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MODELOWANIE DYNAMICZNO-NIEZAWODNOŚCIOWE SYSTEMÓW Z USZKODZENIAMI O WSPÓLNEJ PRZYCZYNIE W WARUNKACH OBCIĄŻENIA LOSOWEGO

DYNAMIC RELIABILITY MODELING OF SYSTEMS WITH COMMON CAUSE FAILURE UNDER RANDOM LOAD

Artykuł przedstawia nową metodę tworzenia modeli dynamiczno-niezawodnościowych systemów, w których niezawodność i stopa ryzyka wyrażane są jako funkcje obciążenia, wytrzymałości i czasu. W pierwszej części artykułu przedstawiono sposób tworzenia modeli niezawodnościowych systemów z uszkodzeniami o wspólnej przyczynie stosując model interferencji pomiędzy obciążeniem a wytrzymałością, oraz wyprowadzono funkcje rozkładu kumulacyjnego oraz gęstości prawdopodobieństwa wytrzymałości dla różnych systemów. Utworzono także modele niezawodnościowe systemów w warunkach cyklicznego obciążenia losowego. Następnie opisano proces obciążania jako proces stochastyczny Poissona oraz wyprowadzono dynamiczne modele niezawodnościowe systemów o nie zmniejszającej się i zmniejszającej się wytrzymałości. Na koniec omówiono związek pomiędzy niezawodnością i czasem oraz stopę ryzyka systemów. Wyniki pokazują, że nawet przy nie zmniejszającej się wytrzymałości, niezawodność systemów zmniejsza się wraz z upływem czasu, podobnie jak ich stopa ryzyka. Gdy spada wytrzymałość, niezawodność systemów zmniejsza się szybciej wraz z upływającym czasem. Proponowane modele można wykorzystywać przy ustalaniu czasu trwania pracy próbnej, czasu niezawodnej pracy oraz harmonogramu eksploatacyjnego. Są one pomocne w zarządzaniu cyklem życia systemów.

***Słowa kluczowe**: dynamiczna niezawodność; uszkodzenie o wspólnej przyczynie; niezawodność systemu; stopa ryzyka; interferencja pomiędzy obciążeniem a wytrzymałością*

This paper presents a new method for developing the dynamic reliability model of systems, in which reliability and hazard rate of systems are expressed as functions of load, strength and time. First, reliability models of systems with common cause failure are developed by applying the load-strength interference model, and the cumulative distribution function and the probability density function of strength for different systems are derived. Reliability models of systems under repeated random load are developed. Then, the loading process is described as a Poisson stochastic process, the dynamic reliability models of systems without strength degeneration and those with strength degeneration are derived. Finally, the relationship between reliability and time, and the hazard rate of systems, are discussed. The results show that even if strength does not degenerate, the reliability of systems decreases over time, and the hazard rate of systems decreases over time, too. When strength degenerates, the reliability of systems decreases over time more rapidly, and the hazard rate curves of systems are bathtub-shaped. The models proposed can be applied to determine the duration of a trial run, the reliable operation life and the maintenance schedule. It is helpful for the life cycle management of systems.

***Keywords:** dynamic reliability; common cause failure; system reliability; hazard rate; load-strength interference*

1. Introduction

Reliability, as the probability for products to perform its intended functions satisfactorily under specified conditions for a specified period of time, has long been treated as one of the most important performance attributes of products. It has been embodied in all stages of product life cycle, such as design, manufacture, service and maintenance [9-10,16].

Common cause failure (CCF), as one of the important failure modes, exists in many engineering systems, especially in nuclear plants, aviation & astronavigation systems, where high reliability is demanded [14,15,28,30]. Many researchers have investigated the effect of CCF on reliability, and introduced a variety of reliability models of systems with CCF, such as common load model (CLM) [17], basic parameter model (BPM) [24], binomial failure rate (BFR) model [2], α factor model (AFM) [18], stochastic reliability analysis (SRA) model [6], and random probability shock (RPS) model [8]. Some of these models have been applied successfully in reliability analysis and probability risk assessment of engineering systems [4,25].

On the other hand, reliability calculated by conventional reliability models, is the reliability when random load is applied only once or for a specified number of times [1,13, 19,26]. In other words, these models cannot reflect the relationship between reliability and the number of load applications explicitly [26]. Although fatigue reliability models can be used to calculate the reliability of components or systems corresponding to a different number of load cycles, they can only reflect the effect of strength degeneration caused by the repetition of load, but cannot reflect the effect of load itself on reliability.

Recently, several researchers have investigated the effect of time-dependent factors on reliability and proposed several dynamic reliability models. Torres & Ruiz [23] proposed an approach to evaluating structural reliability that takes into account capacity degradation over time by means of closed mathematical expressions. They considered events of different intensities by means of environmental hazard curves and assumed that the structural capacity decreased linearly with time. Huang & Chang [10] applied a modularization algorithm on a fault tree, and presented an enhanced approach for sensitivity analysis of dynamic failure tree models with dependencies. Becker et al [7] proposed a theory of dynamic reliability that incorporates random changes of the state variables at the time points of transition between the discrete states of the Markovian component of the model. Czarnecki & Nowak [3] developed a time-variant reliability-based model for evaluation of steel highway bridges with regard to corrosion, in which load and resistance parameters were treated as time-variant random variables, and the limit state functions were formulated based on the available models. Schoenig et al [20] proposed a quantitative analysis method for reliability of hybrid systems, based on the construction of an aggregated Markov graph and the Petri net model of systems. Streicher & Rackwitz [21] proposed a method for reliability-oriented time-variant structural optimization of independent series systems using the first order reliability methods in standard space. Ionescu et al [13] used the Petri nets formalism to model the reliability of the medium voltage distribution systems for a nuclear power plant. Tian & Noore [22] proposed a support vector machine modeling approach for dynamic software reliability prediction.

In the literature reviewed, the hazard rate of components in the systems was usually assumed to be a constant, which was sometimes derived from experimental data indirectly or assumed theoretically, and the relationship among the hazard rate, load, and strength cannot be embodied explicitly. In this paper, we develop a new method for modeling the dynamic reliability of systems, and study the behaviors of reliability and the hazard rate of systems as functions of time.

