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ZASADA ALOKACJI NIEZAWODNOŚCI DLA SYSTEMU WIELKOSKALOWEGO

RELIABILITY ALLOCATION PRINCIPLE FOR LARGE SCALE SYSTEM

Przy alokacji niezawodności wielkoskalowego seryjnego systemu mechanicznego o dużej liczbie komponentów, niezawodności elementów składowych alokowane zgodnie z modelem niezawodnościowym dla systemów klasycznych są nierealnie wysokie, mimo że docelowa niezawodność wyznaczona dla danego systemu jest niska. Najogólniej rzecz biorąc, tradycyjny model niezawodnościowy systemu nie może poprawnie wyrażać związku pomiędzy niezawodnością systemu a niezawodnościami elementów składowych z powodu zależności statystycznej zachodzącej pomiędzy uszkodzeniami komponentów. Z tej samej przyczyny, nie można po prostu alokować niezawodności systemu na poszczególne elementy składowe zgodnie z tradycyjnym modelem niezawodności systemu. W oparciu o obszerną analizę czynników kontrolujących zależność uszkodzeniową pomiędzy elementami składowymi, artykuł przedstawia nową definicję złożoności systemu/podsystemu oraz złożoności elementów składowych, zwraca uwagę na nierówność z przewagą niepewności obciążenia i przedstawia zasadę alokacji niezawodności systemu opartą na nierówności obciążenia. Zgodnie z tą zasadą, wymóg niezawodności systemu może być z powodzeniem alokowany na poszczególne elementy składowe i ostatecznie wyznaczany na poziomie rozkładu wytrzymałości elementów składowych..

Słowa kluczowe: *Alokacja niezawodności, złożoność systemu, złożoność elementu składowego, nierówność obciążenia, zależność uszkodzeniowa.*

For reliability allocation of a large scale series mechanical system composed of a great number of components, the component reliabilities allocated according to classical system reliability model are unrealistically high, even though the assigned target reliability for the system is quite low. Generally, the traditional system reliability model can not properly express the relationship between system reliability and component reliabilities, owing to the statistical dependence among component failures. For the same reason, system reliability can not be simply allocated to the individual components according to traditional system reliability model. Based on comprehensive analysis to the controlling factors for component failure dependence, the present paper introduces a new definition of system/subsystem complexity and component complexity, highlights load uncertainty dominated asperity and presents load roughness based principle for system reliability allocation. According to such a principle, system reliability requirement can be reasonably allocated to components, and totally determined at the level of component strength distribution.

Keywords: *Reliability allocation, system complexity, component complexity, load roughness, failure dependence.*

1. Introduction

Reliability allocation is a process to transfer system reliability requirement to lower level modules such as sub-systems and components. The lower level modules will be called as units in the present paper. Reliability allocation methods are numerous and variety, from the simplest equal allocation to advanced optimization algorithm [16]. As the foundation, an appropriate system reliability model is necessary to correctly express the relationship between system reliability and unit reliabilities or unit load/property distribution parameters.

Generally speaking, reliability allocation is, within the confines of specified cost, weight or size, to find a reasonable solution for the individual units to satisfy system reliability requirement, which can be mathematically express as [15],

$$f(R_1,R_2,\dots,R_n)\geq R_s \qquad (1)$$

where, R_i (i=1, 2, …, n) - required reliability for the ith unit, n - number of units to which system reliability requirement is shared, $f(\cdot)$ - function relationship between system reliability and unit reliabilities, R_s - assigned system reliability.

Obviously, system reliability allocation is a multi-solution problem. Besides the general goal of low cost and high reliability, the normal considerations for reliability allocation policy include manufacturing technique, complexity of the individual unit, loading condition and criticality, likelihood of failure, etc.

Recently, the majority of investigations concerning system reliability allocation are addressed to reliability oriented optimization of different system configurations. Yalaoui [14] showed theoretical and practical results for the reliability allocation of a series-parallel system. Liang [6] introduced a meta-heuristic algorithm to redundancy allocation problem. Limbourg [7] presented a feature modeling approach to reliability optimization, which can not only describe arbitrary reliability allocation problems but also much more complex design problems. Tavakkoli-Moghaddam [9] proposed a genetic algorithm for a redundancy allocation of series-parallel systems when the redundancy strategy can be chosen for individual subsystems.

