

# ANALIZA FUNKCJI OPISUJĄCEJ KOSZTY W DYNAMICZNYM PROJEKTOWANIU W WARUNKACH NIEPEWNOŚCI

## COST-TYPE FUNCTION ANALYSIS IN DYNAMIC DESIGN UNDER UNCERTAINTY

*Dynamiczne projektowanie w warunkach niepewności uwzględniające problem cyklu życia staje się coraz bardziej atrakcyjnym podejściem w projektowaniu inżynierskim. Jednym z ważniejszych zadań jest obliczenie na etapie projektowania kosztu cyklu życia, który można wykorzystać jako funkcję celu lub jako ograniczenie. Koszt cyklu życia to suma wszystkich kosztów poniesionych podczas cyklu życia produktu, wliczając w to koszty projektu, rozwoju, produkcji, eksploatacji, obsługi, wspomagania obsługi oraz likwidacji. W artykule analizujemy kilka modeli kosztu cyklu życia i ilustrujemy ich zastosowania. Następnie, biorąc pod uwagę modele kosztu cyklu życia, budujemy modele projektowania zorientowanego na niezawodność (design-for-reliability models), projektowania w granicach zadanej niezawodności (design-to-reliability models) oraz projektowania odpornego (robust design model). Modele te, poprzez uwzględnienie problemów związanych z cyklem życia, mogą pomagać inżynierom w przygotowaniu niezawodnych i odpornych projektów produktów bądź systemów.*

**Słowa kluczowe:** Dynamika, niezawodność, koszt cyklu życia, projektowanie w warunkach niepewności.

*Dynamic design under uncertainty considering lifecycle issue has become more and more attractive in the engineering design. One of more important task is to calculate the life cycle cost in the design, which used as objective function or constraints. Life cycle cost is the total cost incurred in the life cycle of a product, including design, development, production, operation, maintenance, support and disposal cost. In this paper, we analyze several life cycle cost models and illustrate their applications. Then we build dynamic design-for-reliability, design-to-reliability and robust design models by considering the life cycle cost models. These models can help engineers to make a reliable and robust design for a product or system by considering the life cycle issues.*

**Keywords:** Dynamic, reliability, life cycle cost, design under uncertainty.

### 1. Introduction

One of more important task of dynamic design under uncertainty is to calculate the life cycle cost from design to disposal of a product or system. Due to the characteristics of different products or systems, many life cycle cost analysis methods and models have been developed. Commonly used methods comprise four categories: (1) intuitive methods; (2) analogical methods; (3) parametric methods; and (4) analytical methods [3, 4, 12]. Inspections and maintenance are important interventions to maintain or restore the product or systems in an operating state [6, 7, 8] and produce cost. Since maintenance can affect the reliability during the operating stage, life cycle cost under inspection and maintenance should be accounted for at the early design stage [9, 14].

The organization of the paper is as follows. In Section 2, life cycle cost definition and functions are given. Several life cycle cost methods are analyzed in Section 3. In Section 4, some life cycle cost models and their applications are provided. Some dynamic design models under uncertainty considering life cycle cost are given in Section 5. In Section 6, conclusions are given.

### 2. Life cycle cost definition and functions

The life cycle cost (LCC) concept was originally applied by U.S. Department of Defense (DoD) [1]. LCC is the summation of the costs incurred during the whole life cycle including the

process and logistic support life cycle. LCC is categorized into: Company Cost, Users Cost, and Society Cost according to different life cycle stages [2], shown in Tab. 1.

### 3. Cost estimation method

Duverlie and Castelain propose four different methods for estimating cost [4].

The intuitive method. The cost estimation is based on the experience of the estimators. As different estimators have different experience and knowledge, a big difference may be produced among the estimation results.

The analogical method. The cost estimation is conducted in terms of the similarities between a set or system and the existing sets or systems.

The parametric method. Parametric estimating employs equations that describe relationships between cost and measurable attributes of a set or system. Parametric estimating is often referred to as a 'top-down' estimating method [3].

