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# PROGNOZOWANIE TRWAŁOŚCI ŚRODKÓW TECHNICZNYCH Z WYKORZYSTANIEM WIELU WSKAŹNIKÓW DEGRADACJI I ZDARZEŃ AWARYJNYCH W UJĘCIU MODELU CIĄGŁEJ PRZESTRZENI STANÓW

# ASSET LIFE PREDICTION USING MULTIPLE DEGRADATION INDICATORS AND FAILURE EVENTS: A CONTINUOUS STATE SPACE MODEL APPROACH

*Prognozowanie trwałości środków z wykorzystaniem wskaźników degradacji wiąże się z dwoma zagadnieniami praktycznymi: (1) identyfikacją progów niepewnego uszkodzenia dla wskaźników degradacji oraz (2) łączeniem licznych wskaźników degradacji ortzymanych na podstawie danych z monitorowania stanu. Model degradacji w przestrzeni stanów stanowi efektywne podejście do tych dwóch zagadnień. Jednakże dotychczasowe badania dotyczące tego modelu w dużej mierze przyjmują założenie dyskretnego czasu lub dyskretnych stanów, które wymaga równych odstępów między przeglądami lub dyskretyzacji ciągłych wskaźników degradacji. Aby uniknąć konieczności zakładania dyskretnego czasu i dyskretnych stanów, w niniejszej pracy zaproponowano model przestrzeni stanów oparty na procesie Gamma. Proces Gamma charakteryzuje własność monotoniczna rosnącą, która odpowiada nieodwracalnym procesom degradacji środków technicznych w trakcie jednego cyklu serwisowego. Własność monotoniczna rosnąca ułatwia również ustalenie funkcji prawdopodobieństwa, gdy brane są pod uwagę czasy uszkodzeń. W artykule sformułowano algorytmy estymacji parametrów oraz prognozowania czasu życia dla modelu przestrzeni stanów opartego na procesie Gamma. Dodatkowo określono metodę oceny efektywności wskaźników w modelowaniu degradacji. Proponowany model przestrzeni stanów oparty na procesie Gamma oraz jego algorytmy weryfikowano przy użyciu danych symulacyjnych oraz danych terenowych pozyskanych z przedsiębiorstwa zajmującego się ciekłym gazem ziemnym.*

***Słowa kluczowe:*** *Prognozowanie trwałości środków, model degradacji, algorytm maksymalizacji wartości oczekiwanej, model przestrzeni stanów.*

*Two practical issues are involved in asset life prediction using degradation indicators: (1) identifying uncertain failure thresholds of degradation indicators and (2) fusing multiple degradation indicators extracted from condition monitoring data. The state space degradation model provides an effective approach to address these two issues. However, existing research on the state space degradation model largely adopts a discrete time or states assumption which requires equal inspection intervals or discretising continuous degradation indicators. To remove the discrete time and states assumptions, this paper proposes a Gamma-based state space model. The Gamma process has a monotonically increasing property that is consistent with the irreversible degradation processes of engineering assets within a single maintenance cycle. The monotonically increasing property also makes the establishment of the likelihood function more straightforward when failure times are considered. In this paper, parameter estimation and lifetime prediction algorithms for the Gamma-based state space model are developed. In addition, an effectiveness evaluation approach for indicators in degradation modelling is established. The proposed Gamma-based state space model and algorithms are validated using both simulated data and a field dataset from a liquefied natural gas company.*

***Keywords:*** *Asset life prediction, Degradation model, Expectation-Maximisation algorithm, State space model.*

## 1. Introduction

An unexpected failure of a critical engineering asset can reduce the productivity of a whole plant [7, 15, 20]. Therefore the performance of a plant can be enhanced by accurately predicting the lifetimes of its critical engineering assets. Up to the early nineties, most asset life prediction methods were based on lifetime distributions. However, statistically sufficient failure records are often difficult to obtain in practice due to preventive maintenance and the small population of similar engineering assets. Therefore, asset life prediction methods using degradation indicators extracted from condition monitoring data becomes more preferable than those relying on failure events only.

Two practical issues are often involved when using degradation indicators to predict failure times. The first issue is identifying uncertain failure thresholds of degradation indicators. In reality, a degradation indicator directly relating to a failure mechanism is often difficult to monitor. For example, the measurement of crack length on a metal part relies on special equipment such as the ultrasonogram. Most degradation indicators (e.g. indicators extracted from vibration signals or oil analysis data) only partially reflect failure mechanisms. Setting up a fixed failure threshold for these indicators can cause excessive unexpected failures or false alarms. The second issue is fusing multiple degradation indicators extracted from condition monitoring data. In practice, there is often more than one degradation indicator revealing an asset degradation process. Therefore, information from these degradation indicators should be fused properly, and the effectiveness of these indicators in life prediction should be evaluated.

Some researchers have conducted preliminary investigations on the two issues discussed above. The Proportional Hazard Model (PHM) [5] can describe the relationship between multiple degradation indicators (i.e., covariates) and time dependent failure rates [2, 12, 14, 16]. However, asset life prediction using PHM is based on modelling of time dependent degradation indicators. To make the asset life prediction mathematically tractable, some assumptions are often made about time dependent degradation indicators. Liao et al. assumed that degradation indicators follow some deterministic functions of time [12]. Banjevic et al. discretised continuous degradation indicators before applying the PHM [2]. The PHM whose covariates follow stochastic processes continuous in time and state is yet to be investigated. Another approach to dealing with multiple degradation indicators and uncertain failure thresholds is the logistic regression model. The logistic regression model can identify the relationship between probabilities of failure and values of multiple degradation indicators [26]. However, the logistic regression model only considers degradation indicators at failure or censoring times. Liao's research showed that the results given by logistic regression model are often less accurate than those obtained using the PHM [12]. Similar to the PHM, the logistic regression model cannot perform asset life prediction without a specific model for degradation indicators. Another degradation model considering multiple indicators is the composite scale model [8]. In the composite scale model, linear or multiplicative combinations of indicators or usages are adopted instead of chronological time to signify the age of an asset. Thus, effects of multiple indicators can be considered simultaneously. However, most papers only use the composite scale model to estimate current health states of assets. To predict upcoming health states, mathematical models are still needed to predict the composite scale, which requires further research.

The state space model is an additional mathematical model that can handle multiple degradation indicators and uncertain failure thresholds. The state space model presumes the existence of an underlying degradation process. When the underlying degradation process crosses a predetermined threshold, a failure happens. The underlying degradation process cannot be observed directly; instead, it is partially revealed by degradation indicators. Compared with other degradation models discussed above, the state space model considers both stochastic asset degradation processes and uncertain relationships between degradation indicators and health states. Therefore, degradation indicators are used more efficiently, and no additional mathematical models for time dependent degradation indicators are needed when predicting asset lives. Moreover, the state space model is an effective tool for indicators fusion. Compared with commonly used multivariate statistical approaches and multivariate time series analysis methods, the state space model can analyse degradation indicators with uneven sampling intervals.

