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OPTYMALNA STRATEGIA KONTROLI ELEMENTÓW SKŁADOWYCH Z MOŻLIWOŚCIĄ ODROCZENIA NAPRAWY

AN OPTIMAL POLICY OF INSPECTION FOR A COMPONENT WITH DELAYED REPAIR

Niniejszy artykuł przedstawia zintegrowaną metodologię optymalizacji kontroli i eksploatacji elementów składowych z uwzględnieniem możliwości odroczonej naprawy jako jednej z kilku istniejących opcji. Modele uszkodzeń, ryzyka i kosztu cyklu życia utworzono wykorzystując pojęcia prawdopodobieństwa, procesu stochastycznego i czasu zwłoki. Model ma na celu optymalizację częstotliwości kontroli i czasu odroczenia naprawy a następnie stworzenie optymalnej strategii eksploatacji dla części składowych w ramach kontroli nieokresowej. Działanie zaproponowanego modelu zilustrowano przykładem oraz przedstawiono wpływ kontroli oraz czasu zwłoki w wykonaniu naprawy na koszt cyklu życia.

Słowa kluczowe: strategia kontroli, uszkodzenie, eksploatacja, optymalizacja.

In this paper, an integrated methodology is developed for optimising inspection and maintenance of a component where delayed repair is considered to be one of a few feasible options. The models of failures, risk and life cycle cost are developed using probability, stochastic process and the delay time concept. The model is intended to optimise the inspection intervals and the delay of repair together and then develop an optimal maintenance policy for a component under non-periodic inspection regime. The performance of the proposed model is illustrated by an example, and the effects of inspection and time of delay for repair on life cycle cost are shown.

***Keywords**: inspection policy, failure, maintenance, optimisation.*

1. Introduction

A potential failure (or defect) is a definable state of a component before it deteriorates to a functional failure. As functional failures of a component may have more serious effect on the operation of a system and present greater economic loss, in practice inspections are usually conducted to detect the potential failures so as to prevent the functional failures of the component [1-5, 7]. To this end, models and algorithms are to be developed to determine the optimal intervals of inspections. In most of existing models [1, 6, 8-11], it was assumed that repair follows immediately after a potential failure has been detected. However, in practice, the repair may be delayed from technical and management consideration. In fact, the progresses and consequences of identified defects are different. For those developing slowly or resulting in non-serious consequence, repair can be delayed and be conducted later at a scheduled time of maintenance or when the system involving the component is not busy. In the case that a lot of economic loss may be caused if a running component is shutdown immediately for repair, delayed repair may be considered as one of the options in the development of its maintenance strategy. For example, some defects in railway are allowed to remain in the rail if they do not result in rail failures immediately. However, the delay of repair may increase the risk of system failures and of course, the risk should be evaluated and effectively controlled.

Optimal inspection policies have been the subject of a great deal of research. Christer et al. [1] analyzed the time duration in the state of defect and hence developed the so-called delay time model. A number of theoretical studies and applications have been conducted based on the concept of delay time. Recently, the delay time model has been extended to optimise the scheduling of perfect inspections for multi-component systems [9]. In Ref. [10], imperfect inspection was considered using NHPP and an optimization algorithm was developed in order to obtain the solution of the problem. Podofillini et al [8] developed a Markovian model to calculate the costs and risks of the system operation, and the performance of the developed model was shown with a railway case study. However, in these studies, not much work has been done to investigate the effect of delayed repair in development of inspection strategies. Therefore, this paper develops an integrated methodology for optimising inspection and maintenance of a component where delayed repair is considered to be one of the feasible options.

2. Development of Model

It is assumed that a component may be in one of three states: good, defective and failed, where a defect (i.e., potential failure) is a definable state before a functional failure happens to the component. The occurrence of defects is assumed to follow a non-homogeneous Poisson process (NHPP). The time interval between the occurrence of a defect and when it deteriorates to cause a failure is referred to as delay time [1, 9] or is called P-F interval [6]. Inspections are assumed to be conducted at scheduled times. The inspections may be imperfect (the inspection can not detect all the existing defects) and the inspection intervals may be non-constant. If a defect is identified by an inspection, it will be repaired with a time delay by minimal maintenance. It is also assumed that a defect's presence can only be identified by inspection with a probability $\beta(0<\beta\leq 1)$. In addition, the repair of the defect detected by the current inspection will not be deferred beyond the next inspection.