The analytical method. This method is called detailed model in Ref. [3]. A description that the direct cost of a set or system is calculated by estimating the labor time, labor rate, material quantities, and material prices is given in Ref. [12]. This method is known as the bottom-up technique.

Different methods cover different stages in the life cycle, see Table 2.

Tab. 1. Life cycle stages and costs

	Company Cost	User Cost	Society Cost
Design	Market Recognition, Development		
Production	Materials, Energy, Facilities, Wages, Salaries		Waste, Pollution, Health Damages
Usage	Transportation, Storage, Waste, Breakage, Warranty Service	Transportation, Storage, Energy, Materials, Maintenance	Packing, Waste, Pollution, Health Damages
Disposal/Recycling		Disposal/ Recycling Dues	Waste, Disposal, Pollution, Health Damages

Tab. 2. Use of cost estimation methods

Feasibility	Definition	Development	Production	Utilization	After sale service
	Analogical				
←		→			Analogical
←	→		Analytical		←
		←			→

Scanlan *et al.* propose a cost model for aircraft optimization [11]. The design space is partitioned into three discrete levels of abstraction. A parametric cost estimating method is used and a crude estimation of cost is generated at level 1. At level 3, an analytical method is used and process times are generated with the detailed product definition. The hybrid between a parametric and analytical method where cost expressions are derived from level 3 is used at level 2. Furthermore, the cost expressions can be delivered to level 1 from level 2. Then this cost estimating system forms a bottom-up configuration.

**4. Several cost models and their applications**

Under the assumption that the number of structural limit states is small and the severe hazards causing the limit states occurring are not frequent, the expected total cost during time *t* can be expressed by [14]

$$E[C(t, X)] = C_0(X) + \left[ E \left\{ \sum_{i=1}^{N(t)} \sum_{j=1}^k C_j e^{-\lambda_j t} P_j(X, t_i) \right\} \right] + \int_0^t C_m(X) e^{-rt} dt \quad (1)$$

where X is the vector of design variables; *C*<sub>0</sub> is used to represent the initial cost, which is a function of X; *i* represents the number of severe loading occurrences; *t*<sub>*i*</sub> is used to represent the loading occurrence time; *N*(*t*) is the total number of severe loading occurrences in *t*; *C*<sub>*j*</sub> is the cost in present dollar value of the *j*th limit state being reached; *e*<sup>-λ<sub>*j*</sub>*t*</sup> is the discounting factor over time *t*; *P*<sub>*j*</sub> is the probability of the *j*th limit state being exceeded given the *i*th occurrence of a single hazard or the joint occurrence of different hazards; *k* is the total number of limit states under consideration; and *C*<sub>*m*</sub> is the operation and maintenance cost per year.

Sarma and Adeli propose a life cost model based on steel structures and a fuzzy discrete multi-criterion method is implemented to deal with life cycle cost optimization [10].

$$C_{LC} = C_I + \sum \frac{1}{(1+r)^{y_{n1}}} C_M + \sum \frac{1}{(1+r)^{y_{n2}}} C_{IN} + \sum \frac{1}{(1+r)^{y_{n3}}} C_R + \sum \frac{1}{(1+r)^{y_{n4}}} C_O + \frac{1}{(1+r)^{y_{n5}}} C_F + \frac{1}{(1+r)^{y_{n6}}} C_D \quad (2)$$

where *C*<sub>LC</sub>, *C*<sub>*I*</sub>, *C*<sub>*M*</sub>, *C*<sub>IN</sub>, *C*<sub>*R*</sub>, *C*<sub>*O*</sub>, *C*<sub>*F*</sub>, and *C*<sub>*D*</sub> are the total life cycle, initial, annual maintenance, inspection, repair, operating, probable failure and dismantling costs of the steel structure, respectively; *r* is the discount rate considering the time value of money; and *y*<sub>*n1*</sub>, *y*<sub>*n2*</sub>, *y*<sub>*n3*</sub>, *y*<sub>*n4*</sub>, *y*<sub>*n5*</sub>, and *y*<sub>*n6*</sub> are the years when the associated costs are incurred.