Existing research of the state space degradation models largely adopts discrete time or states assumptions. Christer et al. developed a discrete time state space model to estimate and predict the erosion status of a furnace through its conductance ratios [3]. The discrete time model assumes fixed inspection intervals, which is often not the case in reality. Recently, Wang proposed a new state space model by assuming increments of underlying health states follow a beta distribution [22]. Subsequently, Wang's new model has a monotonically increasing underlying degradation process that is more similar to irreversible engineering asset wear processes. However, Wang's new model is again discrete in time. Makis and Jiang developed a state space model based on a continuous time discrete state Markov process [17]. The discrete state assumption requires discretising continuous degradation processes, which needs expert knowledge and may introduce additional errors. To remove discrete time and states assumptions, state space models continuous in time and states have also been developed. Wang et al. developed a state space model to predict the remaining useful life (RUL) of bearings using root mean square (RMS) values of vibration signals [23]. Wang's model uses values of RUL as underlying health states. This deterministic underlying degradation process does not consider stochastic heterogeneous degradation processes of different individuals. Whitmore et al. proposed a bivariate Wiener process [24] to model a partially revealed degradation process. However, the bivariate Wiener process only considers the covariates collected at failure or censoring times, while degradation indicators at other occasions are ignored.

To address the limitations in the existing models, this paper proposes a continuous state space model based on the Gamma process. The proposed model considers both stochastic asset degradation processes and uncertain relationships between degradation indicators and underlying health states. Continuous time property enables the proposed model to process irregular inspection intervals. Continuous states, on the other hand, avoid discretising indicators with continuous values. This paper develops Monte Carlo based parameter estimation and lifetime prediction algorithms for the proposed model. The censored failure data problem which has been ignored by most existing state space degradation models [3, 17, 22, 23] is considered. In addition, a parametric Bootstrap algorithm is developed to evaluate the effectiveness of different indicators in asset degradation modelling. The proposed algorithms are validated by both simulated data and field data.

The body of this paper is organised as follows: Section 2 discusses the structure and assumptions of the Gamma-based state space model. Section 3 develops a parameter estimation method for the proposed model. Section 4 presents lifetime prediction algorithms. Section 5 proposes an effectiveness evaluation approach for the degradation indicators used in the proposed model. The performance of the proposed algorithms is investigated using simulated data in Section 6. Section 7 conducts a case study using condition monitoring data from liquefied natural gas pumps.

## 2. Model development

The Gamma-based state space model contains two components. The first component is termed as the system equation which represents the underlying degradation process of an asset. The second component, namely the observation equation, is used to model relationships between underlying health states and degradation indicators.

In this research, the system equation given by (1) is assumed to follow a Gamma process. The scalar variable $\Lambda(t) \geq 0$ denotes the underlying health state at time $t \geq 0$. A larger value of $\Lambda(t)$ indicates a worse health state, and a failure is assumed to happen when $\Lambda(t)$ crosses a predetermined threshold. An asset is assumed to be non-defective at the initial time, i.e., $\Lambda(0) = 0$. The increments of $\Lambda(t)$ follow a Gamma distribution given by (1), where $Ga(a\cdot\Delta t,\xi)$ denotes the Gamma distribution with shape parameter $a\cdot\Delta t$ and scale parameter $\xi$.

$$\Lambda(t+\Delta t) - \Lambda(t) \sim Ga(a\cdot\Delta t,\xi) \qquad (1)$$

There are three reasons why this research chooses the Gamma process instead of the commonly used Gaussian process: Firstly, the monotonically increasing property of the Gamma process makes the establishment of the likelihood function easier when a failure is considered. For example, when calculating the likelihood function for a state space model based on the Gaussian process, conditional probability density functions (PDF) are required to ensure that the underlying degradation process does not drift across the failure threshold between two normal states [24]. Integrals are needed in these conditional PDFs, which increase difficulties in establishing and evaluating likelihood functions. Secondly, the monotonically increasing property of the Gamma process is also consistent with irreversible degradation processes of engineering assets. Consequently, the Gamma process has been widely applied in engineering asset degradation modelling [9, 11, 13, 18, 19, 27]. Finally, existing research on the Gamma process also provides approaches to consider operation conditions and unit-specific random effects during degradation modelling [11]. These approaches enable the proposed Gamma-based state space model to be applied to more complicated reality.

The second component of the Gamma-based state space model is the observation equation. In this research the degradation indicators are assumed to follow a multivariate normal distribution given by (2), where $\vec{x}(t)$ denotes the degradation indicator vector at time $t$, and $N(\vec{c}\cdot\Lambda(t),\vec{\Sigma})$ denotes the multivariate normal distribution with mean vector $\vec{c}\cdot\Lambda(t)$ and covariance matrix $\vec{\Sigma}$.

$$\vec{x}(t) \sim N\left(\vec{c}\cdot\Lambda(t),\vec{\Sigma}\right) \qquad (2)$$

To formulate the parameter estimation algorithm more concisely, only degradation indicators from one degradation process are considered in the present paper. The formulations in this paper can be extended to a multiple degradation processes situation without much theoretical difficulties. Inspection times are denoted as $t_i$; $i = 1,2,...,n$, where $n$ is the number of inspections. The values of the underlying health state and degradation indicator vector at the $i$-th inspection are denoted as $\lambda_i$ and $\vec{x}_i$ respectively. The failure time and the failure threshold of the underlying degradation state are denoted as $t_f$ and $\Lambda_f$. Note that $\Lambda_f$ is assumed equal to 1, because the identical life time distribution can be obtained by changing the scale parameter $\xi$ for different values of $\Lambda_f$. For an asset preventively replaced before failure, the censoring time is denoted as $t_s$. Unlike the PHM, the degradation indicators at $t_f$ or $t_s$ are not indispensible during parameter estimation.

## 3. Parameter estimation

This paper uses the Expectation-Maximisation (EM) algorithm [6] to estimate the parameters of the Gamma-based state space model. Dissimilar to the maximum likelihood estimation (MLE) method, the EM algorithm iteratively maximises the expectation of the complete likelihood function instead of directly optimising the marginal likelihood function. For the proposed Gamma-based state space model, the marginal likelihood function involves numeric integral and cannot be evaluated efficiently. Therefore, the EM algorithm is adopted to estimate the parameters. The EM algorithm is carried out by four steps: The first step is to estimate initial parameters. Inappropriate initial parameters may cause the final estimation result trapped in a local maximum point, or even make the EM algorithm divergent [25]. The second step, namely the E step, is to estimate the expectation of the complete likelihood function. Subsequently, the expected complete likelihood function is maximised during the M step. The final step is checking the convergence of the EM loop. If the convergence condition is satisfied, the final result of parameter estimation is obtained. Otherwise, another EM iteration begins. These four steps are discussed in detail as follows:

