^0)$ and

draws R_1^1 from $R^0 = (R_1^0, R_2^0, \cdots, R_{m+1}^0)$

draws R_i^1 from $\pi(R_i \mid R_1^0, \cdots R_{i-1}^0, R_{i+1}^0 \cdots R_{m+1}^0)$

draws R_{m+1}^1 from $\pi(R_{m+1} \mid R_1^0, R_2^0 \cdots R_m^0)$

If this iteration is performed k times (i.e., next starts with R^1 and iterate to R^2, and so on), the Gibbs sequence is R^1, R^2, $\cdots$,R^k.

$$E(R_j) \approx \frac{1}{k}\sum_{t=1}^{k} R_j^t \tag{8}$$

Reliability Growth Estimation for the UAV During Flight-Testing Phases

UAVs are designed and produced for cultivating the innovation capability of students at Northwestern Polytechnical University every year. The UAV is designed, produced, and tested in carrying capacity competition. The data used here were obtained in the aviation and spaceflight in 2005 in China. During the flight-testing phases, The UAV underwent the test/fault-finding/redesign/retest process as shown in Fig. 1 and Fig. 2. In order to ensure the mission reliability of the UAV, a reliability growth model is used to monitor the reliability growth process. The detailed reliability growth estimation is presented as follows.

3.1. UAV mission reliability index allocation

According to the UAV carrying capacity competition rules, two UAVs are permitted for every participating team. Thus, it is necessary that two mature UAVs are designed for the carrying capacity competition. In order to ensure a successful mission, the mission reliability diagram of the UAV system is developed and shown in Fig. 3. According the Fig. 3, two factors should be considered to make the competing mission successful, one is the

Fig. 1. The UAV field installation

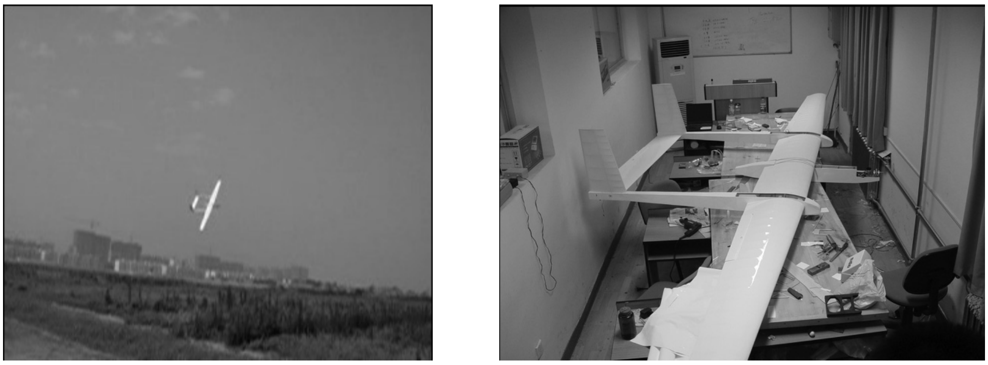

Fig. 2. Flight testing and design modification

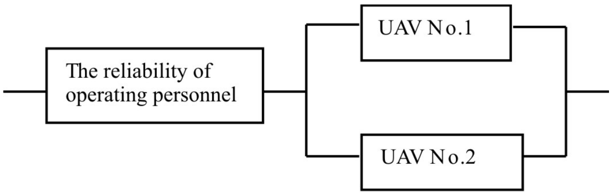

Fig. 3. The UAV total system mission reliability diagram

reliability of the ground operating personnel, and the other is the basic reliability of the airplane. Considering the effect of time and cost, the design target of the mission reliability for the whole system R_s is set at 90%. According to the Fig. 3, the mission reliability can be presented by

$$R_s = R_{op} \times (R_{UAV} + R_{UAV} - R_{UAV} \times R_{UAV}) \tag{9}$$

According to the experience, the reliability of the operating personnel R_{op} is assumed to be 0.94. The reliability index of each airplane R_{UAV} is 80% by using the Formula (8). In view of uncertainty of the flight and transportation processes, a design margin is needed, and the reliability object of each airplane is set at 0.9 in the design process. Using the diagram in Fig. 3, the reliability of the mission is expected to be 0.9024.

3.2. Failure criterion

Some failure modes were observed during reliability growth testing. The failure modes can be divided into two types: failure category A and failure category B. Correction action must be done for failure category B, and only repair action is done for failure category A. Only failure category B needs to be recorded in the reliability growth process. Thus, we need a clear criterion for failure category B. After discussion with the designers and experts, the failure category B is composed of failures due to airplane design deficiencies (such as the unreasonable airplane configuration, insufficient structural rigidity and unsteadiness of joint), which are shown as follows.

