

ALGORYTM DO OCENY I ANALIZY STACJONARNEJ DOSTĘPNOŚCI OPERACYJNEJ OPARTY NA SPECYFIKACJI WYMAGAŃ

AN ALGORITHM FOR EVALUATION AND ANALYSIS OF STATIONARY OPERATIONAL AVAILABILITY BASING ON MISSION REQUIREMENTS

Zarówno metody matematyczne jak i symulacyjne mają ograniczenia jeśli chodzi o ocenę stacjonarnej dostępności operacyjnej. Te pierwsze zakładają, że popyt jest niezależny od systemu operacyjnego, co może skutkować niedoszacowaniem dostępności operacyjnej. Te drugie wymagają dużej liczby prób, aby uzyskać wyniki o wystarczającym stopniu ufności w warunkach wcześniej określonych scenariuszy. Niniejszy artykuł zajmuje się problemem określenia stacjonarnej dostępności operacyjnej na podstawie modeli matematycznych. Proponowany model bierze pod uwagę wiele czynników, wliczając w to pasywację systemu, specyfikację wymagań, parametry projektowe systemu, liczbę działających systemów, czas realizacji oraz czas obsługi. Artykuł przedstawia metodę aproksymacji dostępności operacyjnej. Użyty przykład ilustruje związek pomiędzy wyżej wspomnianymi czynnikami. Doświadczenia numeryczne pokazują, że model ten jest zgodny z wynikami symulacji Monte Carlo, potwierdzając realność i racjonalność proponowanej metody.

Słowa kluczowe: Dostępność operacyjna, stan stacjonarny, niezawodność, obsługiwalność, części zapasowe.

Both mathematical and simulation methods have limitations for evaluation of stationary operational availability. The former assumes that demand is independent of the operating system, which can result in underestimation of the operational availability. The latter requires a large number of trials to obtain the results with a sufficient degree of confidence under the pre-specified scenarios. This paper addresses the issue of determining the stationary operational availability based on mathematical models. The proposed model considers many factors including system passivation, mission requirements, system design parameters, the number of working systems, lead time, and maintenance time. An approximation method to the operational availability is given. Specific example is used to illustrate the relationship among the aforementioned factors. Numerical experiments show that the model agrees well with Monte Carlo simulation results and the feasibility and rationality of the proposed method are validated.

Keywords: Operational availability, stationary state, reliability, maintainability, spare parts

1. Introduction

Operational availability (A_o) is an important measure of system performance. It is a function of system design characteristics, mission requirements, and maintenance scheme. The methods commonly used for calculating A_o can be classified into two types. One is the simulation method, which is established based on the system function model, the mission scenario model, and the maintenance and support model. Software packages like Simlox2.0 [13] and SMMS [2] have such functions. Though the simulation method has many merits, its efficiency is low. It is thus useful more for evaluating the dynamic availability rather than the stationary availability. The other one is the model based method, which constructs the relationship between A_o and system mission requirements and maintenance scheme. Software packages like METRIC [10], VARI-METRIC [11], SPAREL [9] and OPRAL [1] have such functions. Both types of models can be used to calculate stationary availability. However, passivation is not considered in these models. When considering passivation, the actual demand rate is dependent on the number of working systems and this number is lower than when passivation is not considered. Thus, the availability obtained by considering passivation is higher than when passivation is not considered. Lau

et al. [7] studied dynamic availability considering passivation. In [7], the authors examined the relationship between utilization and availability instead of that between mission scenario and availability. Lau et al. [8] also investigated system availability considering the system damage.

This paper develops a method for evaluation of system stationary availability considering mission scenario. We outline a technique for transforming mission scenario to system requirements. This technique is demonstrated in an example on calculation of system availability based on the feedback theory [5, 6]. From the example, we can see that the method works well with different parameters.

2. Basic Principle

The real time value of $A_o(t)$ is determined by factors like system design characteristics, operation and maintenance policy. Based on the experience of simulation, we have observed that the real time value of $A_o(t)$ dynamically changes with time and its final value tapers off to a stationary value as $t \rightarrow \infty$. When not considering management delays, $A_o(t)$ can be expressed as follows:

$$A_o(t_n) = f(\lambda(t_n), r(t_n), b(t_n)) \quad (1)$$

where $\lambda(t_n)$ is the failure rate at time t_n ; $r(t_n)$ is the repair rate at t_n ; $b(t_n)$ is the number of items backordered at t_n ; and $n \geq 1$ is the time interval taking integer values.