2. Reliability models of systems with common cause failure

In this section, through introduction of the conditional reliability, the reliability models of systems with CCF, are developed with the load-strength interference (LSI) model at the system level.

The LSI model has been applied widely for the calculation and analysis of reliability, when the cumulative distribution function (CDF) and probability density function (PDF) of strength δ are $F_\delta(\delta)$ and $f_\delta(\delta)$, respectively, and the CDF and PDF of load s are $F_s(s)$ and $f_s(s)$, respectively. The reliability under strength δ and load s can be expressed as

$$R = \int_{-\infty}^{+\infty} f_s(s) \int_s^{+\infty} f_\delta(\delta)\mathrm{d}\delta\mathrm{d}s = \int_{-\infty}^{+\infty} f_s(s)\left(1 - F_\delta(s)\right)\mathrm{d}s \quad (1)$$

Eq. (1) is usually used as the reliability model for components, however, it can also be used to model the reliability of systems directly [30, 31].

Considering a special case that load s is deterministic, the reliability is the probability that the random strength exceeds the deterministic load. In this case, the failure of components in the system is independent, because the failure of each component is completely determined by its own strength [15, 30]. The reliability when load is deterministic can be taken as the conditional reliability of systems.

For a system with n identical components, when load s is deterministic, the conditional reliability of the system can be expressed as

$$R_{\text{ser}}(s) = \left(\int_s^{+\infty} f_\delta(\delta)\mathrm{d}\delta\right)^n = \left[1 - F_\delta(s)\right]^n \quad (2)$$

$$R_{\text{par}}(s) = 1 - \left(\int_{-\infty}^{s} f_\delta(\delta)\mathrm{d}\delta\right)^n = 1 - \left[F_\delta(s)\right]^n \quad (3)$$

$$\begin{aligned} R_{k/n}(s) &= \sum_{i=k}^{n} C_n^i \left(\int_s^{+\infty} f_\delta(\delta)\mathrm{d}\delta\right)^i \left(\int_{-\infty}^{s} f_\delta(\delta)\mathrm{d}\delta\right)^{n-i} \\ &= \sum_{i=k}^{n} C_n^i \left[1 - F_\delta(s)\right]^i \left[F_\delta(s)\right]^{n-i} \end{aligned} \quad (4)$$

Now we consider the case when load s is a random variable with PDF $f_s(s)$, the reliability of systems can be written as

$$R_{\text{ser}} = \int_{-\infty}^{+\infty} \left(\int_s^{+\infty} f_\delta(\delta)\mathrm{d}\delta\right)^n f_s(s)\mathrm{d}s = \int_{-\infty}^{+\infty} \left[1 - F_\delta(s)\right]^n f_s(s)\mathrm{d}s \quad (5)$$

$$R_{\text{par}} = \int_{-\infty}^{+\infty} \left[1 - \left(\int_{-\infty}^{s} f_\delta(\delta)\mathrm{d}\delta\right)^n\right] f_s(s)\mathrm{d}s = \int_{-\infty}^{+\infty} \left\{1 - \left[F_\delta(s)\right]^n\right\} f_s(s)\mathrm{d}s \quad (6)$$

$$\begin{aligned} R_{k/n} &= \int_{-\infty}^{+\infty} \left\{\sum_{i=k}^{n} C_n^i \left[\int_s^{+\infty} f_\delta(\delta)\mathrm{d}\delta\right]^i \left[\int_{-\infty}^{s} f_\delta(\delta)\mathrm{d}\delta\right]^{n-i}\right\} f_s(s)\mathrm{d}s \\ &= \int_{-\infty}^{+\infty} \left\{\sum_{i=k}^{n} C_n^i \left[1 - F_\delta(s)\right]^i \left[F_\delta(s)\right]^{n-i}\right\} f_s(s)\mathrm{d}s \end{aligned} \quad (7)$$

Eqs. (5)- (7) are derived through the concept of conditional reliability and without the assumption that failures of components are independent, and they have the capability of reflecting the effect of CCF on reliability.

3. System strength and its probability distribution

In this section, the structure of the reliability models of systems developed above is studied, and the PDFs and CDFs of strength of systems are derived.

According to Eqs. (5)-(7), the reliability models of systems can be unified as

$$R_{\text{S}} = \int_{-\infty}^{+\infty} f_s(s) \int_s^{+\infty} f_{\text{e}}(\delta)\mathrm{d}\delta\mathrm{d}s = \int_{-\infty}^{+\infty} f_s(s)\left(1 - F_{\text{e}}(s)\right)\mathrm{d}s \quad (8)$$

where $F_e(\delta)$ and $f_e(\delta)$ are the CDF and PDF of system strength, respectively. For different system structures, they have different expressions, namely

For the series system, $F_e(s)$ is

$$F_{\text{e}}(s) = 1 - \left[1 - F_\delta(s)\right]^n \quad (9)$$

For the parallel system, $F_e(s)$ is

$$F_{\text{e}}(s) = 1 - \left\{1 - \left[F_\delta(s)\right]^n\right\} \quad (10)$$

For the k-out-of-n system, $F_e(s)$ is

$$F_{\text{e}}(s) = 1 - \sum_{i=k}^{n} C_n^i \left[1 - F_\delta(s)\right]^i \left[F_\delta(s)\right]^{n-i} \quad (11)$$

Obviously, Eq. (8) has the same structure as Eq. (1), in which $F_\delta(s)$ is the CDF of the strength of a component. Similarly, $F_\delta(s)$ in (8) can be regarded as the CDF of the strength of a system. Correspondingly, Eqs. (9)-(11) can be defined as the

CDF of strength of the series system, the parallel system, and the k-out-of-n system, respectively.

