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PRZYSPIESZONE BADANIA DEGRADACJI PRZY STAŁYM NAPRĘŻENIU W ANALIZIE DIOD SUPERELEKTROLUMINESCENCYJNYCH I WRAŻLIWOŚCI PARAMETRYCZNEJ

CONSTANT STRESS ADT FOR SUPERLUMINESCENT DIODE AND PARAMETER SENSITIVITY ANALYSIS

Ruchy Browna są jednym z najpotężniejszych procesów stochastycznych w ciągłym czasie i ciągłej przestrzeni, który ma też mocne podstawy fizyczne. W analizie przyspieszonych badań degradacji (ADT), rozkład odwrotny gaussowski, będący rozkładem czasu pierwszego przejścia ruchu Browna z dryfem (drift Brownian motion), staje się bardzo popularnym modelem predykcji statystycznej życia i niezawodności produktów. Diody superelektroluminescencyjne (SLD) o długiej żywotności i wysokiej niezawodności mają wiele zalet fizycznych, które sprawiają, że zastępują one diody laserowe (LD) oraz diody elektroluminescencyjne (świecące) (LED) i mają szerokie zastosowanie w czujnikach światłowodowych. W niniejszym artykule przeprowadzono badania ADT diody SLD przy stałym naprężeniu. Aby ocenić możliwość zastosowania rozkładu odwrotnego gaussowskiego do badań diod SLD, określono najpierw trwałość i niezawodność SLD na podstawie danych o spadku mocy optycznej uzyskanych z badania ADT prowadzonego przy stałym naprężeniu. Następnie przeprowadzono analizy wrażliwości parametrycznej w trzech wymiarach: niezawodności, czasu życia i parametru analitycznego. Wreszcie, kierując się wynikami analizy wrażliwościowej, przedstawiono niektóre zasady planowania i przeprowadzania testów ADT przy stałym naprężeniu.

Słowa kluczowe: *Analiza wrażliwościowa, ruchy Browna, obniżenie charakterystyk, badania przyspieszone, SLD.*

Brownian motion is one of the most powerful stochastic processes in continuous time and continuous space and has a good physics background. For the analysis of accelerated degradation testing (ADT), the inverse Gaussian (IG) distribution, which is the first passage time distribution of the drift Brownian motion (DBM), becomes a very popular statistical prediction model of product life and reliability. Instead of laser diode (LD) and Light Emitting Diode (LED), long-life and high-reliability super-luminescent diode (SLD) has many physical advantages and has been widely used in optical fiber sensors. In this paper, the constant stress ADT (CSADT) of SLD was conducted. In order to evaluate the applicability of IG distribution to SLD, we first estimate the life and reliability of SLD based on the optical power degradation data collected in CSADT. Then parameter sensitivity analyses are conducted in the 3-dimensions of reliability, lifetime and the analytic parameter. Finally, according to the sensitive analysis results, some CASDT planning and testing principles are presented.

Keywords: *Sensitivity analysis, Brownian motion, performance degradation, accelerated testing, SLD.*

1. Introduction

As the rapid development of technology, industrial companies have to manufacture high-reliability and long-life products in order to survive in the competitive market. One of the most important works in product design is to evaluate the lifetime and reliability of the product [3-6]. Now, there are two kinds of accelerated testing which can give these results: Accelerated life testing (ALT) and accelerated degradation testing (ADT). ALT involves collecting data of time-to-failures at different austere stress levels. But it is difficult to obtain adequate failure-time data to satisfy the requirement of ALT because of the highly reliable property of products. To overcome this problem, accelerated degradation testing (ADT) has been used. The feasibility of ADT is based on the fact that some physical characteristics or performance degrading value of a product will eventually lead to failures when the degradation measurement exceeds a pre-specified threshold. Thus, conducting accelerated degradation testing to identify a performance measurement would exhibit degradation and to monitor it over time, and product life and reliability can then be inferred from the degradation paths without the need of observing actual failures.

As far as we know, the earliest reported study on ADT was carried out on film resistance by two Japanese researchers, Shiomi and Yanagisawa, in 1979. From then on, Suzuki[15], Carey [1, 2], Tseng[17, 18], and Padgett [12, 13], respectively presented their DoE (Design of Experiment) and data analysis methods of ADT based on film resistance, MOS, logic chip, LED, carbolic fiber, etc.. All these achievements were successful and prompted the following ADT studies.