Form formula (1), we can see that $Ao(t_n)$ is determined by the values of system failure rate, repair rate and the expected backorders at t_n . When the failure rate and the repair rate have constant values, $Ao(t_n)$ is a function only of $b(t_n)$. Actually, $b(t_n)$ is determined by $Ao(t_{n-1})$ at time t_{n-1} . So formula (1) can be written as:

$$Ao(t_n) = g(\lambda, r, Ao(t_{n-1})) \quad (2)$$

In equation (2), t_0 denotes the initial time when $n=1$. Suppose that $Ao(t)$ in stationary state at t_{n-1} . If $Ao(t_{n-1})$ has a little increase from interval t_{n-1} to interval t_n , $b(t)$ also increases if $b(t_{n-1}) > 0$. Because the changing tendencies of $Ao(t_n)$ and $b(t_n)$ are opposite, $Ao(t)$ will decrease in the next time interval. On the contrary, if $Ao(t_{n-1})$ has a little decrease from the interval t_{n-1} to the interval t_n , then the decreasing tendency of $Ao(t)$ can be derived. Thus, it can be seen from equation (2) that, along with the increase of n , the difference between $Ao(t_{n-1})$ and $Ao(t_n)$ is gradually decreasing and in the end, the stabilization state is obtained. $Ao(t_{n-1})$ and $Ao(t_n)$ form a negative feedback system, as shown in Fig. 1.



Fig. 1. Feed back sketch map of Ao

In Fig. 1, ‘-’ denotes the comparison operator. The output signal after the comparison represents the difference between $Ao(t_{n-1})$ and $Ao(t_n)$. When the system failure rate is larger than the system repair rate, $Ao(t)$ is decreasing with time. Contrary, if the system failure rate is smaller than the system repair rate, $Ao(t)$ is increasing with time. The systems continuously executing missions constitutes sequential regulation actions which decrease the difference between $Ao(t_{n-1})$ and $Ao(t_n)$ according to the theory mentioned above. In order to final the stationary $Ao(t)$, the direction of adjustment in function (1) should be toward decreasing the difference between $Ao(t_{n-1})$ and $Ao(t_n)$. A sketch of this iterative process is displayed in Fig. 2.

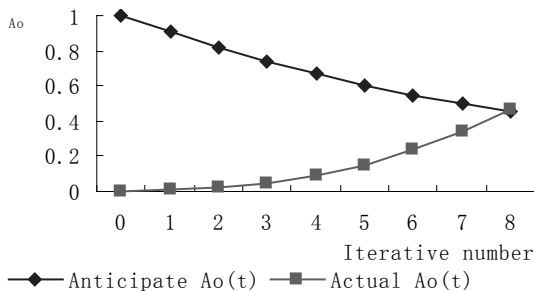


Fig. 2. The iterative processes of Ao

In Fig. 2, the upper curve is the expected value of $Ao(t)$ and the lower line is the current value of $Ao(t)$. As has been noted previously, the expected value of $Ao(t)$ can be seen as $Ao(t_{n-1})$ and the current value of $Ao(t_{n-1})$ can be seen as $Ao(t_n)$. According to the theory mentioned above, the difference between these two values is getting closer and closer. The relationship between

$Ao(t)$ and mission scenarios as well as spares inventory levels can be established, too.

3. Evaluation Model

3.1. Calculating $Ao(t)$

$Ao(t)$ can be seen as the ratio between the number of available systems and the number of nominal systems at time t . When spare backorders take place, a number of systems will be unavailable because of the shortage of spares. Furthermore, spare backorders take place randomly within systems under the condition of no interference. Thus, $Ao(t)$ can be calculated by [12]:

$$Ao(t) = \frac{1}{\frac{1}{A_s(t)} + \frac{1}{A_M(t)} - 1} \quad (3)$$

where $A_s(t)$ is the supply availability and $A_M(t)$ is the maintenance availability. The expression of $A_s(t)$ and $A_M(t)$ are given by [12]:

$$A_s(t) = \prod_{i=1}^I \{1 - EBO_i(t | s_i) / (N \cdot Z_i)\}^{Z_i} \quad (4)$$

$$A_M(t) = \frac{MTBM}{MTBM + MCMT + MPMT} \quad (5)$$

where $MTBM$ is the mean time between maintenance; $MCMT$ is the mean time between corrective maintenance; $MPMT$ is the mean time between preventive maintenance; $EBO_i(t | s_i)$ is the expected backorders of item i at time t ; s_i is the stock level of item i ; N is the number of systems; Z_i is the number of item i per system; i is the index number; and I is the total number of items. $EBO(t)$ can be evaluated by:

$$EBO(t) = \sum_{k=s+1}^{\infty} (k-s) \cdot \frac{e^{-d(t)T} \cdot (d(t) \cdot T_{AT})^k}{k!} \quad (6)$$

where $d(t)$ is the demand rate of spares; T_{AT} is the lead time for repairable items or the turn round time for discardable items and it is referred to as transport time in this paper.