Further, the PDFs of system strength can be derived as

$$f_{\text{ser}}(\delta) = n\left[1 - F_\delta(\delta)\right]^{n-1} f_\delta(\delta) \tag{12}$$

$$f_{\text{par}}(\delta) = n\left[F_\delta(\delta)\right]^{n-1} f_\delta(\delta) \tag{13}$$

$$f_{k/n}(\delta) = \sum_{i=k}^{n} C_n^i \left[1 - F_\delta(\delta)\right]^{i-1}\left[F_\delta(\delta)\right]^{n-i-1}\left[i + nF_\delta(\delta) - n\right] f_\delta(\delta) \tag{14}$$

As a special case, when n=1 and k=1, the CDF and PDF of system strength have the same expressions as those of a component.

4. Reliability models of systems under random repeated load

During the service life of a system, loads experienced by the system are usually random and repetitive. The effect of the number of load applications on system reliability must be considered.

Eq. (1) is the well-known LSI model. However, it is not able to reflect the effect of the number of load applications. It can only be applied to calculate the reliability when the random load is applied once or for a specified number of applications [18, 29]. Similarly, Eqs. (5)-(7) cannot be used to calculate the reliability when random load is applied for an arbitrary number of times.

From the viewpoint of statistics, it can be regarded as extracting w samples of the random load when it is applied for w times. When strength degeneration is not obvious, and thus can be neglected, a system under these w load samples will survive all loads, if it does not fail under the maximum among these w load samples. The maximum load can be defined as the so-called equivalent load.

The maximum among these w times of load applications, is the largest order statistic, which is determined by the samples set ($s_1, s_2, \cdots, s_w$) corresponding to these w times of load applications. Then, the CDF of the equivalent load is

$$F_X(x) = \left[F_s(x)\right]^w \tag{15}$$

The PDF of the equivalent load is

$$f_X(x) = w\left[F_s(x)\right]^{w-1} f_s(x) \tag{16}$$

Further, the reliability model of the system when random load is applied for w times, is expressed as

$$\begin{aligned} R^{(w)} &= \int_{-\infty}^{+\infty} f_e(\delta) \int_{-\infty}^{\delta} f_X(x) \mathrm{d}x \mathrm{d}\delta \\ &= \int_{-\infty}^{+\infty} f_e(\delta) \int_{-\infty}^{\delta} w\left[F_s(x)\right]^{w-1} f_s(x) \mathrm{d}x \mathrm{d}\delta \end{aligned} \tag{17}$$

Rewriting x as s in Eq. (17), we have

$$R^{(w)} = \int_{-\infty}^{+\infty} f_e(\delta) \int_{-\infty}^{\delta} w\left[F_s(s)\right]^{w-1} f_s(s) \mathrm{d}s \mathrm{d}\delta \tag{18}$$

Replacing $f_e(\delta)$ in Eq. (18) with the PDF of system strength, we obtain the reliability models of different systems when load is applied for w times, as follows:

$$R_{\text{ser}}^{(w)} = \int_{-\infty}^{+\infty} n\left[1 - F_\delta(\delta)\right]^{n-1} f_\delta(\delta) \int_{-\infty}^{\delta} w\left[F_s(s)\right]^{w-1} f_s(s) \mathrm{d}s \mathrm{d}\delta \tag{19}$$

$$R_{\text{par}}^{(w)} = \int_{-\infty}^{+\infty} n\left[F_\delta(\delta)\right]^{n-1} f_\delta(\delta) \int_{-\infty}^{\delta} w\left[F_s(s)\right]^{w-1} f_s(s) \mathrm{d}s \mathrm{d}\delta \tag{20}$$

$$\begin{aligned} R_{k/n}^{(w)} = \int_{-\infty}^{+\infty} \sum_{i=k}^{n} C_n^i \left[1 - F_\delta(\delta)\right]^{i-1}\left[F_\delta(\delta)\right]^{n-i-1} \\ \left[i + nF_\delta(\delta) - n\right] f_\delta(\delta) \int_{-\infty}^{\delta} w\left[F_s(s)\right]^{w-1} f_s(s) \mathrm{d}s \mathrm{d}\delta \end{aligned} \tag{21}$$

In Eqs. (19)-(21), the number of load applications is embodied explicitly. Specially, if w=1, Eqs. (19)-(21) have the same expression as Eqs. (5)-(7), respectively.

5. Dynamic reliability model of systems

The Poisson stochastic process, as a counting process, can be applied to describe the relationship between the number of load applications and time [5]. When the loading process is described as a Poisson stochastic process with $\{\lambda(t) > 0\ (t \geq 0)\}$, the probability of load having been applied for w times at time t, is

$$P\left(N(t) - N(0) = w\right) = \frac{\left(\int_0^t \lambda(t)\mathrm{d}t\right)^w}{w!} \mathrm{e}^{-\int_0^t \lambda(t)\mathrm{d}t} \tag{22}$$

where $N(t)$ is the number of times that load has been applied by time t.

For each time load is applied, there are two characteristics of load, one is the time point when load appears, and the other is its magnitude. Here, the two-dimensional description is introduced to describe these two characteristics of load. Namely, the Poisson stochastic process is applied to describe the relationship between the number of load applications and time, and the PDF is used to describe the characteristics of the magnitude of load.

Based on the two-dimensional description of the loading process, we develop the dynamic reliability models of systems first assuming that strength does not degenerate and then assuming that strength degenerates in the following subsections.