### Initial parameter estimation

The initial parameters for the EM algorithm are estimated by the method of moments. Because inspection intervals are uneven, the increments of degradation indicator vectors should be scaled before treated by the method of moments. The method of moments used in this research is motivated by that adopted in [4]. Firstly, Equation (3) can be obtained according to the property of the Gamma process. Then the first-order and second-order moments of the scaled increments of degradation indicator vectors can be calculated as (4) and (5). After that, given an initial value of $\hat{a}$, the estimate of $\xi$, $\vec{c}$, and $\hat{\vec{\Sigma}}$ are estimated using (6), (7), and (8). $\hat{a}$ is obtained by experience. When any diagonal element of $\hat{\vec{\Sigma}}$ is negative, a bigger value of $\hat{a}$ is required.

$$\begin{cases} E\left(\Lambda_f\right) = a\cdot\xi\cdot T_f \\ E\left(\lambda_i - \lambda_{i-1}\right) = a\cdot\left(t_i - t_{i-1}\right)\cdot\xi \\ E\left(\lambda_i - \lambda_{i-1}\right)^2 = a\cdot\left(t_i - t_{i-1}\right)\cdot\xi^2 + a^2\cdot\left(t_i - t_{i-1}\right)^2\cdot\xi^2 \end{cases} \qquad (3)$$

$$E\left(\sum_{i=1}^{n-1}\frac{\vec{x}_i - \vec{x}_{i-1}}{E\left(\lambda_i - \lambda_{i-1}\right)}\right) = \sum_{i=1}^{n-1}\frac{\vec{c}\cdot E\left(\lambda_i - \lambda_{i-1}\right) + E\left(\varepsilon_i\right) - E\left(\varepsilon_{i-1}\right)}{E\left(\lambda_i - \lambda_{i-1}\right)} = (n-1)\cdot\vec{c} \qquad (4)$$

$$\begin{aligned} E\left(\sum_{i=1}^{n-1}\frac{\left(\vec{x}_i - \vec{x}_{i-1}\right)\cdot\left(\vec{x}_i - \vec{x}_{i-1}\right)'}{\left(E\left(\lambda_i - \lambda_{i-1}\right)\right)^2}\right) &= \\ = \sum_{i=1}^{n-1}\frac{\vec{c}\cdot\vec{c}'\cdot E\left(\lambda_i - \lambda_{i-1}\right)^2 + E\left(\left(\vec{\varepsilon}_i - \vec{\varepsilon}_{i-1}\right)\cdot\left(\vec{\varepsilon}_i - \vec{\varepsilon}_{i-1}\right)'\right)}{\left(E\left(\lambda_i - \lambda_{i-1}\right)\right)^2} \\ = \sum_{i=1}^{n-1}\frac{\vec{c}\cdot\vec{c}'\cdot\left(a\cdot\left(t_i - t_{i-1}\right)\cdot\xi^2 + a^2\cdot\left(t_i - t_{i-1}\right)^2\cdot\xi^2\right) + 2\vec{\Sigma}}{a^2\cdot\left(t_i - t_{i-1}\right)^2\cdot\xi^2} \end{aligned} \qquad (5)$$

$$\hat{\xi} = \Lambda_f \big/ \left(T_f\cdot\hat{a}\right) \qquad (6)$$

$$\hat{\vec{c}} = \frac{1}{n-1}\sum_{i=1}^{n-1}\frac{\vec{x}_i-\vec{x}_{i-1}}{E(\lambda_i-\lambda_{i-1})} = \frac{1}{n-1}\sum_{i=1}^{n-1}\frac{\vec{x}_i-\vec{x}_{i-1}}{\Lambda_f(t_i-t_{i-1})/T_f} = \frac{1}{n-1}\frac{T_f}{\Lambda_f}\sum_{i=1}^{n-1}\frac{\vec{x}_i-\vec{x}_{i-1}}{t_i-t_{i-1}} \quad (7)$$

$$\hat{\Sigma} = \frac{\sum_{i=2}^{n}\frac{(\vec{x}_i-\vec{x}_{i-1})\cdot(\vec{x}_i-\vec{x}_{i-1})'}{(t_i-t_{i-1})^2} - \frac{1}{\hat{a}}\cdot\hat{\vec{c}}\cdot\hat{\vec{c}}'\cdot\frac{\Lambda_f^2}{T_f^2}\sum_{i=2}^{n}\frac{1}{t_i-t_{i-1}} - (n-1)\cdot\hat{\vec{c}}\cdot\hat{\vec{c}}'\cdot\frac{\Lambda_f^2}{T_f^2}}{2\sum_{i=2}^{n}(t_i-t_{i-1})^{-2}} \quad (8)$$

### E step

The E step is to estimate the expectation of the complete likelihood function. In this section, both complete and censored failure data are considered. When complete failure data are available, the expected complete likelihood function given degradation indicators and the failure time can be written as (9), where $\theta=\{a \quad \xi \quad \vec{c} \quad \vec{\Sigma}\}$, $\theta_1=\{a \quad \xi\}$, and $\theta_2=\{\vec{c} \quad \vec{\Sigma}\}$ represent the model parameters to estimate. To make the equations more concise, in this paper, $\{\vec{x}_i; i=u,u+1,\dots,v\}$ is denoted by $\vec{x}_{u:v}$; similarly $\{\lambda_i; i=u,u+1,\dots,v\}$ is denoted by $\lambda_{u:v}$.

$$E_{|\vec{x}_{1:n},t_f}\left(\log f\left(\vec{x}_{1:n},\lambda_{1:n},t_f|\theta\right)\right) = E_{|\vec{x}_{1:n},t_f}\left(\log f\left(\lambda_{1:n},t_f|\theta_1\right)\right) + E_{|\vec{x}_{1:n},t_f}\left(\log f\left(\vec{x}_{1:n}|\lambda_{1:n},\theta_2\right)\right) \quad (9)$$

The two components of (9) can be written as (10) and (11) respectively, where $v_i = \lambda_i - \lambda_{i-1}$, $u_i = at_i - at_{i-1}$; $i = 2,3,...,n+1$, and $m$ is the size of the degradation indicator vector. To achieve a shorter equation, $\lambda_{n+1}$ denotes $\Lambda_f$, and $t_{n+1}$ represents $t_f$ in (10). To calculate (10) and (11), three components (i.e., $E_{|\vec{x}_{1:n},t_f}(\lambda_i)$, $E_{|\vec{x}_{1:n},t_f}(\lambda_i^2)$, and $E_{|\vec{x}_{1:n},t_f}(\log(v_i))$ should be estimated first. For this particular model, the three components are estimated through the particle smoother algorithm. The particle smoother can approximate conditional distributions of underlying health states given degradation indicators $\vec{x}_{1:n}$ and failure time $t_f$ by a set of random samples $s_{1:n+1}^{1:N_f}=\{s_i^j; i=1,2,\dots,n+1 \quad j=1,2,\dots,N_f\}$ as (12). In (12), $\delta(\bullet)$ is the Dirac delta measure given by (13). Using these smoothing results $s_{1:n+1}^{1:N_f}$, the three components in (10) and (11) can be approximated as (14). This paper adopts the particle smoother using the backwards simulation method proposed by Simon, Arnaud et al. [21]. Full details about particle smoothing are not discussed in this research. However, a key distribution used by the particle smoother is calculated in the next paragraph.

