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PODEJŚCIE NIEPARAMETRYCZNE DO MODELOWANIA PRZYROSTU NIEZAWODNOŚCI ZŁOŻONYCH SYSTEMÓW NAPRAWIALNYCH

A NON-PARAMETRIC APPROACH FOR MODELLING RELIABILITY GROWTH OF COMPLEX REPAIRABLE SYSTEMS

W złożonym systemie naprawialnym, przyczyny uszkodzeń zidentyfikowane przez program do badania przyrostu niezawodności nie zawsze są usuwalne. Duże wyzwanie stanowi ocena niezawodności systemu na podstawie obserwacji z badań oraz działań naprawczych. Niniejszy artykuł przedstawia nieparametryczną metodę oceny niezawodności systemu. Proponowane podejście uwzględnia informacje o częściowej skuteczności działań naprawczych. Wykorzystanie tych częściowych informacji pozwala na zrewidowanie obserwowanego zestawu danych oraz późniejsze włączenie ich do modelu rozkładu. Średni czas pomiędzy uszkodzeniami (MTBF) ocenia się na podstawie dopasowanego modelu. Proponowane podejście zilustrowano przykładem.

Słowa kluczowe: *rozwój produktu, przyrost niezawodności, skuteczność działań naprawczych, średni czas pomiędzy uszkodzeniami MTBF.*

For a complex repairable system, the identified failure modes during a reliability growth test program are not always fixable. It is a challenging issue to estimate the reliability of the system based on the test observations and corrective actions. This paper presents a non-parametric approach for evaluating the system reliability. The proposed approach incorporates partial information about the effectiveness of corrective actions. Using the partial information, the observed data set is revised, which is then fitted into a distribution model. The MTBF is estimated based on the fitted model. The proposed approach is illustrated by an example.

Keywords**:** *product development, reliability growth, effectiveness of corrective action, MTBF.*

1. Introduction

The improvement in reliability can be achieved by a test-analysis-and-fix process. The process begins with the testing of prototypes of a product or component. Product reliability improves during prototype testing by various corrective actions such as design changes and modifying operating procedures and environment. The process ends when the product reliability (e.g., failure rate or MTBF) reaches a pre-specified level [7-10]. Here, a key issue is how to predict the ultimate reliability of the product based on the test observations and taken corrective actions. A number of reliability growth models have been developed in the literature for evaluating and predicting the reliability of the product, e.g., see [4] and [6]. However, the predicted reliability can be inaccurate if the effectiveness of corrective actions is not appropriately modeled (see [11]).

Quantifying or modeling the effectiveness of corrective actions has attracted some attention. Current approaches are mainly based on subjective judgment or/and historic data, and subjected to the following problems:

- It is often difficult for experts to quantify the required information, and
- The past data may not be suitable for the current situation.

This paper quantifies the effectiveness of corrective actions using partial information. It requires the experts specifying whether a corrective action is a full or partial fix. Then, the test data are accordingly revised, and the revised data is fitted into a lifetime distribution using common statistical approaches. As a result, the reliability measures can be derived from the fitted distribution.

The paper is organized as follows. Section 2 presents the problem background, and Section 3 presents the proposed approach. An example is used to illustrate the approach in Section 4. The paper is concluded with a brief summary in Section 5.

2. Problem background

Reliability growth modeling mainly deals with the following problems [11]:

- Planning reliability growth tests, e.g., determining testing procedure, including the total test time.
- Monitoring progress, e.g., determining whether or not the program is progressing as scheduled. This needs to estimate current (instantaneous) reliability.
- Prediction, i.e., forecasting future reliability improvement.

To appropriately estimate or forecast reliability, one needs to appropriately quantify the effectiveness of corrective actions. During a reliability growth test, after a failure occurs, two optional actions are:

- Repair (minimal repair for the repairable products and perfect repair for non-repairable products), and
- Corrective action.