5.1. Dynamic reliability models of systems when strength does not degenerate

When strength does not degenerate, using Eq. (18) and Eq. (22), the reliability at time t can be expressed as

$$\begin{aligned} R(t) = P\left(N(t) - N(0) = w\right) R^{(w)} = \sum_{w=0}^{+\infty} \frac{\left(\int_0^t \lambda(t)\mathrm{d}t\right)^w}{w!} \\ \mathrm{e}^{-\int_0^t \lambda(t)\mathrm{d}t} \int_{-\infty}^{+\infty} f_e(\delta) \int_{-\infty}^{\delta} w\left[F_s(s)\right]^{w-1} f_s(s) \mathrm{d}s \mathrm{d}\delta \end{aligned} \tag{23}$$

Using the Taylor expansion of the exponential function, Eq. (23) can be simplified as

$$R(t) = \int_{-\infty}^{+\infty} f_e(\delta) \mathrm{e}^{\left[F_s(\delta) - 1\right]\int_0^t \lambda(t)\mathrm{d}t} \mathrm{d}\delta \tag{24}$$

Further, the hazard rate of the system $h(t)$ when strength does not degenerate, is derived as

$$h(t) = \frac{\int_{-\infty}^{+\infty} f_e(\delta)\left[1 - F_s(\delta)\right]\lambda(t) \mathrm{e}^{\left[F_s(\delta) - 1\right]\int_0^t \lambda(t)\mathrm{d}t} \mathrm{d}\delta}{\int_{-\infty}^{+\infty} f_e(\delta) \mathrm{e}^{\left[F_s(\delta) - 1\right]\int_0^t \lambda(t)\mathrm{d}t} \mathrm{d}\delta} \tag{25}$$

The mean time to failure (MTTF) of the system, θ, when strength does not degenerate, is expressed as

$$\theta = \int_0^{+\infty} \int_{-\infty}^{+\infty} t\, f_e(\delta) e^{[F_s(\delta)-1]\int_0^t \lambda(t)\mathrm{d}t} \mathrm{d}\delta \mathrm{d}t \quad (26)$$

Replacing $f_e(\delta)$ in Eq. (24) with its PDF of system strength respectively, the reliability models of systems when strength does not degenerate, are derived as

$$R_{\text{series}}(t) = \int_{-\infty}^{+\infty} n\left[1-F_\delta(\delta)\right]^{n-1} f_\delta(\delta) e^{[F_s(\delta)-1]\int_0^t \lambda(t)\mathrm{d}t} \mathrm{d}\delta \quad (27)$$

$$R_{\text{parallel}}(t) = \int_{-\infty}^{+\infty} n\left[F_\delta(\delta)\right]^{n-1} f_\delta(\delta)\, e^{[F_s(\delta)-1]\int_0^t \lambda(t)\mathrm{d}t} \mathrm{d}\delta \quad (28)$$

$$R_{k/n}(t) = \int_{-\infty}^{+\infty} \sum_{i=k}^{n} C_n^{\,i} \left[1-F_\delta(\delta)\right]^{i-1} \left[F_\delta(\delta)\right]^{n-i-1} \left[i + nF_\delta(\delta) - n\right] f_\delta(\delta) e^{[F_s(\delta)-1]\int_0^t \lambda(t)\mathrm{d}t} \mathrm{d}\delta \quad (29)$$

Similarly, replacing $f_e(\delta)$ in Eq. (25) with its PDF, respectively, the hazard rates of the series system, the parallel system, and the k-out-of-n system when strength does not degenerate, are expressed as

$$h_{\text{ser}}(t) = \frac{\int_{-\infty}^{+\infty} n\left[1-F_\delta(\delta)\right]^{n-1} f_\delta(\delta)\left[1-F_s(\delta)\right]\lambda(t) e^{[F_s(\delta)-1]\int_0^t \lambda(t)\mathrm{d}t} \mathrm{d}\delta}{\int_{-\infty}^{+\infty} n\left[1-F_\delta(\delta)\right]^{n-1} f_\delta(\delta) e^{[F_s(\delta)-1]\int_0^t \lambda(t)\mathrm{d}t} \mathrm{d}\delta} \quad (30)$$

$$h_{\text{par}}(t) = \frac{\int_{-\infty}^{+\infty} n\left[F_\delta(\delta)\right]^{n-1} f_\delta(\delta)\left[1-F_s(\delta)\right]\lambda(t) e^{[F_s(\delta)-1]\int_0^t \lambda(t)\mathrm{d}t} \mathrm{d}\delta}{\int_{-\infty}^{+\infty} n\left[F_\delta(\delta)\right]^{n-1} f_\delta(\delta)\lambda(t) e^{[F_s(\delta)-1]\int_0^t \lambda(t)\mathrm{d}t} \mathrm{d}\delta} \quad (31)$$

$$h_{k/n}(t) = \frac{\int_{-\infty}^{+\infty} \sum_{i=k}^{n} C_n^{\,i}\left[1-F_\delta(\delta)\right]^{i-1}\left[F_\delta(\delta)\right]^{n-i-1}\left[i+nF_\delta(\delta)-n\right] f_\delta(\delta)\left[1-F_s(\delta)\right]\lambda(t) e^{[F_s(\delta)-1]\int_0^t \lambda(t)\mathrm{d}t} \mathrm{d}\delta}{\int_{-\infty}^{+\infty} \sum_{i=k}^{n} C_n^{\,i}\left[1-F_\delta(\delta)\right]^{i-1}\left[F_\delta(\delta)\right]^{n-i-1}\left[i+nF_\delta(\delta)-n\right] f_\delta(\delta) e^{[F_s(\delta)-1]\int_0^t \lambda(t)\mathrm{d}t} \mathrm{d}\delta} \quad (32)$$

Replacing $f_e(\delta)$ in Eq. (26) with its PDF, respectively, MTTFs of series system, parallel system, and k-out-of-n system when strength does not degenerate, are expressed as

$$\theta_{\text{ser}} = \int_0^{+\infty} \int_{-\infty}^{+\infty} t\, n\left[1-F_\delta(\delta)\right]^{n-1} f_\delta(\delta) e^{[F_s(\delta)-1]\int_0^t \lambda(t)\mathrm{d}t} \mathrm{d}\delta \mathrm{d}t \quad (33)$$

$$\theta_{\text{par}} = \int_0^{+\infty} \int_{-\infty}^{+\infty} t\, n\left[F_\delta(\delta)\right]^{n-1} f_\delta(\delta) e^{[F_s(\delta)-1]\int_0^t \lambda(t)\mathrm{d}t} \mathrm{d}\delta \mathrm{d}t \quad (34)$$

$$\theta_{k/n} = \int_0^{+\infty} \int_{-\infty}^{+\infty} t \sum_{i=k}^{n} C_n^{\,i}\left[1-F_\delta(\delta)\right]^{i-1}\left[F_\delta(\delta)\right]^{n-i-1} \left[i+nF_\delta(\delta)-n\right] f_\delta(\delta) e^{[F_s(\delta)-1]\int_0^t \lambda(t)\mathrm{d}t} \mathrm{d}\delta \mathrm{d}t \quad (35)$$

Take a series system with three identical components, the parallel system with three identical components, and the k-out-of-n system with k=2, n=3 as examples. When $\lambda(t)=0.5h^{-1}$, the strength of components follows the normal distribution with mean 600MPa and standard deviation 60MPa, and the load follows the normal distribution with mean 400MPa and standard deviation 40MPa, the reliability and the hazard rate of the systems when strength does not degenerate, are shown as Figs.1-4.