$$E_{|\vec{x}_{1:n},t_f}\left(\log f\left(\lambda_{1:n},t_f|\theta_1\right)\right) = \sum_{i=2}^{n+1}\left(-u_i\log\xi - \log\Gamma(u_i) + (u_i-1)E_{|\vec{x}_{1:n},t_f}\left(\log(v_i)\right) - \frac{1}{\xi}\left(E_{|\vec{x}_{1:n},t_f}(\lambda_i) - E_{|\vec{x}_{1:n},t_f}(\lambda_{i-1})\right)\right) \quad (10)$$

$$E_{|\vec{x}_{1:n},t_f}\left(\log f\left(\vec{x}_{1:n}|\lambda_{1:n},\theta_2\right)\right) = \frac{-n}{(2\pi)^{m/2}} - \frac{n}{2}\ln|\vec{\Sigma}| - \frac{1}{2}tr\left(\vec{\Sigma}^{-1}\sum_{i=1}^{n}E\left(\left(\vec{x}_i-\vec{c}\cdot\lambda_i\right)\cdot\left(\vec{x}_i-\vec{c}\cdot\lambda_i\right)'|\vec{x}_{1:n},\lambda_{n+1}\right)\right) \quad (11)$$

$$f\left(\lambda_i|x_{1:n},t_f\right) \approx \sum_{k=1}^{N_f} w_i^k\delta(\lambda_i - s_i^k) \quad i=1,2,\dots,n \quad (12)$$

$$\delta\left(\lambda_i - s_i^j\right) = \begin{cases}0, & \lambda_i \neq s_i^j \\ 1, & \lambda_i = s_i^j\end{cases} \quad (13)$$

$$\begin{cases} E_{|\vec{x}_{1:n},t_f}(\lambda_i) = \frac{1}{N_f}\sum_{j=1}^{N_f}s_i^j \quad E_{|\vec{x}_{1:n},t_f}(\log(v_i)) = \frac{1}{N_f}\sum_{j=1}^{N_f}\ln\left(s_i^j - s_{i-1}^j\right) \\ E_{|\vec{x}_{1:n},t_f}(\lambda_i^2) = \frac{1}{N_f}\sum_{j=1}^{N_f}\left(s_i^j - E_{|\vec{x}_{1:n},t_f}(\lambda_i)\right)^2 + \left(E_{|\vec{x}_{1:n},t_f}(\lambda_i)\right)^2 \end{cases} \quad (14)$$

To conduct the particle smoother, the conditional PDF of the underling health state at the next inspection time given the failure time and the current heath state should be calculated first. In the proposed model, the failure time is assumed as the first crossing time of the underlying Gamma process $\{\Lambda(t); t > 0\}$ to a predetermined failure threshold $\Lambda_f$. Therefore, the conditional PDF of the underling health state at the next inspection time can be written as (15) according to the Gamma bridge property.

$$f\left(\lambda_{i+1}|\lambda_i,t_f\right) = Be\left(\frac{\lambda_{i+1}-\lambda_i}{\Lambda_f-\lambda_i}; a(t_{i+1}-t_i), a(t_f-t_{i+1})\right); \; i=1,\dots,n-1 \quad (15)$$

For censored data, the expected complete likelihood function is similar to (9), except replacing the failure time $t_f$ with the censored time $t_s$. The expected complete likelihood function for censored data is also approximated by the results of particle smoothing. When conducting particle smoothing, the conditional PDF of the underlying health state at $i + 1$ th inspection point is modified from (15) to (16). The derivation process of (16) is demonstrated in Appendix.

$$f\left(\lambda_{i+1}|\lambda_i,\Lambda(t_s)<\Lambda_f\right) = Gam\left(\lambda_{i+1}-\lambda_i; a\cdot(t_{i+1}-t_i),\xi\right) \cdot\frac{1-\Gamma\left(a\cdot(t_s-t_{i+1}),(\Lambda_f-\lambda_{i+1})/\xi\right)/\Gamma\left(a\cdot(t_s-t_{i+1})\right)}{1-\Gamma\left(a\cdot(t_s-t_i),(\Lambda_f-\lambda_i)/\xi\right)/\Gamma\left(a\cdot(t_s-t_i)\right)}; i=1,\dots,n-1 \quad (16)$$

### M step

After the expected complete likelihood function has been estimated, a new set of parameters is obtained by maximising the expected complete likelihood function. For (10) and (11), the maximisation process can be performed by derivative based methods and is not discussed in this paper.

### Convergence check

The convergence check strategy used in this paper follows another paper by the authors [28]: The EM algorithm used in this paper can be divided into two stages. During the first stage, 1000 particles are used and the development processes of parameter estimates are used as the criteria of convergence. At the second stage, 2000 particles are used and the relative likelihood function [10] given by (17) is used to check the convergence. The details of the convergence check method are not discussed in this paper.

$$\log\left(\frac{f_{\theta^{(l)}}(x_{1:n},t_f)}{f_{\theta^{(l-1)}}(x_{1:n},t_f)}\right) = \log\left(E_{\theta^{(l-1)}}\left(\frac{f_{\theta^{(l)}}(x_{1:n},\lambda_{1:n})}{f_{\theta^{(l-1)}}(x_{1:n},\lambda_{1:n})}\bigg| x_{1:n},t_f\right)\right) = \log\left(\frac{1}{N_f}\sum_{j=1}^{N_f}\frac{f_{\theta^{(l)}}\left(s_{1:n}^{j(l)},x_{1:n}\right)}{f_{\theta^{(l-1)}}\left(s_{1:n}^{j(l)},x_{1:n}\right)}\right) \quad (17)$$

## 4. Lifetime prediction

After the parameters of the proposed model have been estimated, the lifetime can be predicted. In this section, both the cumulative density function (CDF) of the lifetime (i.e. the survival function) and the PDF of the lifetime are calculated.

The survival function given by (18) consists of two components. The first component is the PDF of the current underlying health state $\lambda_c$ given degradation indicators up to the current inspection, and the fact that the failure has not yet happened, i.e., $f\left(\lambda_c \middle| \vec{x}_{1:c}, \lambda_c < \Lambda_f\right)$, where $c$ denotes the current inspection index. Due to the non-Gaussian property of the Gamma-based state space model, $f\left(\lambda_c \middle| \vec{x}_{1:c}, \lambda_c < \Lambda_f\right)$ is calculated using the Monte Carlo-based particle filter. For details of the particle filter algorithm, readers can refer to [1]. The particle filter is conducted according to the conditional PDF given by (16). After particle filtering, $f\left(\lambda_c \middle| \vec{x}_{1:c}, \lambda_c < \Lambda_f\right)$ can be approximated by the filtering particles $f_c^{1:N_f} = \left\{f_c^j; j = 1,2,\ldots,N_f\right\}$. The second component of (18) is the survival function given the current degradation state, i.e., $\Pr\left(t \geq t_f \middle| \lambda_c\right)$. According to the property of the Gamma process, $\Pr\left(t \geq t_f \middle| \lambda_c\right)$ can be obtained as (19), where $I_A(\bullet)$ is the indicator function given by (20). After substituting (19) into (18), and using the particle filtering results $f_c^{1:N_f}$, the survival function is calculated as (21). After differentiating (21), the PDF of the lifetime is obtained as (22).