Since not all failure causes can be detected and corrected in a reliability growth test, failures are usually classified as inherent (or non-assignable) and fixable (or assignable) (e.g., see [1]).

For fixable failures, the corrective actions can be carried out instantly or delayed until the end of the test. The former results in test-fix-test type data and the latter results in test-find-test type data. A common case is their combination and results in test-fix-find-test type data ([2], [3]).

For the test-fix-test type data, a classical model is the Duane model (also termed as learning curve model, see [7]) given by:

$$\mu_j = \lambda t_j^{\delta} \tag{1}$$

where $\mu_j = t_j / j$ is called the cumulative MTBF.

A well-known variant of the Duane model is the Crow model (e.g., see [3]) with the failure intensity function given by

$$r(t) = \frac{\beta}{\eta}(t/\eta)^{\beta-1} \quad \text{or} \quad n(t) = (t/\eta)^{\beta} \tag{2}$$

where η and β are the model parameters to be estimated and $n(t)$ is the expected failure number by t. $\beta < 1$ implies that the system reliability is improving due to corrective actions. If the test ends at $t = T$, the maximum likelihood estimates for η and β are:

$$\beta = \frac{1}{\ln(T) - \frac{1}{n}\sum_{i=1}^{n}\ln(t_i)} \qquad \eta = \frac{T}{n^{1/\beta}} \tag{3}$$

and the MTBF is estimated by $1/r(T)$. The model does not assume that all failures receive a corrective action, and the effectiveness of a corrective action seems to be implicitly reflected in subsequent failure observations.

For the test-fix-test case, the observed failure modes can be divided into two types – Type A (no corrective action assigned to them) and Type B (a design change associated with them). Assuming that each failure mode has constant failure intensity, the reliability is estimated by incorporating the test observations with subjective judgment for the effectiveness of corrective action. An effectiveness factor is usually used to represent the effectiveness of a corrective action. Let λ_0 and λ denote the failure rates before and after a corrective action, the effectiveness factor is defined as below (e.g., see [6]):

$$d = \frac{\lambda_0 - \lambda}{\lambda_0} \quad \text{or} \quad \lambda = (1-d)\lambda_0 \tag{4}$$

It is noted that λ^{-1} is MTBF, the approach actually assigns a new MTBF to replace the original MTBF for a certain failure mode. For example, when d=0.7, the corrective action results in a new MTBF equal to 3.33 times of the original MTBF. It appears somewhat difficult to specify such information in an explicit way.

Since each failure corresponds to a failure mode with a specified factor, a test-fix-test process can be represented by a marked point process given by:

$$\{t_i, m_i, a_i, i = 1, 2, ..., n\} \tag{5}$$

where t_i is the time that the i-th failure occurs, m_i is the failure mode, and a_i represents the effectiveness of the corrective action. The failure modes can reoccur unless d=1.

3. Proposed approach

Suppose that a complex repairable system has two types of failure modes: inherent and fixable. The reliability growth test is either test-fix-test or test-find-test process. We estimate the MTBF at the end of each corrective action or test stage based on the following procedures:

- Step 1: Revise the test data based on engineering judgment for the effectiveness of corrective actions. The effectiveness of a corrective action can be one of the three cases: ineffective (i.e., no improvement), imperfect with a partial fix and perfect with a full fix.
- Step 2: Fit the revised test data into a lifetime distribution. We consider the Weibull distribution.
- Step 3: Evaluate the system reliability (e.g., MTBF, failure rate, etc.) based on the fitted model.
- Step 4: Fit the reliability measure as a function of the total test time or the number of stages for the purpose of prediction or decision analysis.

Here, the key steps are Steps 1 and 4. We focus on them in this section.

3.1. Revision of observed data

We discuss how to revise the test data for each of the three possible correction cases as follows.