Based on the cost-benefit-risk analysis on a bridge, the total life cycle cost up to time *t*<sub>*N*</sub> and the average annuity cost during the design life of the structure are presented by [13].

$$LCC(t_N) = C_I + C_{QA} + \frac{\sum_{i=1}^{t_N} [C_{IN}(t_i) + C_M(t_i) + C_R(t_i) + \sum_{LS=1}^M P_{jLS}(t_i) C_{jLS}]}{(1+r^i)} \quad (3)$$

and

$$C_A = \sum_{j=1}^N \frac{P_j(t_j) r [C_I + C_{QA} + C_{IN}(t_j) + C_M(t_j) + C_R(t_j)]}{1 - (1+r)^{-t_j}} \quad (4)$$

where *C*<sub>*I*</sub> is the design and construction cost; *C*<sub>QA</sub> is the cost of quality assurance/control; *C*<sub>IN</sub>(*t*) is the expected cost of inspection; *C*<sub>*M*</sub>(*t*) is the expected maintenance costs; *C*<sub>*R*</sub>(*t*) is the expected repair costs; *M* is the number of failure limit states; *P*<sub>*jLS*</sub>(*t*) is the annual probability of failure for each limit states; *C*<sub>*jLS*</sub> is the failure cost, and *r* is the discount rate.

Noortwijk models maintenance interventions with discrete and continuous renewal process respectively [9]. When maintenance (as good as new maintenance) is modeled as a discrete renewal process and the renewal times  $T_1, T_2, T_3, \dots$  are non-negative, independent, and identically distributed, the expected average costs per unit time is

$$\lim_{n \rightarrow \infty} \frac{E(k(n))}{n} = \frac{\sum_{i=1}^{\infty} C_i P_i}{\sum_{i=1}^{\infty} i P_i} = \frac{E(C_i)}{E(I)} \quad (5)$$

$$= \frac{E(\text{Cycle Cost})}{E(\text{Cycle Length})} \quad n = 1, 2, \dots$$

where  $E(k(n))$  is the expected cost over the bounded horizon  $(0, n]$ ;  $P_i$  represents the probability of a renewal in unit time  $i$ ; and  $C_i$  is the cost associated with a renewal in unit time  $i$ . With the discount rate being considered, the expected life cycle cost over an unbounded horizon is

$$\lim_{n \rightarrow \infty} E(k(n)) = \frac{\sum_{i=1}^{\infty} \alpha^i C_i P_i}{1 - \sum_{i=1}^{\infty} \alpha^i P_i} = \frac{E(\alpha^2 C_i)}{1 - E(\alpha^2)} = k(\alpha) \quad (6)$$

Because of the uncertainty in design variables and other system parameters, the life cycle cost is uncertain. Norwalk gives the expression of variance of the life cycle cost for the first time. Under the consideration that the maintenance is modeled as a discrete renewal process, the long-run average variance of life cycle cost per unit time is represented by [9]

$$\lim_{n \rightarrow \infty} \frac{\text{var}(k(n))}{n} = \frac{\text{var}(C_i)E^2(I) + \text{var}(I)E^2(C_i) - 2E(I)E(C_i)\text{cov}(I, C_i)}{[E(I)]^3} \quad (7)$$

And the average variance of the discounted cost over an unbounded horizon is

$$\lim_{n \rightarrow \infty} \frac{\text{var}(k(n))}{n} = \lim_{n \rightarrow \infty} \frac{E[k^2(n, \alpha) - E^2(k(n, \alpha))]}{n} \quad (8)$$

where

$$\lim_{n \rightarrow \infty} E(k^2(n, \alpha)) = 2 \frac{\sum_{i=1}^{\infty} \alpha^i C_i P_i}{1 - \sum_{i=1}^{\infty} \alpha^i P_i} \frac{\sum_{i=1}^{\infty} \alpha^{2i} C_i P_i}{1 - \sum_{i=1}^{\infty} \alpha^{2i} P_i} + \frac{\sum_{i=1}^{\infty} \alpha^{2i} C_i^2 P_i}{1 - \sum_{i=1}^{\infty} \alpha^{2i} P_i} \quad (9)$$

$$= 2 \frac{E(\alpha^i C_i)}{1 - E(\alpha^i)} \frac{E(\alpha^{2i} C_i)}{1 - E(\alpha^{2i})} + \frac{E(\alpha^{2i} C_i^2)}{1 - E(\alpha^{2i})}$$