3.2. Calculating $A_s(t)$

Lemma 1 [4]: Suppose that the non-homogeneous Poisson input intensity function has the form $\lambda(t) \geq 0$ ($t \geq 0$) and the non-stationary service time distribution is denoted by $G(t)$. Then the number of arrivals undergoing service at time t has a Poisson distribution with mean $\Lambda(t) = \int_0^t (1 - G(v)) \lambda(v) dv$.

The expected number of backorders of items i at time t can be written as:

$$EBO_i(t) = EBO_i(s | \Lambda_i(t)) = \sum_{k=s+1}^{\infty} (k-s) \cdot \frac{e^{-\Lambda_i(t)} \cdot [\Lambda_i(t)]^k}{k!} \quad (7)$$

$$A_s(t) = \prod_{i=1}^I \{1 - EBO_i(t) / (N \cdot Z_i)\}^{Z_i} \quad (8)$$

The value of $A_s(t)$ is dependent on $d(t)$ and $EBO(t)$. Moreover, $EBO(t)$ is dependent on $d(t)$. Thus, when $t \rightarrow \infty$, the stationary value can be expressed by:

$$EBO_i(t|s) = \sum_{k=s+1}^{\infty} (k-s) \cdot \frac{e^{-d_i(t)T_{AT}} \cdot (d_i(t)T_{AT})^k}{k!} \quad (9)$$

$$A_s(t) = \prod_{i=1}^I \left\{ 1 - EBO_i(t|s) / (N \cdot Z_i) \right\}^{Z_i} \quad (10)$$

3.3. Calculating $A_M(t)$

If a system has two possible states: available and unavailable, then the state transition diagram between these two states can be depicted by use of λ and μ . It is shown in Fig. 3.

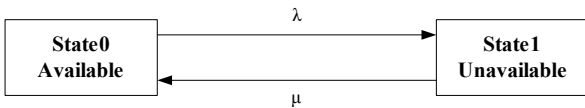


Fig. 3. The state transition diagram of the system

According to Fig. 3, $P_o(t)$ can be calculated by [3]:

$$P_o(t) = \frac{\mu}{\mu + \lambda} + \frac{e^{-(\lambda + \mu)t}}{\lambda + \mu} [\lambda P_o(0) - P_D(0)] \quad (11)$$

where $P_o(t)$ is system availability at time t and $P_D(0)$ is the system un-availability at time 0.

$$P_o(0) + P_D(0) = 1 \quad (12)$$

If $P_D(0)=1$, then

$$P_o(t) = \frac{\mu}{\mu + \lambda} + \frac{\lambda e^{-(\lambda + \mu)t}}{\lambda + \mu} \quad (13)$$

$$= \frac{\mu}{\mu + \lambda} + \left(P_o(0) - \frac{\mu}{\lambda + \mu} \right) e^{-(\lambda + \mu)t}$$

The value of $A_M(t)$ with nonhomogeneous Poisson process is:

$$A_M(t + \Delta t) = \frac{\mu}{\mu + \lambda(t + \Delta t)} \quad (14)$$

$$+ \left(A_M(t) - \frac{\mu}{\lambda(t + \Delta t) + \mu} \right) e^{-[\lambda(t + \Delta t) + \mu]\Delta t}$$

3.4. Transforming system mission scenario

Because the stationary availability is obtained on the basis of a given system mission scenario, we need to consider the mission requirements in its evaluation. We can first transform the mission scenario to system utilization rate. If the number of systems required to perform a mission is $M (1 \leq M \leq N)$ and at any time t the number of available systems $N (N \geq 1)$ is known. Since M systems forms a group, the number of groups that can perform missions and the corresponding probability can be obtained. Because N may not be exactly divisible by M , we propose the following analysis technique. Given M and N , there must exist an integer $k (k \geq 1)$ satisfying the following expression:

$$kM \leq N < (k+1)M \quad (15)$$

Obviously, in equation (15), we have

$$k \leq \frac{N}{M} < (k+1) \quad (16)$$

$$0 \leq \frac{N}{M} - k < 1$$

Set $q = NM - k (0 \leq q < 1)$. By the Bernoulli Lemma, the probability distribution of the number of groups is

$$P(g) = \begin{cases} 1-q & g = k \\ q & g = k+1 \\ 0 & other \end{cases} \quad (17)$$

where g is the number of system groups.