Super-luminescent diode (SLD) offers the advantage of broadband, high optical output, immunity to fiber-induced noise, less coherent and less sensitive to reflections. SLD is widely used as the light sources of Fiber Optical Gyroscope (FOG), optical time domain reflectors (OTDR), Optical Communication, Optical Coherence Tomography (OCT), etc. [19]. With the development of engineering and science technology, such optical devices will be designed to function for a long period of time before they fail. It is not an easy task to assess the lifetime distribution of the products by using the traditional life-testing procedures which record only time to failure data. Under this situation, constant stress ADT is to be used in this paper.

In the following, we first introduce the stochastic process for modeling degradation paths and life distribution. Then, CSADT of SLD is performed and its life and reliability estimations are obtained using the models. Thirdly, in order to determine the effects of model parameters on the reliability estimation, sensitivity analyses are conducted in three dimensions. Finally, the principles for SLD's ADT performance are discussed and conclusions are given.

2. Degradation model of CSADT

Degradation model reflects the relationship between some performance indicator and time. There are four typical methods to model degradation processes:

a) deterministic functions;
b) assuming that some parameters of a deterministic function follow some specified distributions [9, 11];
c) in addition to the deterministic functions, there is an error term which follows the normal distribution with mean zero and standard variation σ [20, 22];
d) similar to b), except for the use of stochastic process to describe the error [10, 16, 21-23].

In fact, if the degradation process is described by model a) described above, we will observe a very smooth deterioration curve of product performance. As long as the critical performance value is given, all products will fail at the same time. But there are unit-to-unit variability and variability due to operating and environmental conditions, deterioration curves have fluctuations. Consequently, the randomness must be considered in the model, and thus, models b), c) and d) are preferred. But for a time-dependent degradation process, a stochastic process, i.e. model d) seems to be more appropriate.

Brownian motion is one of the most powerful stochastic processes in continuous time and continuous space. In addition, Brownian motion is an essential ingredient in stochastic calculus and is essential for defining one of the most important classes of Markov processes, the diffusion processes, and solving large sample estimation problems in mathematical statistics. Brownian motion has wide applications in disciplines such as physics, economics, communication theory and reliability theory.

Based on the above analysis, a degradation process $Y(t)$ can be described by the following drift Brownian motion (DBM) model:

$$Y(t) = \sigma B(t) + g(t,s)\cdot t + y_0 \tag{1}$$

where $B(t)$ is a standard Brownian motion on $[0,\infty)$, and y_0 is the initial degradation level at time zero, and σ is the dispersion and describes unit-to-unit variability due to operating and environmental conditions [11]. So it is assumed that the dispersion parameter does not change with stress. In order to establish the relationship between the degradation process $Y(t)$ and the stress loaded on product, we assume that the term $g(t,s)\cdot t$ can be written as the product of the stress function $d(s)$ and the time scale function $\tau(t)$, i.e., $g(t,s)\cdot t = d(s)\cdot\tau(t)$ and $\tau(t)=ln(t)$ in this paper. Hence, we could regard $d(s)$ as the drift parameter of DBM $Y(t)$. In engineering, the drift parameter is commonly known as the degradation rate, so it is an accelerated model.

By linearizing the DBM in equation (1), we can obtain the pdf of its first passage time distribution:

$$f(t;y_0,a) = \frac{a-y_0}{\sigma\sqrt{2\pi\tau^3(t)}}\exp\left\{-\frac{\left[(a-y_0)-d(s)\cdot\tau(t)\right]^2}{2\sigma^2\tau(t)}\right\} \tag{2}$$

We will call equation (4) the inverse Gaussian distribution (IG), and the corresponding reliability function is:

$$R(t)=\Phi\left[\frac{a-y_0-d(s)\tau(t)}{\sigma\sqrt{\tau(t)}}\right]-\exp\left(\frac{2d(s)(a-y_0)}{\sigma^2}\right)\Phi\left[-\frac{a-y_0+d(s)\tau(t)}{\sigma\sqrt{\tau(t)}}\right] \tag{3}$$

where Φ is the standard normal cdf.