1) No corrective action case

If there is no corrective action or the corrective actions are ineffective, then time sequence between failures is:

$$\{x_i = t_i - t_{i-1}, t_0 = 0, i = 1, 2, ..., n\} \tag{6}$$

The sequence $\{x_i\}$ can be directly used to fit a Weibull distribution with parameters η and β, and MTBF is estimated by

$$\mu = \eta\Gamma(1 + 1/\beta) \tag{7}$$

If there are imperfect or perfect corrective actions, (7) will considerably underestimate MTBF.

2) Full fix case

For a full fix, the corrected failure mode will not reoccur. Suppose that there is a corrective action at t_i. Consider two cases: $i<n$ and $i=n$.

- If $i<n$, the effect of a perfect fix is equivalent to removing t_i from the original data set.
- If $i=n$, the effect of a perfect fix is equivalent to changing t_n as a right censored observation, or changing x_n into an interval data (x_n,∞), for which the likelihood function is 1-$F(x_n)$

In such a way, we obtain a new reduced data set, which can be fitted into a Weibull distribution. The estimate of MTBF based on this data set is generally an upper bound of MTBF unless all the corrective actions are perfect.

3) Partial fix case

In this case, we consider the data set $\{ x_i, i = 1,2,...,n\}$. We look at two cases:

- No corrective action at t_i, and
- There is a corrective action at t_i.

a) No corrective action at t_i: The revision depends on whether there is a corrective action at t_{i-1} or not.

- If there is no corrective action at t_{i-1}, x_i keeps unchanged.
- If there is a corrective action at t_{i-1}, change x_i into an interval datum (x_i, t_i- t_{i-k}), where t_{i-k} is the last failure time with no corrective action before t_{i-1} (t_{i-k} = 0 if all the failures before t_{i-1} are corrected). This is because x_i keeps unchanged if the corrective action at t_{i-1} is ineffective and x_i would be t_i- t_{i-k} if all these corrective actions are perfect.

b) There is a corrective action at t_i:

- If there is a corrective action at t_{i-1}, change x_i into an interval datum (x_i,∞).
- If there is no corrective action at t_{i-1}, we need to look at two case: $i<n$ and $i=n$. If $i<n$, remove x_i (its effect will be reflected in revised x_{i+1}, which is an interval data with the lower bound is x_i, see the second point of "No corrective action at t_i"); if $i=n$, change x_i into an interval datum (x_i,∞).

In such a way, we obtain a new data set, which will be fitted into a Weibull distribution. The estimated MTBF based on this revised data set is generally an underestimate since some of the corrective actions can be perfect. Thus, the two estimates for MTBF associated with the full fix and partial fix will form a reasonable interval estimate for MTBF, and this will be confirmed in Section IV.

Since the corrective actions can be either perfect or imperfect, it is more reasonable to revise the test data set according to the judgment for the effectiveness of each corrective action. This can be done according to the rules outlined above.

3.2. Fit of the reliability measure

For the test-fix-test case, using the above approach we can obtain a sequence of μ_i versus t_i, where t_i is the time after each corrective action. The sequence (μ_i,t_i) can be fitted into an empirical relation, such as (1), for relevant decision analysis.

For the test-find-test case, we can estimate μ_i at the end of the i-th stage. As such, we can obtain a sequence of μ_i versus i. The sequence $(\mu_i,i,)$ can be fitted into an empirical relation for relevant decisions. An optional relation is as below:

$$\mu_i = \lambda i^{\delta} \tag{8}$$

4. Example

The data shown in Table 1 come from [11]. Among 17 failures, only 4 are corrected. The failure modes are divided into three types: random (9 observations without any corrective actions), hardware (6 observations with 3 corrected), and manufacturing process (2 observations with 1 corrected). The sample is not big. The reliability measure is MTBF.

4.1. MTBF estimates from demonstration test and reference models

We first look at the three cases:

- MTBF estimate from reliability demonstration test
- MTBF estimate from the Duane model
- MTBF estimate from the Crow model.