Consider the $(j\text{-}1)$th inspection at time s_{j-1}. As illustrated in Figure 1, the failures induced by the delay of repair must be the one that is caused by a defect initiating from an earlier interval (s_{k-1}, s_k) where $k<j$, and the defect has not been detected by inspections at times $s_k, \ldots, s_{j-2}$, but detected by the inspection at s_{j-1}, and results in a failure within $(s_{j-1}, s_{j-1}+y)$. The expected number of failures in $(s_{j-1}, s_{j-1}+y)$ which are induced by delayed

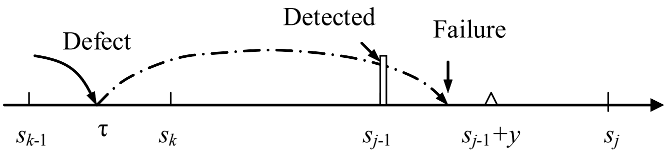

Fig. 1. Illustration of occurrence of failure induced by delayed repair

repair and caused by the defects occurring in (s_{k-1}, s_k) can be given by:

$$n(k,j)=(1-\beta)^{j-k-1}\beta\int_{s_{k-1}}^{s_k}\lambda(\tau)[G(s_{j-1}+y-\tau)-G(s_{j-1}-\tau)]d\tau \quad (1)$$

where, $\lambda(\tau)$ is hazard rate of defect at time τ, and $G(\bullet)$ is cumulative distribution function of P-F interval for a defect.

Let S_m denote inspection policy with instances of inspections being at $s_1,s_2,\ldots,s_m$. Hence, in the interval (0,T), the expected number of failures induced by delayed repair, $N_D(S_m,y)$ can be obtained by summing up all failures for all the inspection intervals. That is:

$$N_D(\mathbf{S}_m,y)=\sum_{j=1}^{m+1}\sum_{k=1}^{j}n(k,j) \quad (2)$$

In addition, failures may happen if defects can not be detected by inspections. Similarly, we have that the expected number of failures due to imperfect inspections:

$$N_F(\mathbf{S}_m)=\sum_{j=1}^{m+1}\sum_{k=1}^{j}\{(1-\beta)^{j-k}\int_{s_{k-1}}^{s_k}\lambda(\tau)[G(s_j-\tau)-G(s_{j-1}-\tau)]d\tau\} \quad (3)$$

Therefore, the risk of failure can be evaluated by:

$$R=\gamma[N_D(\mathbf{S}_m,y)+N_F(\mathbf{S}_m)] \quad (4)$$

where, γ is average of loss caused by a failure of the component.

Consider a life cycle of the component (0, T). The costs associated to failures and maintenance in the period include:

(1) Renewal cost

During a life cycle the component is only renewed once. Thus, the cost of renewal is: c_R

(2) Cost induced by failures

When failures occur to the component, they may lead to the need for unplanned maintenance or even cause an accident. The failures may be caused by the delay of repair and result from the defects which are not detected by inspections. Hence, the cost induced by failures can be evaluated by: $c_F[N_D(S_m,y)+N_F(S_m)]$.

(3) Planned maintenance

If defects are identified through inspections, they can be removed by planned maintenance (PM). Denote $N_{PM}(S_m,y)$ as the expected number of defects which have been detected by inspections in the period, and then the cost associated with this during the interval (0, T) is: $c_{PM}(y)N_{PM}(S_m,y)$. Similar to the derivation to Eq.(3), we have:

$$N_{PM}(\mathbf{S}_m,y)=\sum_{j=1}^{m+1}\sum_{k=1}^{j-1}(1-\beta)^{j-k-1}\beta\int_{s_{k-1}}^{s_k}\lambda(\tau)[1-G(s_{j-1}+y-\tau)]d\tau \quad (5)$$

The other maintenance activity in this category is inspection, and the inspection cost in the period (0, T) is: $c_I m$. Therefore, the average cost in a life cycle of the component can be given by:

$$C(\mathbf{S}_m,y)=\frac{c_R+c_I m+c_F[N_D(\mathbf{S}_m,y)+N_F(\mathbf{S}_m)]+c_{PM}(y)N_{PM}(\mathbf{S}_m,y)}{T} \quad (6)$$

3. Optimal Policy of Inspection and Repair

The optimisation model is

$$\text{Min: } C(S_m, y) \quad (7)$$

$$\text{s.t. } R<R_0 \quad (8)$$

where R_0 is permitted risk level for the reliable and safe operation of the component.

3.1. Optimising the time of delay for repair

The necessary condition for optimal time of delay for repair can be given by:

$$\frac{dC(\mathbf{S}_m,y)}{dy}=0 \quad (9)$$

Using Eq.(6), it can be derived that

$$[c_F-c_{PM}(y)]\frac{dN_D(\mathbf{S}_m,y)}{dy}+c'_{PM}(y)N_{PM}(\mathbf{S}_m,y)=0 \quad (10)$$

3.2. Optimising the intervals of inspections

The necessary condition for optimal instance of inspection is:

$$\frac{\partial C(\mathbf{S}_m,y)}{\partial s_x}=0 \quad (11)$$

It can be deduced from Eq.(6) that

$$c_F[\frac{\partial N_D(\mathbf{S}_m,y)}{\partial s_x}+\frac{\partial N_F(\mathbf{S}_m)}{\partial s_x}]+c_{PM}(y)\frac{\partial N_{PM}(\mathbf{S}_m,y)}{\partial s_x}=0 \quad (12)$$