According to the conclusions of FMEA and FTA, we know that under the effect of temperature, the failure of a chip and the alignment of the chip to pigtail will lead to the degradation of the optical power of SLD. And this is the critical degradation failure mode of SLD [24].

Based on the above analyses, we will assume that the drift parameter $d(s)$ follows Arrhenius reaction rate model and with the symbol T substituting s, we obtain:

$$d(T) = A\cdot\exp\left(\frac{E_a}{kT}\right) \tag{4}$$

where T is the absolute Kelvin temperature with unit K; k is Boltzmann's constant, $k = 8.6171\times10^{-5}$ eV/K; E_a is the activation energy with unit eV, and A is a constant. Generally, we write $B=E_a/k$, then $d(T) = A\cdot\exp(-B/T)$.

3. CSADT and Analysis of SLD

3.1. CSADT of SLD

3-level CSADT of SLD is constituted of the following [7]:

a. Determining equipment capacity for temperature control.
In order to make an efficient and reasonable testing plan, we should obtain the accuracies of temperature control, rate of temperature change, stable time of temperature, temperature distribution, etc.

b. Determining SLD temperature operation limits (TOL).
Provide evidence to SLD CSADT, including temperature characteristics and the operation limits of SLD.

c. CSADT of SLD.
There are 15 SLDs and they are divided into 3 groups in 3-level CSADT. The corresponding testing parameters are given in Tab. 1.

Tab.1. CSADT parameters of SLD

Temperature	Value	Number of Measure	Value	Sample Size	Value
T_1	60°C	M_1	4000	n_1	5
T_2	80°C	M_2	3000	n_2	5
T_3	100°C	M_3	2000	n_3	5

During CSADT of SLD, we recorded the optical power every 0.5h, i.e., the interval of performance inspection Δt is 0.5h.

3.2. Result of SLD CSADT

By testing of SLD TOL, temperature characteristics of SLDs have been collected and shown in Figure 1.

The optical power of SLD on different stress levels 60ºC, 80ºC and 100ºC are shown in Figure 2.

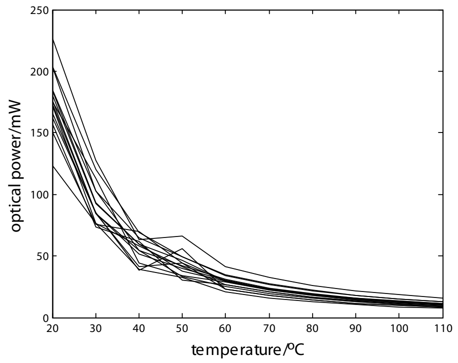

Fig. 1. Temperature characteristics of SLDs

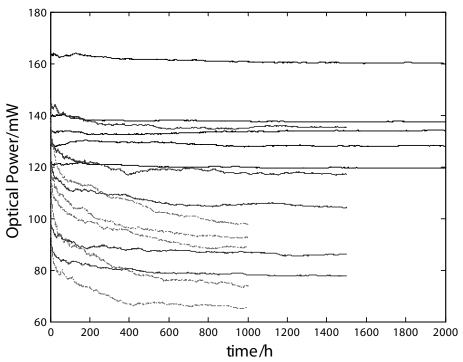

Fig. 2. The optical power of SLDs in CSADT (a)

From Figure 2, on the effect of temperature characteristics, we can note that there are different initial degradation levels at time zero. Through determination of SLD TOL, we need to eliminate this effect. And the eliminated optical powers are shown in Figure 3.

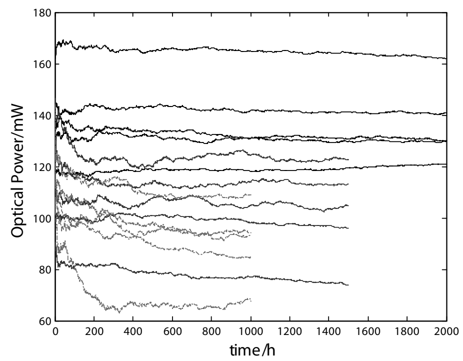

Fig. 3. The optical power of SLD in CSADT (b)

Armed with the above models and the methods of Li [8], we can obtain the following parameter estimations.

Tab. 2. Parameter estimations

parameter	A	Ea	σ
estimations	9.295	0.487	0.4904

Then, the estimated reliability of SLD is shown in Figure 4.