Equation (17) shows that the probability of having k groups is $1-q$ and the probability of having $k+1$ groups is q . Suppose that the stationary value of $A_o(t)$ under each of these two numbers of groups can be obtained. Using the probability values derived in equation (17), the weighted average of $A_o(t)$ can be obtained by summing the products of the stationary value of $A_o(t)$ and its corresponding probability.

4. Example

4.1. Given data

Suppose that there are 8 airplanes that may be used to perform a mission and the mission profile requires 3 airplanes for each trip and 4 trips per day. The pertinent data and the inventory levels of the items are shown in Tab. 1.

Tab. 1. Given data

Item ID	1/λ (hours)	1/μ (hours)	T _{AT} (days)	Number of Items per System	Inventory Level
LRU1	829	1	45	4	6
LRU2	850	1	30	4	7
LRU3	829	1	45	3	6
LRU4	364	1	45	6	12
LRU5	1020	1	45	4	5
LRU6	753	1	45	3	8
LRU7	1262	1	30	3	6
LRU8	700	1	30	4	8

4.2. Results

Through equations (3)~(17), the values of stationary $A_o(t)$ changing with the number of systems and the value of T_{AT} can be obtained. Based on the data in Tab. 1, the result of stationary $\lim_{t \rightarrow \infty} A_o(t) = 0.394$, in other words, there are about 3.15 airplanes available on average. Using Simlox 2.0, the value of dynamic $A_o(t)$ can be obtained and it is shown in Fig. 4. In Fig. 4, the curve denotes the real value of $A_o(t)$ and it can be seen that this curve fluctuates with time and its final value approaches its stationary value. By comparing with simulation results, we find that the results derived by analytical calculation and simulation are pretty close and the relative difference between these two results is about 20%.

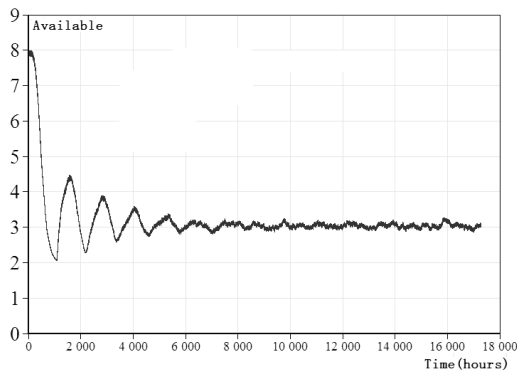


Fig. 4. The A_o changes with time

4.3. Sensitivity analysis

Based on the theory mentioned above, the relationship between stationary $A_o(t)$ and T_{AT} , number of systems, and repair time can be established. The effect of changing one or two of these variables on the stationary $A_o(t)$ can be analyzed. In this paper, the T_{AT} and the number of systems are selected as the required parameters and T_{AT} is adjusted through scaling, that is, the new value of T_{AT} equals to the old value of T_{AT} multiplied by the scale factor. The stationary values of $A_o(t)$ changing with the adjustment of these two factor values are shown in Tab. 2.

Tab. 2. The calculation results of stationary A_o

A_o		the Number of Systems performing mission					
		8	10	12	14	16	18
scale factor of T_{AT}	2.0	0.095	0.112	0.128	0.143	0.158	0.172
	1.8	0.120	0.139	0.158	0.177	0.193	0.208
	1.6	0.158	0.181	0.200	0.223	0.243	0.261
	1.4	0.208	0.235	0.261	0.283	0.305	0.326
	1.2	0.282	0.315	0.344	0.369	0.393	0.416
	1.0	0.394	0.432	0.467	0.492	0.516	0.540
	0.8	0.564	0.602	0.632	0.657	0.679	0.699
	0.6	0.785	0.815	0.840	0.855	0.868	0.879