From Figs.1-4, it can be concluded that even if strength does not degenerate, the reliability of systems decreases as time goes. The hazard rate of systems also decreases over time. For systems with identical components, the hazard rate of the series system is the highest, and that of the parallel system the lowest. The hazard rate curves have the partial feature of a bathtub curve, including only the initial failure period and the random failure period, but not the wearout period. This is because strength degeneration is not considered.

5.2. Dynamic reliability model of systems when strength degenerates

In the following, the dynamic reliability models of systems with strength degeneration are developed through differential equations.

According to Eqs. (12)-(14), it is known that system strength can be expressed as a function of the strengths of the components. Generally, component strength at time t is dependent on its initial strength and time t, therefore, system strength δ_t at time t can be written as a function of the initial strength δ and time t. When the loading process is describes as a Poison stochastic process, the probability that load appears in interval (t, $t+\Delta t$) is $\lambda(t)\Delta t$. Therefore, $R(t+\Delta t)$ can be expressed as

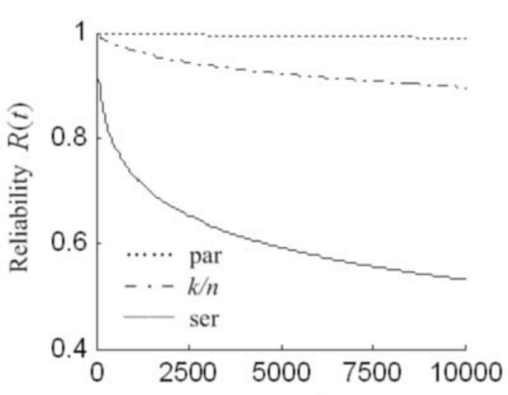

Fig. 1. Relationship between reliability of systems and time without strength degeneration

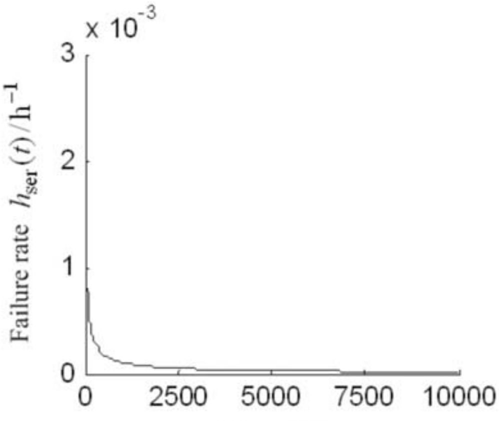

Fig. 2. Hazard rate curve of the series system (n=3) without strength degeneration

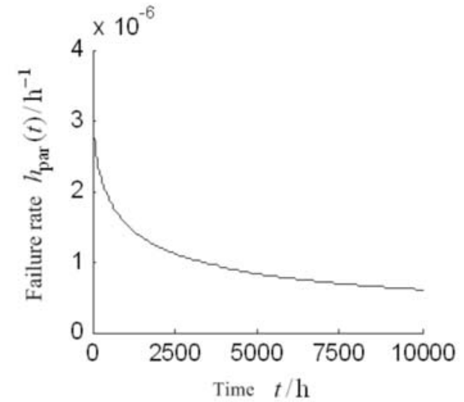

Fig. 3. Hazard rate curve of the parallel system (n=3) without strength degeneration

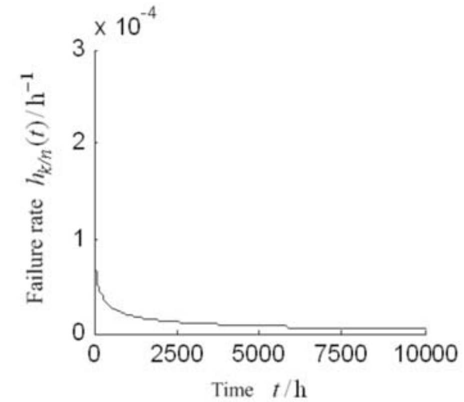

Fig. 4. Hazard rate curve of the k/n system (k=2, n=3) without strength degeneration

$$\begin{aligned} R(t+\Delta t) &= R(t)P\left(\delta_e(\tau) > s, \forall \tau \in [t, t+\Delta t]\right)\lambda(t)\Delta t + R(t)\left(1-\lambda(t)\Delta t\right) \\ &= R(t) + R(t)\lambda(t)\Delta t\left(\int_{-\infty}^{\delta_e(\tau)} f_s(s)\mathrm{d}s - 1\right) \\ &= R(t) + R(t)\lambda(t)\Delta t\left[F_s\left(\delta_e(\tau)\right)-1\right] \end{aligned} \tag{36}$$

Here, $\delta_e(\tau)$ can be expressed as a function of the initial strength δ and time t, therefore, Eq. (36) can be rewritten as

$$R(t+\Delta t) - R(t) = R(t)\lambda(t)\Delta t\left[F_s(\delta,\ \tau)-1\right] \tag{37}$$

Divided by Δt and $\Delta t \to 0$, $\tau \to t$, Eq. (37) yields

$$\frac{\mathrm{d}R(t)}{\mathrm{d}t} = R(t)\lambda(t)\left[F_s(\delta,\ t)-1\right] \tag{38}$$

Solving Eq. (38) yields

$$\ln R(t) = \int_0^t \lambda(t)\left[F_s(\delta,\ t)-1\right]\mathrm{d}t + C \tag{39}$$