$$\Pr\left(t \geq T_f \middle| \vec{x}_{1:c}, \lambda_c < \Lambda_f\right) = = \int_0^{\Lambda_f} \Pr\left(t \geq T_f \middle| \lambda_c\right) f\left(\lambda_c \middle| \vec{x}_{1:c}, \lambda_c < \Lambda_f\right) d\lambda_c \tag{18}$$

$$\Pr\left(t \geq T_f \middle| \lambda_c\right) = \Pr\left(\Lambda(t) \geq \Lambda_f \middle| \lambda_c\right) = = \frac{\Gamma\left(a \cdot (t - t_c), \left(\Lambda_f - \lambda_c\right)/\xi\right)}{\Gamma\left(a \cdot (t - t_c)\right)} I_{(t_c\ \infty)}(t) \tag{19}$$

$$I_A(x) = \begin{cases} 0, & x \notin A \\ 1, & x \in A \end{cases} \tag{20}$$

$$\Pr\left(t \geq T_f \middle| \vec{x}_{1:c}, \lambda_c < \Lambda_f\right) = \int \Pr\left(\Lambda(t) \geq \Lambda_f \middle| \lambda_c\right) f\left(\lambda_c \middle| \vec{x}_{1:c}\right) d\lambda_c \approx \frac{1}{N_f} \sum_{i=1}^{N_f} \left( \frac{\Gamma\left(a \cdot (t - t_c), \left(\Lambda_f - f_c^i\right)/\xi\right)}{\Gamma\left(a \cdot (t - t_c)\right)} \cdot I_{(t_c\ \infty)}(t) \right) \tag{21}$$

$$f\left(t = T_f \middle| \vec{x}_{1:c}, \lambda_c < \Lambda_f\right) \approx \frac{1}{N_f} \sum_{i=1}^{N_f} \left( \frac{a}{\Gamma\left(a(t - t_c)\right)} \int_{\left(\Lambda_f - f_c^i\right)/\xi}^{\infty} \left\{\log(z) - \psi\left(a(t - t_c)\right)\right\} z^{a(t-t_c)-1} e^{-z} dz \cdot I_{(t_c\ \infty)}(t) \right) \tag{22}$$

## 5. Effectiveness evaluation of degradation indicators

In real applications, it is important to evaluate the relative effectiveness of different degradation indicators. After effective indicators have been identified, a more cost-effective condition monitoring system can be built by only installing necessary sensors. Moreover, the size of the database storing condition monitoring data can also be reduced. In addition, the over-fitting problem when applying a degradation model to a real dataset can be overcome by ignoring unnecessary degradation indicators. Some degradation models can identify the effectiveness of different degradation indicators. For example, the importance of different covariates of the PHM can be revealed by regression coefficients of the covariates. For the composite scale model, the effectiveness of different degradation indicators can be disclosed by weight parameters and mean values of degradation indicators [8]. In the proposed Gamma-based state space model, the relationships between degradation indicators and underlying health states are modelled by the observation equation in various formulations. Consequently, the effectiveness of a degradation indicator cannot be simply revealed by a certain parameter.

This research develops a parametric bootstrap method to evaluate the effectiveness of indicators by comparing their influences on the result of particle filtering. An indicator that affects the particle filtering results significantly can have a considerable impact on the result of parameter estimation, because the estimation of the expected complete likelihood function during the EM algorithm is based on the particle filtering and smoothing. In addition, the asset life prediction method also relies on the particle filter. Therefore, the influence of an indicator during particle filtering reveals the effectiveness of the indicator in degradation modelling and life prediction.

The process of the proposed indicators effectiveness evaluation method is as follows: Firstly, the proposed model is fitted to a training dataset and the parameters are estimated as $\hat{\theta}$. Then, $k$ sequences of simulated data are generated using the parameter estimates $\hat{\theta}$. After that, the particle filter is carried out to estimate underlying health states of the $k$ simulated degradation sequences. During particle filtering, each degradation indicator is omitted in turn, and the mean square error (MSE) of the estimation results is calculated. Thus $m$ MSEs are calculated as $MSE_j$ ($j$ = 1,...,$m$), where $MSE_j$ denotes the MSE of underlying health state estimates when the $j$-th indicator is omitted, and $m$ is the size of a degradation indicator vector. After that a particle filter considering all the indicators is applied to the simulated data, and the MSE of the underlying health state estimates is estimated as $MSE_0$. A relative contribution ratio is calculated as $r_j = MSE_j/MSE_0$ (obviously $r_j \in (1\ +\infty)$) for the $j$-th degradation indicator. A bigger value of $r_j$ indicates that the $j$-th degradation indicator is more important. On the contrary, if $r_j$ is close to one, the $j$-th degradation indicator can be omitted. However, degradation indicators which are highly correlated to each other may have relative contribution ratios close to one simultaneously. These indicators cannot be removed altogether. One solution is only omitting the indicator with the smallest relative contribution ratio, and then calculating the relative contribution ratios of the rest indicators again.

## 6. Simulation study

### 6.1. Simulation data generation

To investigate the performance of the proposed algorithms, a simulation study was conducted. First of all, a set of simulation data was generated. The simulation dataset consisted of two complete degradation sequences and two censored degradation sequences of degradation indicators. The parameters adopted to generate a simulation dataset were as follows:

$a_s = 0.005,\ \xi_s = 0.05,\ \vec{c}_s = \begin{pmatrix}2 & 2.5 & 3\end{pmatrix}',$

$$\vec{\Sigma}_s = \left(\begin{pmatrix}5 & -1 & 1\end{pmatrix}' \quad \begin{pmatrix}-1 & 5 & 2\end{pmatrix}' \quad \begin{pmatrix}1 & 2 & 6\end{pmatrix}'\right)\times 10^{-3}.$$

These parameters are illustrative only and without any particular meaning. The inspection interval is assumed to be 60 hours, i.e. $t_i - t_{i-1} = 60$. One of the four sequences of degradation indicators is shown in Fig. 1.