We first consider $N_D(S_m,y)$. It is evident that the term E_x, which contains variable s_x in $N_D(S_m,y)$, is given by:

$$E_x=\sum_{k=1}^{x}n(k,x+1)+\sum_{j=x}^{m+1}n(x,j)+\sum_{j=x+1}^{m+1}n(x+1,j) \quad (13)$$

Using Eqs.(1), (2)and (13), we have

$$\frac{\partial N_D(\mathbf{S}_m,y)}{\partial s_x}=\frac{\partial E_x}{\partial s_x}$$

$$=\frac{\partial[\sum_{k=1}^{x}n(k,x+1)]}{\partial s_x}+\frac{\partial[\sum_{j=x+1}^{m+1}n(x,j)]}{\partial s_x}+\frac{\partial[\sum_{j=x+2}^{m+1}n(x+1,j)]}{\partial s_x} \quad (14)$$

It can be deduced that

$$\frac{\partial[\sum_{k=1}^{x-1} n(k,x+1)]}{\partial s_x} = \sum_{k=1}^{x-1} \beta(1-\beta)^{x-k} \int_{s_{k-1}}^{s_k} \lambda(\tau)[g(s_x+y-\tau)-g(s_x-\tau)]d\tau \quad (15)$$

$$\frac{\partial[\sum_{j=x+1}^{m+1} n(x,j)]}{\partial s_x} = \beta \int_{s_{x-1}}^{s_x} \lambda(\tau)[g(s_x+y-\tau)-g(s_x-\tau)]d\tau + \sum_{j=x+1}^{m} \beta(1-\beta)^{j-x-1}\lambda(s_x)[G(s_{j-1}+y-s_x)-G(s_{j-1}-s_x)] \quad (16)$$

$$\frac{\partial[\sum_{j=x+1}^{m+1} n(x+1,j)]}{\partial s_x} = -\sum_{j=x+2}^{m} \beta(1-\beta)^{j-x}\lambda(s_x)[G(s_j+y-s_x)-G(s_j-s_x)] \quad (17)$$

Thus,

$$\frac{\partial N_D(\mathbf{S}_m,y)}{\partial s_x} = \sum_{k=1}^{x} \beta(1-\beta)^{x-k} \int_{s_{k-1}}^{s_k} \lambda(\tau)[g(s_x+y-\tau)-g(s_x-\tau)]d\tau + \beta\lambda(s_x)G(y) - \sum_{j=x+2}^{m+1} \beta^2(1-\beta)^{j-x-1}\lambda(s_x)[G(s_{j-1}+y-s_x)-G(s_{j-1}-s_x)] \quad (18)$$

Similarly, we have

$$\frac{\partial N_F(\mathbf{S}_m)}{\partial s_x} = \sum_{k=1}^{x} \beta(1-\beta)^{x-k} \int_{s_{k-1}}^{s_k} \lambda(\tau)g(s_x-\tau)d\tau - \sum_{j=x+2}^{m+1} \beta^2(1-\beta)^{j-x-1}\lambda(s_x)G(s_{j-1}-s_x) \quad (19)$$

And

$$\frac{\partial N_{PM}(\mathbf{S}_m,y)}{\partial s_x} = -\sum_{k=1}^{x} \beta(1-\beta)^{x-k} \int_{s_{k-1}}^{s_k} \lambda(\tau)g(s_x+y-\tau)d\tau + \beta\lambda(s_x)[1-G(y)] - \sum_{j=x+2}^{m+1} \beta^2(1-\beta)^{j-x-1}\lambda(s_x)[1-G(s_j+y-s_x)] \quad (20)$$

Therefore, the optimal instance for x'th inspection can be obtained using Eq.(12) and Eqs.(18)-(20) as:

$$[c_F-c_{PM}(y)]\{\sum_{k=1}^{x}(1-\beta)^{x-k}\int_{s_{k-1}}^{s_k}\lambda(\tau)g(s_x+y-\tau)d\tau+\lambda(s_x)G(y)\}$$

$$=[c_F-c_{PM}(y)]\sum_{j=x+1}^{m}\beta^2(1-\beta)^{j-x-1}\lambda(s_x)G(s_j+y-s_x)+c_{PM}(1-\beta)^{m-x}\lambda(s_x) \quad (21)$$

3.3. Algorithm

An iterative algorithm is presented here to optimise the inspection instances and the time allowed for a delay of repair. The procedure of the algorithm is described as follows.

(a) Start from m=1, set the initial intervals of inspections for m times of inspections within T by $\Delta s=s_k-s_{k-1}=T/(m+1)$, and let the initial delay of repair $y=0.5\,\Delta s$.