Noting that $R(0)=1$, and $C=0$, we get

$$R(t) = \mathrm{e}^{\int_0^t [F_s(\delta,\ t)-1]\lambda(t)\mathrm{d}t} \tag{40}$$

Generally, when the initial strength δ is random, the reliability of systems with strength degeneration is

$$R(t) = \int_{-\infty}^{+\infty} f_e(\delta)\mathrm{e}^{\int_0^t [F_s(\delta,\ t)-1]\lambda(t)\,\mathrm{d}t}\mathrm{d}\delta \tag{41}$$

Further, the hazard rate of systems with strength degeneration is

$$h(t) = \frac{\int_{-\infty}^{+\infty} f_e(\delta)\left[1-F_s(\delta,\ t)\right]\lambda(t)\mathrm{e}^{\int_0^t [F_s(\delta,\ t)-1]\lambda(t)\,\mathrm{d}t}\mathrm{d}\delta}{\int_{-\infty}^{+\infty} f_e(\delta)\mathrm{e}^{\int_0^t [F_s(\delta,\ t)-1]\lambda(t)\,\mathrm{d}t}\mathrm{d}\delta} \tag{42}$$

The MTTF of systems when strength degenerate, is expressed as

$$\theta = \int_0^{+\infty}\int_{-\infty}^{+\infty} t\, f_e(\delta)\mathrm{e}^{\int_0^t [F_s(\delta,\ t)-1]\lambda(t)\,\mathrm{d}t}\mathrm{d}\delta\mathrm{d}t \tag{43}$$

As a step of validation, if we now assume that strength does not degenerate, namely, $F_s(\delta, t)$ is independent of t, Eq. (41) and Eq. (24), Eq. (42) and Eq. (25), and Eq. (43) and Eq. (26), give the same expression, respectively.

Further, replacing $f_e(\delta)$ in Eq. (41) with its PDF respectively, the reliability models of series system, parallel system, and k/n system when strength degenerates, are derived as

$$R_{\mathrm{ser}}(t) = \int_{-\infty}^{+\infty} n\left[1-F_\delta(\delta)\right]^{n-1} f_\delta(\delta)\ \mathrm{e}^{\int_0^t [F_s(\delta,\ t)-1]\lambda(t)\mathrm{d}t}\mathrm{d}\delta \tag{44}$$

$$R_{\mathrm{par}}(t) = \int_{-\infty}^{+\infty} n\left[F_\delta(\delta)\right]^{n-1} f_\delta(\delta)\ \mathrm{e}^{\int_0^t [F_s(\delta,\ t)-1]\lambda(t)\mathrm{d}t}\mathrm{d}\delta \tag{45}$$

$$\begin{aligned} R_{k/n}(t) = \int_{-\infty}^{+\infty}\sum_{i=k}^{n} C_n^{\,i}\left[1-F_\delta(\delta)\right]^{i-1}\left[F_\delta(\delta)\right]^{n-i-1} \\ \left[i+nF_\delta(\delta)-n\right] f_\delta(\delta)\ \mathrm{e}^{\int_0^t [F_s(\delta,\ t)-1]\lambda(t)\mathrm{d}t}\mathrm{d}\delta \end{aligned} \tag{46}$$

Similarly, replacing $f_e(\delta)$ in Eq. (42) with its PDF, respectively, the hazard rate functions of series system, parallel system and k/n system with strength degeneration, are derived as

$$h_{\mathrm{ser}}(t) = \frac{\int_{-\infty}^{+\infty} n\left[1-F_\delta(\delta)\right]^{n-1} f_\delta(\delta)\left[1-F_s(\delta,\ t)\right]\lambda(t)\mathrm{e}^{\int_0^t [F_s(\delta,\ t)-1]\lambda(t)\,\mathrm{d}t}\mathrm{d}\delta}{\int_{-\infty}^{+\infty} n\left[1-F_\delta(\delta)\right]^{n-1} f_\delta(\delta)\mathrm{e}^{\int_0^t [F_s(\delta,\ t)-1]\lambda(t)\,\mathrm{d}t}\mathrm{d}\delta} \tag{47}$$

$$h_{\mathrm{par}}(t) = \frac{\int_{-\infty}^{+\infty} n\left[F_\delta(\delta)\right]^{n-1} f_\delta(\delta)\left[1-F_s(\delta,\ t)\right]\lambda(t)\mathrm{e}^{\int_0^t [F_s(\delta,\ t)-1]\lambda(t)\,\mathrm{d}t}\mathrm{d}\delta}{\int_{-\infty}^{+\infty} n\left[F_\delta(\delta)\right]^{n-1} f_\delta(\delta)\mathrm{e}^{\int_0^t [F_s(\delta,\ t)-1]\lambda(t)\,\mathrm{d}t}\mathrm{d}\delta} \tag{48}$$

$$h_{k/n}(t) = \frac{\int_{-\infty}^{+\infty}\sum_{i=k}^{n} C_n^{\,i}\left[1-F_\delta(\delta)\right]^{i-1}\left[F_\delta(\delta)\right]^{n-i-1}\left[i+nF_\delta(\delta)-n\right] f_\delta(\delta)\left[1-F_s(\delta,\ t)\right]\lambda(t)\mathrm{e}^{\int_0^t [F_s(\delta,\ t)-1]\lambda(t)\,\mathrm{d}t}\mathrm{d}\delta}{\int_{-\infty}^{+\infty}\sum_{i=k}^{n} C_n^{\,i}\left[1-F_\delta(\delta)\right]^{i-1}\left[F_\delta(\delta)\right]^{n-i-1}\left[i+nF_\delta(\delta)-n\right] f_\delta(\delta)\mathrm{e}^{\int_0^t [F_s(\delta,\ t)-1]\lambda(t)\,\mathrm{d}t}\mathrm{d}\delta} \tag{49}$$