Reliability optimization techniques can be classified as linear programming, dynamic programming, integer programming, geometric programming, heuristic method, Lagrangean multiplier method, genetic algorithm, and so on [1]. These techniques differ in methodology and application condition, but all are based on traditional system reliability model to verify reliability allocation result, in which no failure dependence effect can be reflected.

Ramirez-Marquez et al [8] pointed out that recognition of common cause failures (CCFs) for optimal configuration is important due to the significant impact of these failures on the overall system reliability. It is well known that for a specified system configuration, system reliability depends on not only component reliability, but also the degree of the dependence among component failures. In the same way, system or subsystem complexity, in the sense of reliability allocation, depends on both the number of components that the system or subsystem contains and the dependence among component failures.

2. Component failure dependence and system reliability models

Since common cause failure (CCF in short) exists in the majority of systems and the failure dependence among components plays an important role for system reliability and safety [2-5, 11], the conventional assumption of "component failures are statistically independent of each other in a system" is not usually valid. Therefore, the traditional system reliability models, developed under the assumption of independent component failures, are not applicable to reliability evaluation or reliability allocation for systems composed of components of which the failures are statistically dependent of each other, which cover the majority of electronic systems and almost all of mechanical systems.

System reliability model incorporating CCF effect can be developed by means of the load-strength interference analysis at system-level. In the condition that component strengths (denoted by X) are independent and identically distributed random variables, and all the components are subjected to the same random load Y, series system reliability model writes [12, 13]:

$$R^n_{series} = \int_{-\infty}^{\infty} g(y)\left[\int_y^{\infty} f(x)dx\right]^n dy \tag{2}$$

And parallel system reliability model writes:

$$R^n_{parallel} = 1 - \int_0^{\infty} g(y)\left[\int_0^{y} f(x)\mathrm{d}x\right]^n \mathrm{d}y \tag{3}$$

where, $g(y)$ and $f(x)$ denote the probability density functions of stress Y and strength X, respectively, n denotes the number of components in system.

For a system composed of components of which strengths are independent and non-identically distributed variables, let X_1, X_2, ..., X_n stand for the strengths of the n components respectively, and $F_i(x)$ the distribution function of X_i (i=1~n). With the notations of $X=min\{X_1, X_2, ..., X_n\}$, the distribution functions of the minimum statistic is

$$F_N(x) = 1 - \prod_{i=1}^{n}[1 - F_i(x)] \tag{4}$$

By means of the interference analysis between the minimum strength statistic and the applied load, series system reliability, which is equal to the probability that the minimum strength statistic exceeds the applied load, can be expressed as

$$R^n_{series} = \int_{-\infty}^{\infty} h(y)[1 - F_N(y)]dy = \int_{-\infty}^{\infty} h(y)\prod_{i=1}^{n}[1 - F_i(y)]dy \tag{5}$$

More generally, a system might comprise of various components, and the loads applied to the individual components are different, too.

Considered in this paper is a typical situation in which all the loads subjected to the individual component are linearly correlated random variables, i.e., $Y_i=a_iY_0+b_i$ (i=1~n), where Y_i is the load applied on component i and Y_0 is a unified load random variable, a_i and b_i are constants. For such a loading condition, system reliability models can be developed by means of unification of the linearly correlated random loads.

Suppose that the load applied to the ith component follows the normal distribution with expectation μ_i and standard deviation (std) σ_i, i.e., Y_i~$N(\mu_i, \sigma_i)$, it is easy to get the relationship between Y_i and a standard normal-distributed random variable Y_0 (y_0~$N(0,1)$), i.e.

$$Y_0=(Y_i-\mu_i)/\sigma_i \tag{6}$$

or

$$Y_i=\sigma_iY_0+\mu_i \tag{7}$$

Evidently, the following transformation holds true (referring to Fig.1)

$$\int_{-\infty}^{\infty} h_i(y)\int_y^{\infty} f(x)dxdy = \int_{-\infty}^{\infty} h_0(y)\int_{\sigma_i y+\mu_i}^{\infty} f(x)dxdy \tag{8}$$

Where, $h_i(y)$ is the pdf of the load subjected to the ith component, $h_0(y)$ is the pdf of the standard normally distributed variable.