(b) Start from k=1 to m, determine iteratively the inspection instances s_k using Eq.(21) under the condition of inspection instances $(s_1,\ldots,s_{k-1},s_{k+1},\ldots,s_m)$ being fixed.

(c) If the difference between the instance obtained from the current iteration s_k and that from the last iteration s'_k for any k is so small that $\max_k\left|s_k-s'_k\right|/\Delta s<10^{-4}$, stop the iteration and go to Step (d); Otherwise, go to Step (b) to repeat the iteration.

(d) Determine the optimal delay time of repair, y, using Eq.(10).

(e) Using Eq.(6), calculate the expected cost per unit time, C_m, over a life cycle of the component.

(f) If C_{m+1}- C_m>0, the optimal number of inspections is obtained, and the inspection instances obtained at this stage are considered to be optimum; otherwise, $m\leftarrow m+1$, go back to Step (a) to repeat the procedure.

4. Example

An example is given to illustrate the performance of the model and algorithm. It is supposed that the defect of a component follows NHPP, and the failure rate is $\lambda(t)=0.0016\times(t/1000)^{0.6}$. Let the P-F interval to be an exponential distribution, and probability density function is $f(\tau)=0.005e^{-0.005\tau}$. The other parameters related to failures and maintenance are given in Table 1.

Table 1. Parameters related to failures and maintenance

Parameter	*Value*
C_R	5000 (Yuan)
C_I	40 (Yuan)
C_F	1000 (Yuan)
$c_P(y)$	$200+800e^{-5y}$ (Yuan)
T	5000 (hours)

Using the model and optimisation algorithm presented in the paper, the optimal policy of maintenance can be obtained. Figure 2 shows the cost per unit time in a life cycle of the component as a function of number of inspections. It can be seen that for the case of CI=40 and time of delay being 50 hours, the optimal cycles of inspections are 50 and minimal life cycle cost is 16475 Yuan. It can also be seen from Figure 2 that the cost of inspection for one time has considerable effects not only on life cycle cost of the component, but also on the times of inspections in a life cycle. When inspection cost per time increases up to 50 and 60 respectively, the life cycle cost will be 17035 and 17515 Yuan, and times of inspections in a life cycle will be reduced to 32 and 27 respectively. Figure 3 shows the effect of delayed repair on average cost per unit time. In the example, the optimal delay of repair is 50 hours. When the time of delay is longer, the average cost will increase as a result of probability

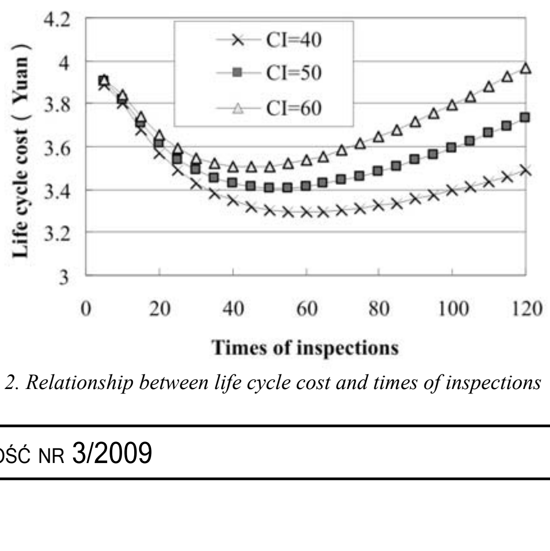

Fig. 2. Relationship between life cycle cost and times of inspections

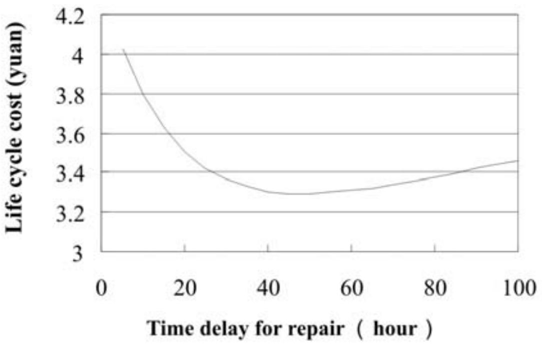

Fig. 3. Effect of delayed repair on life cycle cost

of failure becoming larger. As the shorter time of delay means that the repair may be conducted in an emergent manner, a higher average cost will be resulted in.

5. Concluding Remarks

In this paper, an integrated model is developed for optimising inspection and maintenance of a component where delayed repair is considered to be one of the feasible options. The methodology is capable of optimising the inspection intervals and the delay of repair together under a non-periodic inspection regime and imperfact inspection condition. As future work, we intend to apply the methodology to railway industry where delayed repair is one of important issues to be considered for asset management.

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