Inspection is a useful tool to obtain the current condition of a deteriorating structure. Whether repair is needed or not depends on the observed condition of the structure. Frangopol employs an event tree model to evaluate the repair possibilities related to an uncertain inspection/repair environment [5].

As the number of inspections,  $m$ , increases, the number of branches,  $2^m$ , in the event tree increases exponentially. In order to

illustrate the event tree model easily,  $m=3$  is adopted here and the event tree is shown in Fig. 1.

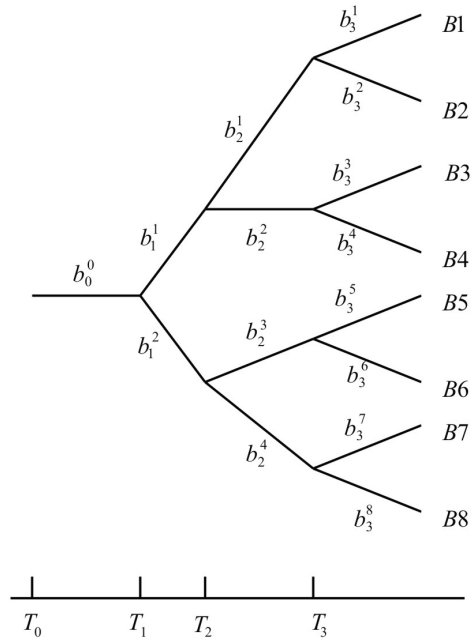


Fig. 1. Event tree with three inspections

The expected life cycle cost  $C_{LC}$  can be expressed by

$$C_{LC} = C_T + C_{PM} + C_{IN} + C_R + C_F \quad (10)$$

where  $C_T$  is the initial cost of the structure.

The preventive maintenance cost  $C_{PM}$  can be expressed by

$$C_{PM} = \sum_{i=1}^n C_{main,iT} \frac{1}{(1+r)^{iT}} \quad (11)$$

where  $n$  is the number of preventive maintenances;  $T$  is the preventive maintenance interval;  $C_{main,iT}$  is the cost of preventive maintenance at time  $iT$ ;  $r$  is the discount rate.

The inspection cost  $C_{IN}$  is

$$C_{IN} = \sum_{i=1}^m C_{ins} \frac{1}{(1+r)^{T_i}} \quad (12)$$

where  $m$  is the number of inspections;  $C_{ins}$  represents the inspection cost based on the inspection method;  $T_i$  is the time of occurrence of the  $i$ th inspection.

The repair cost is represented by

$$C_R = \sum_{i=1}^{2^m} C_{r,i} P(B_i) \quad (13)$$

where  $C_{r,i}$  is the cost associated with the  $i$ th branch; and  $P(B_i)$  is the probability of the  $i$ th branch.

$C_F$  is the expected failure cost and can be expressed by

$$C_F = \sum_{i=1}^{2^m} C_f P_{f,life,i} P(B_i) \quad (14)$$

where  $C_f$  is the cost associated with one failure; and  $P_{f,life,i}$  is the lifetime probability for the branch  $i$ .

**5. Dynamic design under uncertainty considering life cycle issues**

Since the initial reliability is determined by design variables, the life cycle cost is a function of design variables. In addition, random design variables usually exist in optimal design. Hence, a design-to-cost and design-for-cost optimization model under uncertainty based on a reasonable life cycle cost model are given by

$$\begin{aligned} \min R(\mathbf{X}, t) \\ \text{s.t. } \mu_{C(\mathbf{X}, t)} \leq [C] \end{aligned} \quad (15)$$

and

$$\begin{aligned} \min \mu_{C(\mathbf{X}, t)} \\ \text{s.t. } R(\mathbf{X}, t) \geq [R] \end{aligned} \quad (16)$$

where  $C(\mathbf{X}, t)$  is the time-dependent life cycle cost;  $R(\mathbf{X}, t)$  is the time-dependent reliability;  $[C]$  is the financial budget,  $[R]$  is the required reliability level, and  $\mu_{C(\mathbf{X}, t)}$  is the mean value of the life-cycle cost.