From the above reliability curve, we know that the reliability corresponding to the operation life 5×10^4 hours is 0.9158.

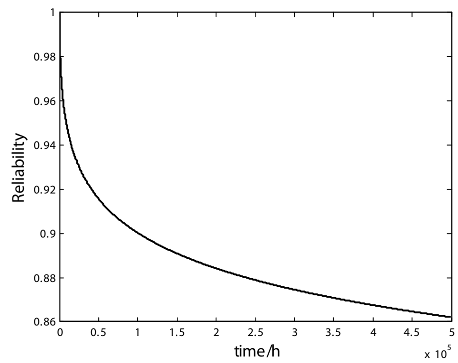

Fig. 4. The estimated reliability of SLD

4. Parameter Sensitivity Analysis

Generally, we will obtain different estimated results by different estimation methods, samples, performance measuring techniques, etc. In this paper, we utilize the DBM to describe the optical power degradation of SLD. In order to determine the stability of the DBM on SLD and propose some useful advice for the development and the CSADT implementation of SLD, we conduct parameter sensitivity analysis.

4.1. Sensitivity analysis of the parameter A

In this section, we consider the effect of the parameter *A* on the reliability of SLD. Let the activation energy E_a and the dispersion σ remain at the original estimation values as shown in Table II. Among the 3 dimensions of the parameter *A*, time *t,* and reliability of SLD, there is a curved surface which depict the reliability change with the parameter *A* and time *t* simultaneously, as shown in Figures 5 & 6.

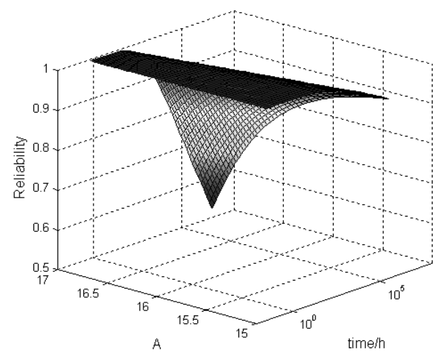

Fig. 5. The curved surface which depicts the reliability change with parameter A and time t simultaneously (Absolute change)

To understand better the sensitivity of the reliability to the parameter *A*, the reliabilities of SLD when *A* decreases and increases by 50% from the value listed in Table 2 are given in Table 3.

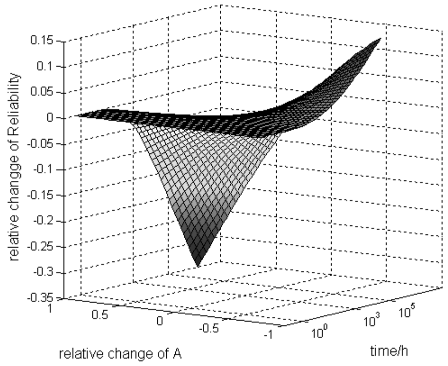

Fig. 6. The curved surface which depicts the reliability change with parameter A and time t simultaneously (Relative change)

Tab.3. Parameter A and SLD reliability

A	50% $\hat{A}$ =4.6475	150% $\hat{A}$ =13.9425
R(50000)	0.9573	0.8492

Based on Figures 5 and 6, SLD reliability increases with the parameter A decreasing. And this trend enlarges with time increasing. But between -100%A~+100%A, the relative change of SLD reliability is between +15%~-35%. Thus, the parameter A slightly affects SLD reliability.

4.2. Sensitivity analysis of the activation energy E_a

Similarly, we consider the effect of the activation energy E_a on the reliability of SLD in this section. Let parameter A and the dispersion σ remain at the original estimation values as listed in Table II, among the 3-dimensions of the activation energy E_a, time t and the reliability of SLD, there is a curved surface which depicts the reliability change with the activation energy E_a and time t simultaneously, as shown in Figure 7 & 8.

When the activation energy E_a increases by 50% and decreases by 10%, the corresponding reliabilities of SLD are shown in Table 4. From Figures 7 and 8, we can see that if the activation energy E_a is less than 0.4, the reliability will be approaching zero in a very short time.