1) MTBF estimate from reliability demonstration test

It is noted from Table 1 that no configuration change was made after $t_9 = 286.37$ test hours. This implies that the subsequent test can be viewed as a reliability demonstration test. As a result, the demonstrated MTBF is equal to $\frac{t_{17}-t_9}{8} = 55.36$ hours. We will use it as the "true" value of MTBF.

2) MTBF estimate from the Duane model

Fitting the data into the Duane model given by (1), we have the estimated parameters and MTBF shown in the second column of Table 2. The relative error (RE, see the last row) of the estimated MTBF relative to the demonstrated MTBF equals 18.1%, implying that the Duane model considerably underestimates MTBF. On the other hand, if we remove those observations with corrective actions and then fit the revised data set into the Duane model, we have the estimates shown in the third column. In this case, the Duane model considerably overestimates MTBF. The above results confirm the conclusion that it is unreasonable to estimate MTBF using the Duane model [11].

Tab. 1. A marked failure point process [11]

i	t_i *hours*	*Mode*	*Action*
1	8.00	Random	
2	14.70	Hardware	Change
3	63.57	Random	
4	70.05	Hardware	Change
5	85.72	Random	
6	86.70	Random	
7	170.55	Manufacturing process	
8	241.92	Manufacturing process	Change
9	286.37	Hardware	Change
10	400.00	Random	
11	418.69	Random	
12	424.19	Hardware	
13	506.24	Hardware	
14	569.22	Random	
15	646.70	Random	
16	705.07	Hardware	
17	729.22	Random	

Tab. 2. Results of the Duane model

	Without data removal	*After data removal*	*Average*
λ	3.3148	5.9840	
δ	0.4053	0.3585	
MTBF	45.33	63.58	54.46
RE, %	18.1	14.8	1.6

3) MTBF estimate from the Crow model

Using the maximum likelihood method (MLM) and least squares error (LSE) method, we obtained the estimates shown in Table 3 for the Crow model given by (2). According to the relative errors shown in the last row, the MTBF estimate based on the MLM is fairly accurate but the estimate based on the LSE is very poor. This implies that the Crow model is very sensitive to the parameter estimate method.

Tab.3. Results of the Crow model

	MLM	*LSE*
β	0.7257	0.5973
η	14.70	6.91
MTBF	59.11	75.49
RE, %	6.8	36.4

4.2. MTBF estimates from the proposed model

Using the approach outlined in Section 3.1, we obtained the estimated model parameters and MTBF shown in Table 4 for the three cases. According to the relative errors shown in the last row, the MTBF estimates are quite accurate. In addition we have the following observations:

- The MTBF estimate for the no corrective action case is very close to the cumulative MTBF (i.e., t_n/n=729.22/17=42.90).
- The MTBF estimate associated with the full fix is an overestimate, and the estimate associated with the partial fix is an underestimate. This confirms the earlier analysis. In addition, the two estimates are fairly close to each other. This implies that they can build a relatively tight interval estimate for MTBF. Their average is a good point estimate for MTBF (see the last column in Table 4).
- $\beta \approx 1$ for all the three cases. This seemingly supports the use of the exponential distribution when modeling all the data.

Tab.4. Parameters of the Weibull model

	No action	Full fix	Partial fix	Average
β	1.0740	0.9386	1.0620	
η	43.98	54.54	53.78	
MTBF	42.79	56.14	52.53	54.33
RE, %	22.7	1.4	5.1	1.9

5. Conclusions

In this paper, a non-parametric approach has been proposed for modeling reliability growth for complex repair systems. The proposed approach is actually an algorithm to evaluate the system reliability based on test data and the effectiveness of corrective actions. The required engineering judgment is to specify whether a corrective action is a full fix or a partial fix, and hence the approach is very simple and easily implemented. It makes few assumptions and considers the actual effect of corrective actions, and hence should be able to provide more accurate assessment for the system reliability. The accuracy has been illustrated by an example. In the meantime, the limitations of the Duane and Crow models have been identified.

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