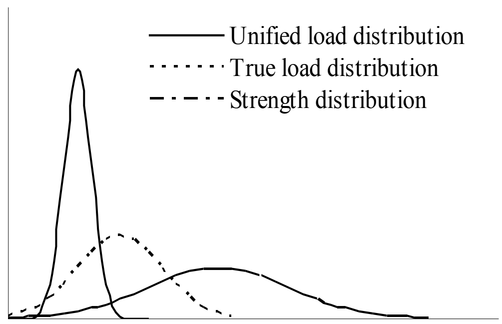

Fig. 1. Illustration of load unification and extended load-strength interference relationship

In such a situation, series system reliability can be presented as

$$R^n_{series} = \int_{-\infty}^{\infty} h_0(y)\prod_{i=1}^{n}[1 - F_i(\sigma_i y + \mu_i)]dy \tag{9}$$

Contrasting to the above system reliability models with CCF effect incorporated, i.e. Eq.2 and Eq.3, the traditional system reliability models for series system and parallel system are, respectively,

$$R^n_{id-series} = \prod_{i=1}^{n} R_i \tag{10}$$

$$R^n_{id-parallel} = 1 - \prod_{i=1}^{n}(1 - R_i) \tag{11}$$

where R_i denotes component reliability and can be calculated by means of the following stress-strength interference equation

$$R = \int_{-\infty}^{\infty} g(y)\int_y^{\infty} f(x)dxdy \tag{12}$$

The numerical differences between the failure-dependent system reliability model (Eq.2 and Eq.3) and the traditional failure-independent system reliability model (Eq.10 and Eq.11) depends, besides component numbers, mainly on the degree of component failure dependence, which can be approximately described by stress roughness factor which is defined as

$$L_R = \frac{\sigma_y}{(\sigma_x^2 + \sigma_y^2)^{1/2}} \tag{13}$$

where, σ_y and σ_x denotes the standard deviations of stress Y and strength X, respectively.

3. Failure dependence and system complexity

Traditionally, it is thought that the number of constitute parts within a system exclusively determines the complexity of the system. For instance, subsystem complexity was defined as the number of modules and their associated circuitry, or the ratio of the number of essential parts within the subsystem to the total number of such essential parts in the entire machine [10]. General subsystem complexity takes the form

$$C_i = \frac{n_i}{\sum_{i=1}^{m} n_i} \tag{14}$$

where, n_i - number of components in the ith subsystem, m - number of subsystems in the entire system.

For failure-dependent system, system reliability depends not only on the number of components, given the reliabilities of the individual components, but also the degree of the dependence among component failures. The degree of component failure dependence is largely determined by load roughness. Subsequently, a more reasonable index to describe subsystem complexity should be "load-uncertainty-based complexity" defined as

$$C_i = \frac{n_i^{1-L_{Ri}}}{\sum_{i=1}^{m} n_i^{1-L_{Ri}}} \tag{15}$$

where, L_{Ri} stands for the roughness factor of the ith subsystem which is determined by the following equation:

$$L_{Ri} = (\prod_{j=1}^{n_i} L_{Rij})^{1/n_i} \tag{16}$$

where, L_{Rij} – roughness factor of the jth component of the ith subsystem.

In a sense, $n_i^{1-L_{Ri}}$ is the equivalent component number of the ith subsystem.

Shown in Fig. 2 are the "equivalent component number - load roughness" curves, where, the real number of components in the subsystem are 10, 20, and 30, respectively. It shows that the equivalent component number or the complexity of a system decreases with the increase of load roughness. The larger the number of component in a system is, the more significantly the system complexity varies. On the other side, system complexity is totally determined by the number of components that the system contains only in the condition of zero load roughness, i.e., deterministic load condition.

Shown in Fig. 3 are the "system reliability - load roughness" curve estimated by means of the failure-dependent system reliability model and that estimated by means of the traditional failure-independent system reliability model, respectively. Where, the

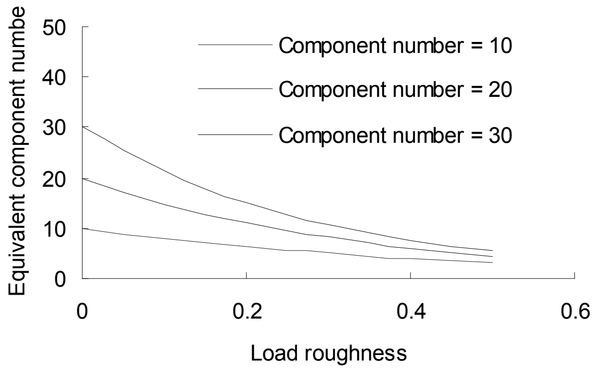

Fig. 2. Equivalent component number - load roughness curves

number of components in the system is 100. The situation is that, all the components are statistically identical with the strength following Gauss distribution with the expectation of 800 MPa and standard deviation of 50 MPa, and all the components subject to the same random stress following Gauss distribution with the expectation of 600 MPa. Different stress standard deviation parameters (100, 200, …, 600 MPa) are considered to show the effect of load roughness on system reliability. With the same component strength distribution and the same stress expectation, the increase in stress uncertainty leads to the decrease in both component reliability and system reliability. However, the effect on dependent system and the effect on independent system are different.