Tab. 4. The activation energy Ea and SLD reliability

E_a	$\hat{E}_a$ ×0.9=0.4383	$\hat{E}_a$ ×1.5=0.7305
R(50000)	0.0087	0.9804

From Figures 7 & 8, SLD reliability increases with the activation energy E_a. This trend changes sharply when E_a is in the range of 0.4~0.6. In this range, with E_a increasing, the decrease of SLD reliability with time slows down. For example, when E_a decreases by 20% from the value given in Table II, the SLD reliability decreases from 1 to 0 in 5 hours. When E_a increases by 20%, the SLD reliability just decreases from 1 to 0.975 in 100 thousand hours. However, the SLD reliability changes very slowly with time when E_a > 0.6. Thus, the SLD reliability is very sensitive to the activation energy E_a in the range of 0.4~0.6.

We can explain this phenomenon from the chemistry perspective. Activation energy, also called midnight energy, is defined as the minimum energy necessary for a specific chemical reaction to occur. Usually one can think of the activation energy as the height of the potential barrier (sometimes called the energy barrier) separating two minima of potential energy (of the reactants and of the products of reaction). For SLD, the activation energy can be denoted as the energy barrier which SLD needs to make a transition from the normal state to the failure state. The smaller the activation energy is, the easier the transition is.

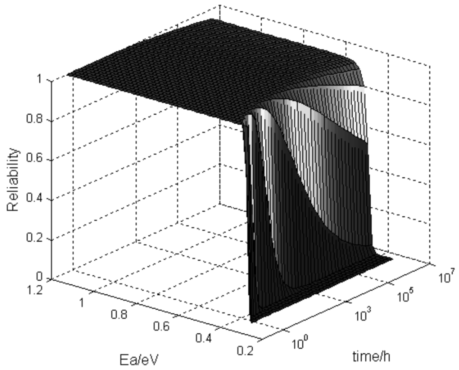

Fig.7. The curved surface which depicts the reliability change with the activation energy Ea and time t simultaneously (Absolute change)

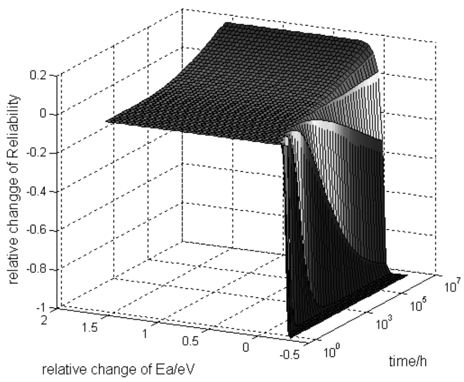

Fig. 8. The curved surface which depicts the reliability change with the activation energy Ea and time t simultaneously (Relative change)

4.3. Sensitivity analysis of dispersion σ

We also consider the effect of the dispersion σ on the reliability of SLD in this section. Let the activation energy E_a and parameter A remain at the original estimated values as shown in Table II. Among the 3-dimensions of the dispersion σ, time t and the reliability of SLD, there is a curved surface which depicts the reliability change with the dispersion σ and time t simultaneously, as shown in Figures 9 & 10.

From Figures 9 &10, the SLD reliability decreases with the increase of the dispersion σ. With the increase of σ, the reliability decreases more rapidly with time.

In this case, we can observe that the dispersion σ does not affect the reliability as much as the other two parameters. This is because the optical power is a function of ln(t). Under this logarithm relationship, the variability of the degradation process which is modeled by the DBM changes very slowly, as shown in Figure 11. From Figure 11, we know that even after 0.5 million hours, $\sigma^2 ln(t)$ is still very close to 3.5. If the relationship between product performance and time is linear (as shown in Figure 12), the variability $\sigma^2 t$ rapidly magnifies to 2.5×10^5!

Hence, the sensitivity of the reliability to the dispersion σ is relative to the relationship between performance and time. The more remarkably performance changes, the more sensitive the reliability is to the dispersion σ.