And a robust design optimization model under uncertainty can be provided

$$\begin{aligned} \min w_1 \mu_{C(\mathbf{X}, t)} + w_2 \sigma_{C(\mathbf{X}, t)} \\ \text{s.t. } R(\mathbf{X}, t) \geq [R] \end{aligned} \quad (17)$$

where  $w_1$  and  $w_2$  are weight factors and  $\sigma_{C(\mathbf{X}, t)}$  is the standard deviation of lifecycle cost, which can be obtained by Eqs. (7) and (8).

**6. Conclusions**

Life cycle cost analysis, as an important component of life cycle design under uncertainty, has been recognized as an effective way to improve competitiveness in the current market. As the development of design methods, life cycle cost should be estimated at the design stage to achieve design-to-cost or design-for-cost and robust design. Time-dependent design can be conducted based on the estimated life cycle cost. Therefore, it is necessary to choose a suitable life cycle cost model. In this paper, several life cycle cost methods and models are analyzed to help designers to choose a suitable one. Then proper design-to-cost or design-for-cost model can be established. A robust design optimization model is also provided.

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**7. References**

1. Aiedu Y, Gu P. Product life cycle cost analysis: state of the art review. *International Journal of Production Research* 1998; 36: 883-908.
2. Alting L. Life-cycle design of products: a new opportunity for manufacturing enterprise. Andrew Kusiak, ed. *Concurrent Engineering: Automation, Tools, and Techniques*. New York: Wiley, 1993.
3. Dean E B. The design-to-cost manifold. San Diego, 1990; The International Academy of Astronautics Symposium on Space System Cost Methodologies and Applications.
4. Duverlie P, Castelain J M. Cost estimation during design step: parametric method versus case based reasoning method. *International Journal of Advanced Manufacturing Technology* 1999; 15: 895-906.
5. Frangopol D M, Lin K Y, Estes A C. Life-cycle cost design of deteriorating structures. *Journal of Structural Engineering* 1997; 123: 1390-1401.
6. Huang H Z, Liu Z J, Murthy D N P. Optimal reliability, warranty and price for new products. *IIE Transactions* 2007; 39: 819-827.
7. Liu Z J, Chen W, Huang H Z, Yang B. A diagnostics design decision model for products under warranty. *International Journal of Production Economics* 2007; 109: 230-240.
8. Liu Y, Huang H Z. Comment on "A framework to practical predictive maintenance modeling for multi-state systems" by Tan C.M. and Raghavan N. [*Reliab Eng Syst Saf* 2008; 93(8): 1138-50]. *Reliability Engineering and System Safety* 2009; 94: 776-780.
9. Noortwijk J M. Explicit formulas for the variance of discounted life-cycle cost. *Reliability Engineering and System Safety* 2003; 80: 185-195.
10. Sarma K C, Adeli H. Life-cycle cost optimization of steel structures. *International Journal for Numerical Method in Engineering* 2002; 55:1451-1462.
11. Scanlan J, Hill T, Marsh R, Bru C, Dunkley M, Cleveley P. Cost modeling for aircraft design optimization. *Journal of Engineering Design* 2002; 13: 261-269.
12. Shields M D, Young S M, Managing product life cycle costs: An organizational model. *Cost Management* 1991; 39-52.
13. Stewart M G, Reliability-based assessment of ageing bridges using risk ranking and life cycle cost decision analysis. *Reliability Engineering and System Safety* 2001; 74: 263-273.
14. Wen Y K. Minimum lifecycle cost design under multiple hazards. *Reliability Engineering and System Safety* 2001; 73: 223-231.

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