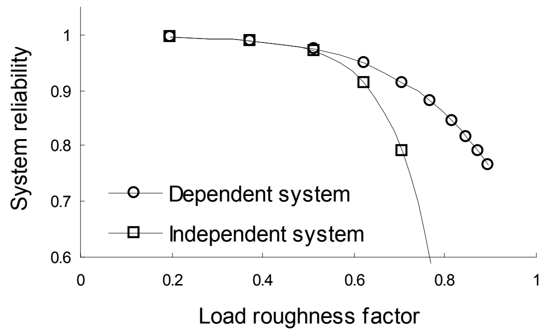

Fig.3. System reliability - load roughness curves

4. Component complexity

Many components are complex in geometry and exist more than one weak areas or high stress zones. These kinds of components include gears, racks, turbine disks, ladder shafts, bearings, and so on, and a variety of plate or shell structures. In the sense of reliability, such a component or structure should be taken as a "system" but not just one "element". Complex component tends to holds low reliability compared to simple component. Thus, different components have different complexities in the sense of reliability.

Here, component complexity is defined as

$$C_{ci} = \frac{n_{ci}^{1-L_{Rci}}}{\sum_{i=1}^{n} n_{ci}^{1-L_{Rci}}} \tag{17}$$

where, n_{ci} - number of weak areas (elements) on the ith component, n - number of components in the entire system, L_{Rci} - roughness factor of the ith component:

$$L_{Rci} = (\prod_{j=1}^{n_{ci}} L_{Rcij})^{1/n_{ci}} \quad (18)$$

where L_{Rcij} – roughness factor of the jth weak area on the ith component.

Based on the fact that a component should be taken as a series system contains many weak areas, and the concept of component complexity, system reliability should be allocated to the individual weak areas of each component, but not simply allocated to components. Moreover, since the same element (one weak area) reliability can be yielded by different stress-strength distribution combinations, reliability allocation should be deployed to the level of strength distribution of the every weak area on all the components composing the system.

5. Factors affecting system reliability allocation

With the scenario of system reliability allocation, the effect of load uncertainty on system reliability and related properties can be sorted as the following.

Load asperity effect – load asperity should be characterized by both the average intensity of the load and its uncertainty degree as well. A subsystem or a complex component with higher load uncertainty should be allocated relatively higher reliability.

Complexity effect – complexity is determined by both the number of constitute elements within a subsystem (or the number of weak areas on a component) and the degree of load uncertainty or the degree of load roughness. Complex component should be allocated to relatively lower reliability.

Owing to factor that the effect of load roughness is determined by both load uncertainty and strength uncertainty, and different stress-strength distribution combinations can produce the same reliability, system reliability allocation should be ended by determined component weak area strength distribution. It is not sufficient to allocate system reliability only at component reliability level, since the same component reliability can be obtained by different stress-strength combinations, while the same component reliability obtained from different stress-strength combinations contributes differently to system reliability.

6. Conclusion

System reliability models developed based on component independent-failure assumption are not applicable to mechanical system subjected to uncertain load environment.

System reliability allocation should be stepwise developed from system level, subsystem level, component level, up to element level (susceptible locations on a component), and ends up with completely determined component strength distribution, i.e. component size and its tolerance as well.

On the fundamental principle and related concepts for reliability allocation, load-uncertainty-based subsystem complexity and component complexity are defined. On the basic rule for reliability allocation, the traditional complexity principle and load asperity principle are revised, a "load uncertainty" based principle is added. The underlying concept is the embodiment of failure dependence effect through reliability allocation process, which makes the reliability index of a large-scale system can be reasonably allocated to component or element level.

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The research work is subsidized with the Hi-Tech Research and Development Program of China (grant No. 2007AA04Z428) and the National Science Foundation of China (grant No.50775030)

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