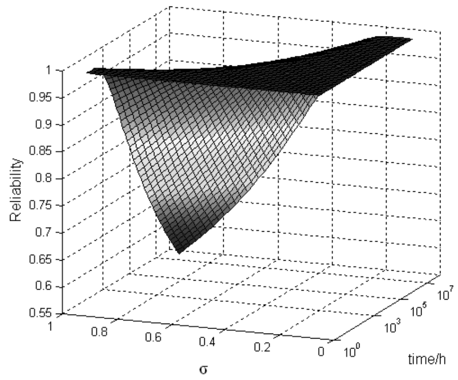

Fig. 9. The curved surface which depicts the reliability change with dispersion σ and time t simultaneously (Absolute change)

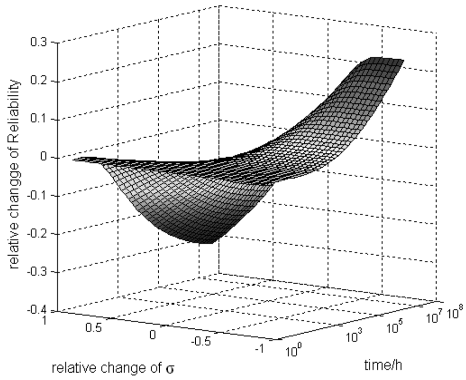

Fig. 10. The curved surface which depicts the reliability change with dispersion σ and time t simultaneously (Relative change)

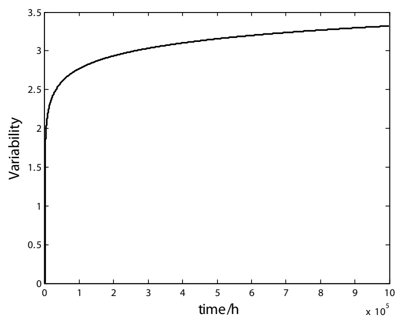

Fig. 11. $\sigma^2 ln(t)$

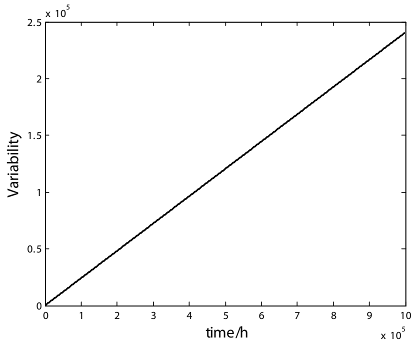

Fig.12. $\sigma^2 t$

5. Concluding Remarks

This paper deals with the statistical analysis and sensitivity analysis for the CSADT of SLD. On the base of the above investigation, we can get the following conclusions:

a. The SLD reliability increases with the activation energy E_a increasing, the dispersion σ decreasing, and the parameter A decreasing. These characteristics could be derived by taking partial derivatives of the IG distribution with respect to E_a, σ and A, respectively.

b. Among the three parameters E_a, σ and A, A has the smallest effect on the SLD reliability. On the direction of the reliability decreasing, the reliability is more sensitive to the activation energy E_a than to the dispersion σ. We can interpret the above conclusions from the perspective of Arrenhius accelerated model. For the parameter A, it is just a constant coefficient without unit and influences the Arrhenius reaction rate i.e. the drift parameter of the DBM by multiplying. The activation energy E_a is an exponential part of the Arrenhius accelerated model. Obviously, the effect of the E_a is greater than that of A. Similarly, in equation (3), when the term $d(s)$ was substituted with equation (4), then E_a has the same exponential effect on the reliability, while the dispersion σ is a divisor.

c. When the activation energy E_a increases to some value, it does not affect the reliability as much any more (as shown in Figures 7 & 8).

d. The reliability sensitivity to the dispersion σ depends on the relationship between the product performance and time. The IG distribution is derived from the first passage time of DBM. The variance of DBM will badly influence the estimation of the IG distribution. If the variance of the degradation data collected during ADT is very big, then it will be enlarged by time because of $Var(Y(t))= \sigma^2\tau(t)$. this conclusion is very easy to understand. We just use the degradation data collected during only 3000h ADT to extrapolate the reliability for the 100000h life of the SLD, so the extrapolation extent is great. Even there is a little variation within the degradation data, it will cause a huge fluctuation in the reliability estimation in the future. Hence, in order to improve the accuracy of reliability prediction, we should take some actions, such as:
   - Ensuring the consistency of SLD. We must attach more importance to manufacturer selection.
   - Controlling the SLD manufacturing process. If there is little room to select a manufacturer, then we can specify some requirement for controlling the manufacturing process;
   - improving the screening technology and being strict with the selecting requirements of SLD;
   - Ensuring the consistency of ADT conditions. These include the monitoring of testing stress, consistent determination of the performance of detecting devices and relative environmental factors.

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6. References

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