(1) Failures resulting in airplane crash and disintegration,
(2) Component failures affecting competition of the airplane flight mission,
(3) Flight instability.

The failures below are not included in failure category B:

(1) Failures due to human errors,
(2) Failures resulting from environmental factors (such as gust of wind, poor runway),
(3) Failures due to unreasonable assembly.

4. Testing Data during Reliability Growth Process

During the flight testing process, according to the failure criterion, the failures were recorded. In all testing phrases, the systemic failures of the airplane are summarized and shown in Table 1.

The success/failure sequential data were collected during the eight testing phrases and shown in Table 2.

5. Determination of the Prior Information Distribution Parameters

The UAV design team has designed many types of UAVs and is experienced in flight-testing. The experts predict that ten phases and almost fifty flight-testing are needed based on their experience. The prior information is mainly relied on the desi-

Tab. 1. Failure modes during the flight-testing phases

Serial number	Failures	Reasons	Improvement measures
1	Nose landing gear bending when landing	inadequate rigidity of Nose landing gear	Improvement of the landing gear
2	output power of engine reduced because of oil pipeline distortion	oil pipeline distortion because of heat of engine	Improvement of the pipe and adding cooling fin
3	Instability flight	Unreasonable airplane center of gravity	Harmonizing the fuselage and mass balance
4	stagger flight	Improper airfoil size	Improvement of airfoil machining precision and plane size adjustment
5	stagger flight	Unreasonable airfoil joint	The hinge joint is changed into the fixed joint
6	Poor longitudinal stability	unreasonable installation angle of stabilator	Improved installation angle of stabilator
7	stabilator flutter	Inadequate stiffness of stabilator	Increasing the stiffness of stabilator
8	Damage of aircraft wheel	Inadequate strength of aircraft wheel	Redesign or strengthen the wheel
9	Difficult adding the stowage	Unreasonable hold design	Redesign
10	Insufficient engine power	No deceleration system	Adding reasonable deceleration system

Tab. 2. Success/failure data during the eight test flight stages

Stage of testing *i*	1	2	3	4	5	6	7	8
Number of successes	0	1	1	2	4	5	6	10
Number of failures	3	2	1	1	1	1	1	0

Tab. 3. The prior distribution parameters

Location Parameters	α_1	α_2	α_3	α_4	α_5	α_6
value	0.2	0.2	0.172	0.1585	0.0928	0.0627
Location Parameters	α_7	α_8	α_9	α_{10}	α_{11}	α_{12}
value	0.0426	0.0225	0.0124	0.0123	0.0122	0.0220

gner's experiences. Therefore, according to the previous airplane test flight data and its design, the parameter of the prior distributions α_i (i=1, …, 12) can be assessed by designers: $\beta = 10$, the values of α_i are shown in Table 3.

6. Reliability growth estimation from attribute data

The airplane's field reliability can be obtained by combining prior information and posterior distribution parameters. The reliability of the UAV in different phases is calculated by using the Gibbs Sampling method, and is shown as below in Table 4.

From Table 4 above, the reliability growth estimation model can predict the reliability in the future testing phrases (R_9, R_{10}). The reliability of UAV has reached 0.9398, which is higher than the original aim of 0.9 after undergoing eight flight-testing phases. The reliability target has been reached by the 9th phase and there is no need to go to phases 10 and 11.

After the Gibbs sampler became convergent, two thousand additional samples were generated and we found that

$$P(R_8 > 0.9) = 0.7944 \qquad (10)$$

If reliability testing is done to validate the mission reliability index of 0.9 for the whole systems with the confidence level of 0.7944, the reliability testing plan parameters for the product considering only safety and failure patterns are given in Table 5.

From Table 5, if reliability testing for validating the reliability index is needed, more flight-testing must be conducted. So the time and cost can be saved by applying the reliability growth testing monitor process.

Tab. 4. Posterior distribution reliability estimation

Reliability Expectation	R_1	R_2	R_3	R_4	R_5	R_6
Values	0.1509	0.3424	0.5076	0.6569	0.7415	0.8078
Reliability Expectation	R_7	R_6	R_9	R_{10}	R_{11}	
Values	0.8832	0.9398	0.9542	0.9656	0.9768	

Tab. 5. Reliability testing plan parameters

Failure Criterion: # of acceptable failures	0	1	2
Flight-testing sampling size	16	30	42

7. Discussion & conclusion

The Bayesian method is applied to reliability growth estimation for the UAV during the testing/failure-finding /improvement-/retesting reliability growth process. Application on the case study shows that the method based on the Dirichlet prior distribution is appropriate in the staged reliability growth testing process. There are three advantages of the method: (1) it is easy to obtain the parameters of the prior distribution by using prior information; (2) it can predict the reliability in the future testing phrases; and (3) it can save time and cost effectively.

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