6. References

1. Alfredsson P. Optimization of multi-echelon repairable item inventory systems with simultaneous location of repair facilities. *European Journal of Operational Research* 1997; 99: 584-595.
2. BHU. SMMS User's Reference, Beijing: Department of System Engineering of Engineering Technology of Beihang University, 2006.
3. Billinton R, Allan R. *Reliability Evaluation of Engineering Systems*. Plenum Press: New York, 1982.
4. Carrillo M J. Extensions of Palm's Theorem: a Review. *Management Science* 199; 37: 739-744.
5. Huang H Z, Gu Y K. Modeling the product development process as a dynamic system with feedback. *Concurrent Engineering: Research and Applications* 2006; 14: 283-291.
6. Huang H Z, Gu Y K. Product development process modeling based on information feedback and requirement cooperation. *Concurrent Engineering: Research and Applications* 2006; 14: 87-98.
7. Lau H C, Song H W, See C T, Cheng S Y. Evaluation of time-varying availability in multi-echelon spare parts systems with

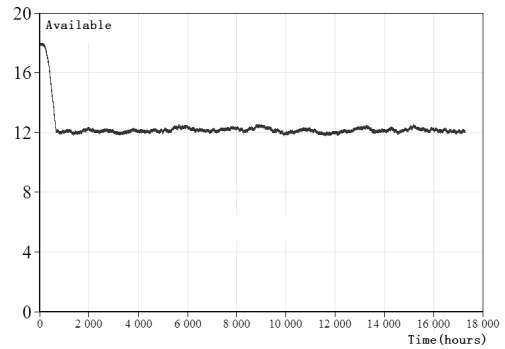


Fig. 5. The $A_o(t)$ changes with time

In Tab. 2, it can be seen that the stationary $A_o(t)$ increases with the increasing of the number of systems and also with the increasing of T_{AT} . If the scale factor of T_{AT} is set at 0.6, according to Tab. 2, the stationary $\lim_{t \rightarrow \infty} A_o(t) = 0.879$, which is shown in

Fig. 5. After a comparison, it can be seen the relative difference between the analytical calculation value and the simulation value is about 15.7%.

Through a sequence of different scale factors, a number of stationary $A_o(t)$ can be obtained by simulation. Comparing analytical results with these simulation results, it can be seen that, first, the differences is larger when the number of systems is smaller and the differences is getting smaller when the number of systems is increasing. Second, the time required to reach the stationary $A_o(t)$ is larger when the T_{AT} is larger. Because the repair time of faulty items is pretty short in this example, it has little effect on the stationary $A_o(t)$. On the contrary, if the repair time is becoming larger, its effect on the stationary $A_o(t)$ can not be neglected.

5. Conclusions

System design parameters, operation requirements and maintenance effects are all critical factors affecting the stationary $A_o(t)$. In this paper, we have evaluated the stationary $A_o(t)$ considering passivation and mission requirements. First, the shortcomings of separate analytical and simulation methods can be avoided. Second, the model presented in this paper can establish the stationary $A_o(t)$ as a function of system design parameters, operation requirements, and maintenance effects. Thus, it is very useful for designers and operators to evaluate stationary $A_o(t)$ considering these factors. At last, for further research, the model can be expanded to include other factors, such as support equipment, and therefore, the model's application scope can be further extended.

- passivation. *European Journal of Operational Research* 2006; 170: 91-105.
8. Lau H C, Song H. Evaluation of Time-Varying Availability in Multi-Echelon Inventory System with Combat Damage. Edmonton, 2005; IEEE Conference on Automation Science and Engineering (CASE), Canada.
 9. Nordin A, Maier F F. SPAREL: A Model for Reliability and Sparing in the World of Redundancies. Atlanta, 1989; The Annual Reliability and Maintainability Symposium.
 10. Sherbrooke C C. Metric: A Multi-Echelon Technique for Recoverable Item Control. *Operational Research* 1968; 16: 122-141.
 11. Sherbrooke C. C. Vari-Metric: Improved Approximations for Multi-Indenture, Multi-Echelon Availability Models. *Operations Research* 1986; 34: 311-319.
 12. Sherbrooke C C. *Optimal Inventory Modeling of Systems: Mutli-Echelon Techniques*, Second Edition. John Wiley & Sons: New York, 2004.
 13. Systecon AB. *Simlox2 User's References (version 2)*, Stockholm: Systecon AB, 2004.

Doc. Naichao WANG, Associate Professor

Prof. Rui KANG

Zhiyu JIA, Ph.D. student

Lichao WANG, Ph.D.

Department of System Engineering of Engineering Technology

BeiHang University

Beijing, 100191, China

email: tian_jia_zhuang@sina.com e-mail: kangrui@buaa.edu.cn