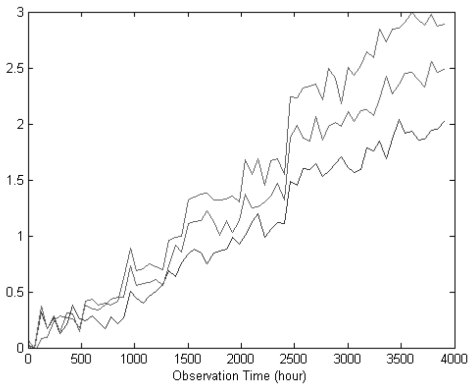


*Fig. 1. Simulated degradation indicators*

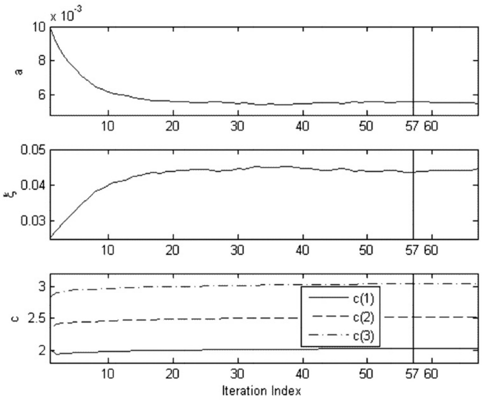


*Fig. 2. The convergence process of the EM algorithm*

## 6.2. Parameter estimation

Given the four degradation sequences, parameter estimation was conducted. First of all, according to (6), (7), and (8), initial parameters were estimated as $\hat{a}_1 = 0.01,\ \hat{\xi}_1 = 0.02535,$ $\hat{\vec{c}}_1 = \begin{pmatrix}2.007 & 2.322 & 2.811\end{pmatrix}$,

$$\hat{\vec{\Sigma}}_1 = \left(\begin{pmatrix}5.057 & -0.317 & 2.298\end{pmatrix}' \begin{pmatrix}-0.317 & 5.624 & 3.572\end{pmatrix}' \begin{pmatrix}2.298 & 3.572 & 7.892\end{pmatrix}'\right)\times 10^{-3}.$$

Then, EM iterations started with this initial parameter set. The EM iterations were conducted in two stages. In the first stage which lasted 57 iterations, 1,000 particles were used to perform particle smoothing. At the second stage, 2,000 particles were adopted for a better estimation results. As shown in Figure 2, the convergence process of parameter estimates became much smoother when 2,000 particles were used. After 67 iterations, the final results were acquired as: $\hat{a}_{67} = 0.005475,$ $\hat{\xi}_{67} = 0.04454,\ \hat{\vec{c}}_{67} = \begin{pmatrix}2.024 & 2.516 & 3.037\end{pmatrix}',$

$$\hat{\vec{\Sigma}}_{67} = \left(\begin{pmatrix}4.729 & -1.256 & 0.927\end{pmatrix}' \begin{pmatrix}-1.256 & 4.481 & 1.965\end{pmatrix}' \begin{pmatrix}0.927 & 1.965 & 6.162\end{pmatrix}'\right)\times 10^{-3}.$$

The parameter estimation results showed that the proposed EM algorithm can detect the unknown parameters.

## 6.3. RUL predication

To test the prediction ability of the proposed model, an additional simulated degradation sequence of indicators was generated. As described in Section 4, the lifetime prediction algorithm is divided into two steps. The first step is estimating the distribution of current underlying health state using the particle filter. As to the simulated data for test, underlying health states at different inspections were estimated as Fig. 3. The second step is predicting the RUL based on the underlying health state estimation results. The life prediction results and corresponding confidence intervals are demonstrated in Fig. 4. As shown in Fig. 4, when more condition monitoring indicators were available, the RUL prediction results became more accurate and the confidence intervals were narrower. The reason is that the prior estimate of the URL was updated by more degradation indicators and the fact that the asset still survived. Therefore, the proposed lifetime prediction algorithm can combine the information from degradation indicators and survived time.

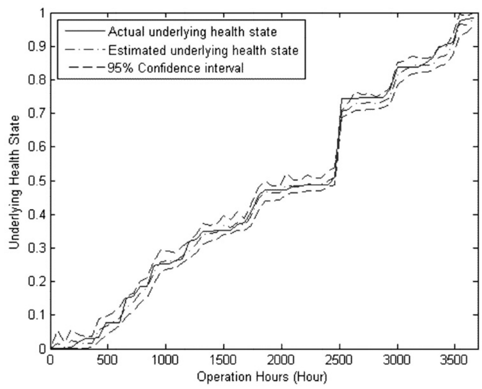


*Fig. 3. Estimation of underlying health states*

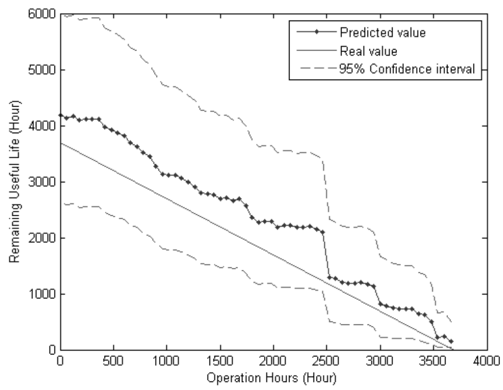


*Fig. 4. RUL prediction results*

### 6.4. Effectiveness evaluation of indicators

The effectiveness evaluation method for indicators was also tested by a simulation study. Firstly, 44 complete sequences of simulated degradation indicators were generated using the parameters: $a_s = 0.005$, $\xi_s = 0.05$, $\vec{c}_s = \begin{pmatrix}0.2 & 2.5 & 3\end{pmatrix}'$,

$$\vec{\Sigma}_s = \begin{pmatrix}\begin{pmatrix}1 & 0 & 0\end{pmatrix}' & \begin{pmatrix}0 & 0.005 & 0\end{pmatrix}' & \begin{pmatrix}0 & 0 & 0.005\end{pmatrix}'\end{pmatrix}.$$

The inspection interval was still assumed to be 60 hours. Four sequences of these simulated degradation indicators were used as training data; the other 40 sequences were used as test data. Based on the training data, the parameters were estimated as: $\hat{a} = 0.004903$, $\hat{\xi} = 0.04979$, $\hat{\vec{c}} = \begin{pmatrix}0.2292 & 2.568 & 3.08\end{pmatrix}'$,

$$\hat{\vec{\Sigma}} = \begin{pmatrix}\begin{pmatrix}1032 & 1.677 & -5.426\end{pmatrix}' \begin{pmatrix}1.677 & 4.622 & -0.4843\end{pmatrix}' \begin{pmatrix}-5.426 & -0.4843 & 4.613\end{pmatrix}'\end{pmatrix}\times 10^{-3}.$$

The bootstrap algorithm developed in Section 5 was then conducted. Forty sequences of simulated indicators were generated during the bootstrap process, and relative contribution ratios of different indicators were calculated as the second row of Table 1.