Replacing $f_e(\delta)$ in Eq. (43) with its PDF respectively, the MTTFs of series system, parallel system, and k-out-of-n system when strength degenerates, are expressed as

$$\theta_{\mathrm{ser}} = \int_0^{+\infty}\int_{-\infty}^{+\infty} t\; n\left[1-F_\delta(\delta)\right]^{n-1} f_\delta(\delta)\mathrm{e}^{\int_0^t [F_s(\delta,\,t)-1]\lambda(t)\,\mathrm{d}t}\,\mathrm{d}\delta\mathrm{d}t \tag{50}$$

$$\theta_{\mathrm{par}} = \int_0^{+\infty}\int_{-\infty}^{+\infty} t\; n\left[F_\delta(\delta)\right]^{n-1} f_\delta(\delta)\mathrm{e}^{\int_0^t [F_s(\delta,\,t)-1]\lambda(t)\,\mathrm{d}t}\,\mathrm{d}\delta\mathrm{d}t \tag{51}$$

$$\theta_{k/n} = \int_0^{+\infty}\int_{-\infty}^{+\infty} t \sum_{i=k}^{n} C_n^{\,i}\left[1-F_\delta(\delta)\right]^{i-1}\left[F_\delta(\delta)\right]^{n-i-1} \left[i+nF_\delta(\delta)-n\right] f_\delta(\delta)\mathrm{e}^{\int_0^t [F_s(\delta,\,t)-1]\lambda(t)\,\mathrm{d}t}\,\mathrm{d}\delta\mathrm{d}t \tag{52}$$

As examples, consider the series system with three identical components, the parallel system with three identical components, and the k-out-of-n system with k=2, n=3. When $\lambda(t)$=0.5h^{-1}, the strength of components follows the normal distribution with mean 600MPa and standard deviation 60MPa, and the load follows the normal distribution with mean 400MPa and standard deviation 40MPa, and the strength degenerates as $\delta_t = \delta\, e^{-0.00002t}$, the reliability and the hazard rates of the systems incorporating strength degeneration, are shown in Figs.5-8.

From Figs.5-8, it can be concluded that when strength degenerates, reliability of system decreases over time more rapidly, and the hazard rate curves of systems are bathtub-shaped.

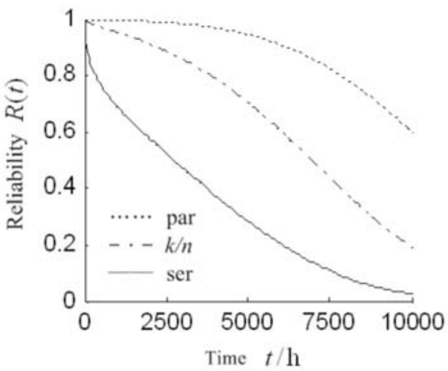

Fig. 5. Relationship between system reliability and time with strength degeneration

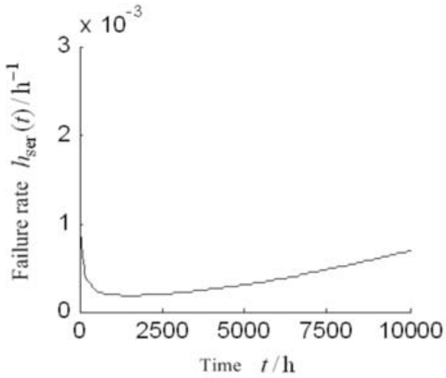

Fig. 6. Hazard rate curve of the series system (n=3) with strength degeneration

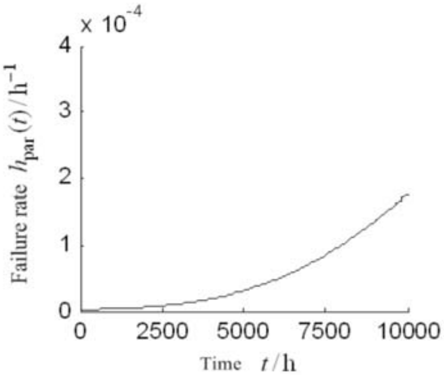

Fig. 7. Hazard rate curve of the parallel system (n=3) with strength degeneration

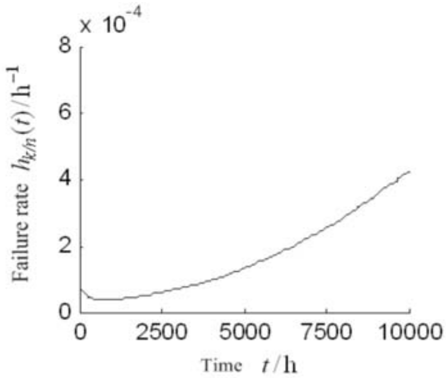

Fig. 8. Hazard rate curve of the k/n system (k=2, n=3) with strength degeneration

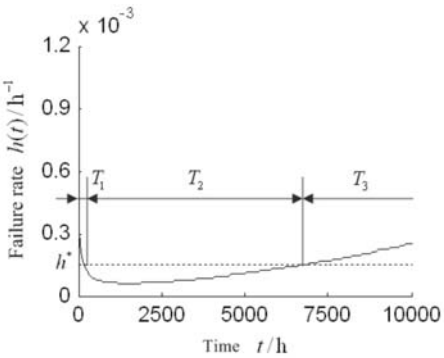

Fig. 9. Application of the hazard rate curve of systems

If we are given a maximum acceptable hazard rate value h^*, the early failure duration T_1, the random failure duration T_2 and the wearout failure duration T_3, can be distinguished objectively using the models proposed, as shown in Fig. 9. Further, we can use this information to make decision on burn-in duration, the reliable operation life, and maintenance schedule.

6. Conclusion

In this paper, a new method is developed for modeling dynamic reliability of systems and the mathematical function of reliability and hazard rate of systems are presented. First, through applying the load-strength interference model at the system level, the reliability models of systems are derived, and the CDFs and PDFs of strength for different systems are developed. Then, the reliability models of systems when random load is applied for multiple times are developed with order statistics. Based on the loading process described as the Poisson stochastic process, the dynamic reliability models of the series system, the parallel system and the *k*-out-of-*n* system without strength degeneration, and those with strength degeneration, are developed, respectively. Further, the relationship between system reliability and time, and the hazard rate of systems are discussed in different cases. The results show that, even when strength does not degenerate, the reliability of systems decreases as time goes, and the hazard rate decreases over time, too. When strength degenerates, the reliability decreases more rapidly over time, and the hazard rate curves are bathtub-shaped.