To investigate the performance of the proposed effectiveness evaluation algorithm for indicators, parameter estimation was conducted using the original training dataset when different indicators were omitted. When the first indicator was not considered the parameters were estimated as: $\hat{a}^{(1)} = 0.004847$, $\hat{\xi}^{(1)} = 0.05037$, $\hat{\vec{c}}^{(1)} = \begin{pmatrix}2.577 & 3.091\end{pmatrix}'$,

$$\hat{\vec{\Sigma}}^{(1)} = \begin{pmatrix}\begin{pmatrix}4.67 & -0.4555\end{pmatrix}' & \begin{pmatrix}-0.4555 & 4.67\end{pmatrix}'\end{pmatrix}\times 10^{-3}.$$

Similarly, when the other two indicators were omitted, the parameter estimates were: $\hat{a}^{(2)} = 0.004746$, $\hat{\xi}^{(2)} = 0.05144$, $\hat{\vec{c}}^{(2)} = \begin{pmatrix}0.2343 & 3.11\end{pmatrix}'$,
$\hat{\vec{\Sigma}}^{(2)} = \begin{pmatrix}\begin{pmatrix}1031 & -8.172\end{pmatrix}' & \begin{pmatrix}-8.172 & 4.631\end{pmatrix}'\end{pmatrix}\times 10^{-3}$, and
$\hat{a}^{(3)} = 0.004288$, $\hat{\xi}^{(3)} = 0.05693$, $\hat{\vec{c}}^{(3)} = \begin{pmatrix}0.2334 & 2.631\end{pmatrix}'$,
$\hat{\vec{\Sigma}}^{(2)} = \begin{pmatrix}\begin{pmatrix}1032 & 2.761\end{pmatrix}' & \begin{pmatrix}2.761 & 4.534\end{pmatrix}'\end{pmatrix}\times 10^{-3}$.

Using these parameter estimates, the particle filter was carried out to test data. The MSEs (denoted by $MSE_j'$) of the underlying health state estimates are given by the third row of Table 1. The MSE of the underlying health state estimates using all the three indicators was also calculated as $3.904\times10^{-4}$. The results displayed in Tab. 1 show that ignoring an indicator with a larger relative contribution ratio during parameter estimation can cause more significant error in underlying health state estimation. On the contrary, considering the first indicator whose relative contribution ratio is near one, can not improve the underlying health estimates significantly. Therefore, the proposed effectiveness evaluation method for indicators can recognize the importance of different indicators.

*Tab. 1. The results of effectiveness evaluation for indicators*

| Index of the indicator $j$ | 1 | 2 | 3 |
|---|---|---|---|
| Relative contribution ratio $r_j$ | 1.012 | 1.655 | 2.205 |
| $MSE_j'$ | $4.511\times10^{-4}$ | $6.729\times10^{-4}$ | $11.58\times10^{-4}$ |

## 7. Case study

### 7.1. Data introduction

Liquefied natural gas (LNG) pumps are critical in the LNG industry. An unexpected breakdown of an LNG pump can reduce the amount of LNG at the receiving terminal and cause performance degradation of the whole plant. The specifications of LNG pumps investigated in this case study are listed in Tab. 2, and the structure of an LNG pump is shown in Fig. 5. The LNG pump is enclosed within a suction vessel and mounted with a vessel top plate. Three ball bearings are installed to support the entire dynamic load of the integrated shaft of a pump and a motor. The three bearings in the LNG pump are self-lubricated at both sides of the rotor shaft and tail using LNG. Due to the low viscous value (about 0.16cP) of LNG, the three bearings are poorly lubricated. In addition, the bearings work at a high speed (3,600rpm). Therefore, bearings installed in these LNG pumps are failure-prone.

*Tab. 2. The specifications of the pump*

| Capacity | Pressure | Impeller Stage | Speed | Voltage | Rating | Current |
|---|---|---|---|---|---|---|
| 241.8 $m^3$/hr | 8.7 kg/$cm^2$· g | 9 | 3,585 RPM | 6,600V | 746 kW | 84.5 A |

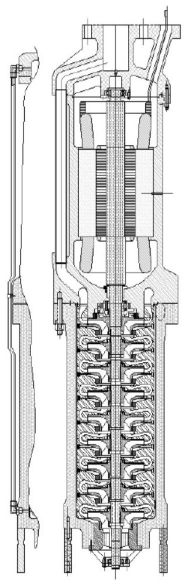

*Fig. 5. Pump schematic*

To monitor the health of the bearings, for each bearing, three accelerometers were installed on housing near the bearing assembly in horizontal, vertical and axial directions respectively. In this case study, vibration signals from two bearings installed on two LNG pumps were investigated. The vibration signals were sampled at irregular intervals. At the beginning and last stage of its lifetime, the vibration signals were measured more frequently; while at the middle stage of life, the vibration signals were collected at relatively larger intervals. This kind of irregular inspection strategy is often used in reality, because it is not necessary to measure vibration signals frequently when a bearing is running smoothly. The vibration signals investigated in this case study were all measured at the horizontal direction.

The overall features of the vibration signals are listed in Tab. 3. The outer raceway spalling and the inner raceway flaking on the bearings are shown in Fig. 6 and Fig. 7. In this case study, vibration signals collected from the bearing installed on Pump P301D were used to estimate the parameters of the proposed model, while the vibration signals collected from the bearing installed on Pump P301C were used to test the lifetime prediction ability of the proposed model.

*Tab. 3 Vibration data features*

| Machine No | Life Time | Failure Mode | Sample Number | Sampling Frequency |
|---|---|---|---|---|
| P301C | 4,698Hrs | Outer raceway spalling | 120 | 12,800 Hz |
| P301D | 3,511Hrs | Inner raceway flaking | 136 | 12,800 Hz |

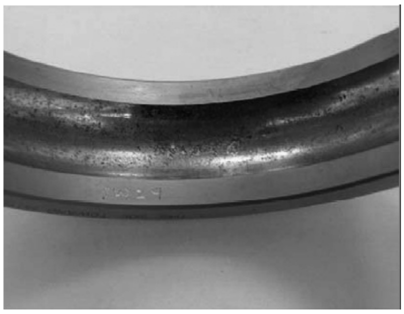

*Fig. 6. Outer raceway spalling of P301C*

*Fig. 7. Inner raceway flaking of P301D*

## 7. 2. Model application

Bearing failures (e.g. inner race crack, outer race crack, and rolling element crack) often generate shock pulses whose energy emanates at a relatively high frequency band. Therefore, a vibration signal, after high pass filtering (HPF), is often more sensitive to early defects of a bearing. For a raw vibration signal, the kurtosis and the crest factor which reveals the number of extreme deviations can also indicate early defects. After investigating different features of the vibration signals used in this case study, three features were adopted as degradation indicators of the proposed model: the entropy of the vibration signal after HPF at 3,000 Hz, the crest factor of the vibration signals after HPF at 2,500 Hz, and the crest factor of the raw vibration signals.

Using vibration signals collected from Pump P301D, the parameters of the proposed model were estimated as

$$\hat{a} = 0.01087,\ \hat{\xi} = 0.02621,\ \hat{c} = \left(1.658 \quad 0.6134 \quad 2.392\right)',$$

$$\hat{\mathbf{\Sigma}} = \left(\left(5.295\ \ \text{-}1.439\ \ \text{-}1.764\right)' \left(\text{-}1.439\ \ 5.356\ \ \text{-}1.099\right)' \left(\text{-}1.764\ \ \text{-}1.099\ \ 5.741\right)'\right) \times 10^{-2}.$$

The effectiveness of the three indicators was also investigated. Tab. 4 shows that the crest factor of the raw signals has the highest relative contribution ratio. However, the relative contribution ratios of the three features are close to each other. Therefore, none of the features can be omitted.

*Tab. 4. Effectiveness evaluation for the three features extracted from the vibration signals*

| Features | Entropy after HPF at 3000 Hz | Crest factor after HPF at 2500 Hz | Crest factor of the raw signal |
|---|---|---|---|
| Relative contribution ratio | 1.594 | 1.305 | 2.155 |

Using the model parameters estimated using the vibration signals collected form P301D, the RUL of the bearing installed on Pump P301C was obtained as Figure 8. At the beginning, the prediction error was significant. This was caused by the difference between the lifetimes of the training dataset and the test dataset. At the beginning, only few condition monitoring observations were collected. The RUL was largely predicted based on the lifetime of the training dataset which was much shorter than that of the test data. Consequently, the predicted RUL was shorter than the actual value. When a longer indicator history was considered, the slower degradation progress of the bearing from P301C was detected. As a result, the prediction error decreased. Especially at the last stage of the life, prediction results were very close to real values. Fig. 8 also illustrates that most actual RUL values fall in the 95% confidence interval, even at the beginning of the life.

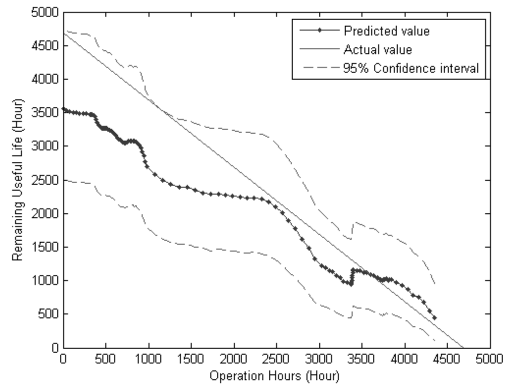


*Fig. 8. RUL prediction results of the bearing on P301C*

## 7. 3. Discussion

The results of this case study show that the proposed Gamma-based state space model can overcome the limitation of failure data by considering degradation processes of multiple indicators. Furthermore, using the particle filtering method, the remaining useful life estimate can be updated recursively by considering the degradation indicators extracted from condition monitoring data.

The continuous property also makes the proposed model an appropriate candidate for this case study. In this case study, the inspection intervals were extremely irregular, which varied from 3 hours to 133 hours. Converting these uneven observation intervals to equal ones by interpolation is extremely difficult. Therefore, degradation models (e.g. [17]) with the discrete time assumption were not used in this case study. Moreover, discretising of the degradation indicators is also difficult due the inadequate knowledge of the degradation process of a bearing on a LNG pump. Therefore, a state space model continuous in time and state is preferable in this case study.

## 8. Conclusions

This research has developed a Gamma-based state space model to predict asset lives using both failure events and degradation indicators. Compared with existing state space degradation models, the proposed model is continuous in time and states, and does not require the Gaussian assumption. This continuous property enables the proposed model to process irregular inspection intervals and avoid discretising continuous degradation indicators. Furthermore, the monotonically increasing Gamma process used in the proposed model is more appropriate to model the irreversible asset health degradation processes than the commonly used Gaussian process. The monotonically increasing property of the Gamma process also makes the construction of the likelihood function easier than non-monotonically increasing stochastic processes when failure events are considered.

To deal with the non-Gaussian property of the proposed model, a Monte Carlo-based EM algorithm has been proposed to estimate the parameters and the censored degradation data have been considered in the parameter estimation algorithm. The asset life prediction algorithm has also been developed using the Monte Carlo method and Bayesian theory. In addition, this paper has developed an effectiveness evaluation method for degradation indicators to identify the relative importance of the degradation indicators adopted in the state space model. The performance of the proposed algorithms has been evaluated in simulation studies and a real application.

## 9. Appendix

The inference of conditional PDF of underlying health states for censored data:

$$f\left(\Lambda(t_{i+1})=\lambda_{i+1}\middle|\Lambda(t_i)=\lambda_i,\Lambda(t_s)<\Lambda_f\right)=\frac{d}{du}\Pr\left(\Lambda(t_{i+1})\le u\middle|\Lambda(t_i)=\lambda_i,\Lambda(t_s)<\Lambda_f\right)\Big|_{u=\lambda_{i+1}}$$

$$=\frac{\frac{d}{du}\Pr\left(\Lambda(t_{i+1})\le u,\Lambda(t_s)<\Lambda_f\middle|\Lambda(t_i)=\lambda_i\right)\Big|_{u=\lambda_{i+1}}}{P\left(\Lambda(t_s)<\Lambda_f\middle|\Lambda(t_i)=\lambda_i\right)}$$

$$=\frac{\frac{d}{du}\int_{\lambda_i}^{\Lambda_f}\Pr\left(\Lambda(t_{i+1})\le u,\Lambda(t_s)=v\middle|\Lambda(t_i)=\lambda_i\right)dv\Big|_{u=\lambda_{i+1}}}{\Pr\left(\Lambda(t_s)<\Lambda_f\middle|\Lambda(t_i)=\lambda_i\right)}$$

$$=\frac{\int_{\lambda_{i+1}}^{\Lambda_f}f\left(\Lambda(t_{i+1})=\lambda_{i+1},\Lambda(t_s)=v\middle|\Lambda(t_i)=\lambda_i\right)dv}{\Pr\left(\Lambda(t_s)<\Lambda_f\middle|\Lambda(t_i)=\lambda_i\right)}$$

$$=\frac{\int_{\lambda_{i+1}}^{\Lambda_f}f\left(\Lambda(t_s)=v\middle|\Lambda(t_{i+1})=\lambda_{i+1}\right)f\left(\Lambda(t_{i+1})=\lambda_{i+1}\middle|\Lambda(t_i)=\lambda_i\right)dv}{\Pr\left(\Lambda(t_s)<\Lambda_f\middle|\Lambda(t_i)=\lambda_i\right)}$$

$$=\frac{f\left(\Lambda(t_{i+1})=\lambda_{i+1}\middle|\Lambda(t_i)=\lambda_i\right)P\left(\Lambda(t_s)<\Lambda_f\middle|\Lambda(t_{i+1})=\lambda_{i+1}\right)}{\Pr\left(\Lambda(t_s)<\Lambda_f\middle|\Lambda(t_i)=\lambda_i\right)}$$

$$=Gam\left(\lambda_{i+1}-\lambda_i;a\cdot(t_{i+1}-t_i),\xi\right)\frac{1-\Gamma\left(a\cdot(t_s-t_{i+1}),(\Lambda_f-\lambda_{i+1})/\xi\right)/\Gamma\left(a\cdot(t_s-t_{i+1})\right)}{1-\Gamma\left(a\cdot(t_s-t_i),(\Lambda_f-\lambda_i)/\xi\right)/\Gamma\left(a\cdot(t_s-t_i)\right)}$$

where $\Gamma(a,x)=\int_{z=x}^{\infty}z^{a-1}e^{-z}dz$ is the incomplete Gamma function.

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