As long as the probability distributions of load and strength and the rule of strength degeneration are known, reliability and hazard rate of systems can be calculated by the models proposed, which can be used to distinguish the early failure period, the random failure period, and the wearout failure period. It is helpful for the life cycle management of systems.

7. References

1. Anatoly L. Extended block diagram method for a multi-state system reliability assessment. Reliability Engineering & System Safety 2007; 92(12): 1601-1607.
2. Atwood C L. The binomial failure rate common cause model. Technometrics 1986; 28(2): 139-148.
3. Czarnecki A. A, Nowak A S. Time-variant reliability profiles for steel girder bridges. Structural Safety 2008; 30(1): 49-64.
4. Dai Y S, Levitin G, Wang X L. Optimal task partition and distribution in grid service system with common cause failures. Future Generation Computer Systems 2007; 23(2): 209-218.
5. Ditlevsen O.Stochastic model for joint wave and wind loads on offshore structures. Structural Safety 2002; 24(2-4): 139-163.
6. Döre P. Basic aspects of stochastic reliability analysis for redundancy systems. Reliability Engineering & System Safety 1989; 24(4): 351-375.
7. Günter B, Leonidas C, Dimitris K. Dynamic reliability under random shocks. Reliability Engineering & System Safety 2002; 77(3): 239-251.
8. Hokstad P. A shock model for common cause failure. Reliability Engineering & System Safety 1988; 23(2): 127-145.
9. Huang C Y, Chang Y R. An improved decomposition scheme for assessing the reliability of embedded systems by using dynamic fault trees. Reliability Engineering & System Safety 2007; 92(10): 1403-1412.
10. Huang H Z, An Z W. A discrete stress-strength interference model with stress dependent strength. IEEE Transactions on Reliability 2008; 58(1): 118-122.
11. Huang H Z, Liu Z J, Murthy D N P. Optimal reliability, warranty and price for new products. IIE Transactions 2007; 39(8): 819-827.
12. Huang H Z, Qu J, Zuo M J. Genetic-algorithm-based optimal apportionment of reliability and redundancy under multiple objectives. IIE Transactions 2009; 41(4): 287-298.
13. Ionescu D C, Ulmeanu A P, Constantinescu A C, et al. Reliability modeling of medium voltage distribution systems of nuclear power plants using generalized stochastic Petri nets. Computers and Mathematics with Applications 2006; 51(2): 285-290.
14. Jose E, Ramirez M, Coit D W. Optimization of system reliability in the presence of common cause failures. Reliability Engineering & System Safety 2007; 92(10): 1421-1434.
15. Levitin G. Incorporating common-cause failures into norepairable multi-state series-parallel system analysis,”. IEEE Transactions on Reliability 2001; 50(4): , vol. 50, no. 4, pp. 380-388, 2001.
16. Liu Y. Huang H Z. Comment on ‘‘A framework to practical predictive maintenance modeling for multi-state systems’’ by Tan C.M. and Raghavan N. [Reliab Eng Syst Saf 2008; 93(8): 1138–50]. Reliability Engineering and System Safety 2009; 4(3): 776-780.
17. Mankamo T, Kosonen M. Dependent failure modeling in highly redundant structures–application to BWR safety valves. Reliability Engineering & System Safety 1992; 35(3): 235-244.
18. Mosleh A, Parry G W, Zikria A F. An approach to the analysis of common cause failure data for plant-specific application. Nuclear Engineering and Design 1994; 150(1-2): 25-47.
19. Nishimura S I, Shimizu H. Reliability-based design of ground improvement for liquefaction mitigation. Structural Safety 2008; 30(3): 200-216.
20. Raphaël S, Aubry J F, Cambois T, et al. An aggregation method of Markov graphs for the reliability analysis of hybrid systems. Reliability Engineering & System Safety 2006; 91(2): 137-148.
21. Streicher H, Rackwitz R. Time-variant reliability-oriented structural optimization and a renewal model for life-cycle costing. Probabilistic Engineering Mechanics 2004; 19(1-2): 171-183.
22. Tian L, Noore A. Dynamic software reliability prediction: an approach based on support vector machines. International Journal

of Reliability, Quality and Safety Engineering 2005; 12(4): 309-321.
23. Torres M A, Ruiz S E. Structural reliability evaluation considering capacity degradation over time. Engineering Structures 2007; 29(9): 2183-2192.
24. Vaurio J K. Availability of redundant safety systems with common mode and undetected failures. Nuclear Engineering and Design 1980; 58(3): 415-424.
25. Vaurio J K. Consistent mapping of common cause failure rates and alpha factors. Reliability Engineering & System Safety 2007; 92(5): 628-645.
26. Wang Z, Xie L Y. Reliability model of component under stochastic ally repeated load. The First International Conference on Maintenance Engineering, Beijing: Science Press 2006; 895-900.
27. Wang Z, Xie L Y, Li B. Time-dependent reliability models of systems with common cause failure. International Journal of Performability Engineering 2007; 3(4): 419-432.
28. Xing L D, Meshkat L, Susan K, Donohue. Reliability analysis of hierarchical computer-based systems subject to common-cause failures. Reliability Engineering & System Safety 2007; 92(3): 351-359.
29. Xie L Y, Wang Z. Reliability degradation of mechanical components and systems. Handbook of Performability Engineering (Misra, Krishna B Ed.), Springer 2008.
30. Xie L Y, Zhou J Y, Hao C Z. System-level load–strength interference based reliability modeling of k-out-of-n system. Reliability Engineering & System Safety 2004; 84(3): 311-317.
31. Xie L Y, Zhou J Y, Wang Y Y,et al. Load-strength order statistics interference models for system reliability evaluation. International Journal of Performance Engineering 2005; 1(1): 22-36.

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The research work is subsidized by China Postdoctoral Science Foundation with the grant No. 20080440294.

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