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PARTICLE SWARM OPTIMIZATION FUZZY SYSTEMS FOR THE AGE REDUCTION IMPERFECT MAINTENANCE MODEL

This research includes two topics: (1) the modeling of periodic preventive maintenance policies over an infinite time span for repairable systems with the reduction of the degradation rate after performing an imperfect preventive maintenance (PM) activity; (2) the parameter estimation of failure distribution and the restoration effect of PM from the proposed PM policy for deteriorating systems. The concept of the improvement factor method is applied to measure the restoration effect on the degradation rate for a system after each PM. An improvement factor is presented as a function of the system's age and the cost of each PM. A periodic PM model is then developed. The optimal PM interval and the optimal replacement time for the proposed model can be obtained by minimizing the objective functions of the cost rate through the algorithms provided by this research. An example of using Weibull failure distribution is provided to investigate the proposed model. The method is proposed to estimate the parameters of the failure process and the improvement effect after each PM by analyzing maintenance and failure log data. In this method, a PSO-based method is proposed for automatically constructing a fuzzy system with an appropriate number of rules to approach the identified system. In the PSO-based method, each individual in the population is constructed to determine the number of fuzzy rules and the premise part of the fuzzy system, and then the recursive least-squares method is used to determine the consequent part of the fuzzy system constructed by the corresponding individual. Consequently, an individual corresponds to a fuzzy system. Subsequently, a fitness function is defined to guide the searching procedure to select an appropriate fuzzy system with the desired performance. Finally, two identification problems of nonlinear systems are utilized to illustrate the effectiveness of the proposed method for fuzzy modeling.

Keywords: *imperfect maintenance, preventive maintenance, reliability, fuzzy modeling, particle swarm optimization.*

1. Introduction

It has been shown that performing PM activities can yield the restoration effect for a deteriorating system to the states between as good as new and as bad as old (Pham and Wang, 1996). Fuzzy modeling is a popular branch of system identification for constructing a fuzzy model to explain the behavior of an identified system described by a set of input-output data. The constructed fuzzy model is a rule-based system consisting of a set of fuzzy rules. Nakagawa (1979) has presented a model to describe the age reduction effects when the PM activity is performed for a system. Chan and Shaw (1993) have studied the modeling of the hazard rate restoration after performing a PM activity. Most of the PM models shown in the literature assume that the restoration effect of PM is occurred on the age or the hazard rate of the system. However, Canfield (1986) has proposed a model by assuming that the PM activity can only relieve stress temporarily and hence slow the rate of system degradation while the hazard rate is still monotonically increased. Park et al. (2000) has extended Canfield's model to determine the optimal PM policy. Malik (1979) has proposed the improvement factor method to measure the restoration effect for a deteriorating system after performing the PM activities. The proposed method considers that the imperfect PM activity can reduce a system's age from t to t/β, where β is the improvement factor, and can result in restoring the system's reliability to $R(t/\beta)$ from $R(t)$. Lie and Chun (1986) have developed an improvement factor to measure the restoration effect, which is affected by the PM cost and the system's actual age. Jayabalan and Chaudhuri (1992b) have also applied the improvement factor method to investigate the age restoration of a system after performing the PM activities. These PM models with using the improvement factor method assume that the improvement factor is a constant. However, Lie and Chun (1986) have considered the improvement factor as a variable for the PM model. Yet, some parameters are not well defined in the proposed improvement factor. In fact, the restoration effect can be affected by several factors after performing the PM activities, such as, system's age (or operating time), the interval of the PM, and the cost of each PM activity. Cheng and Chen (2003) proposed an improvement factor to measure the restoration effect, which is affected by the system's age and the cost of each PM. Yang et al. (2003) have proposed a similar improvement factor which is a function of the number of PM performed and the cost of each PM. Literature survey has shown that many PM models have been developed for the deteriorating and repairable systems. Typically, these models are to determine the optimal interval between PM activities and the number of PM before replacing the system by minimizing the expected average cost over a finite or infinite time span. Nakagawa (1986) has presented periodic and sequential PM models with minimal repair at each failure for the repairable systems and provided the optimal policies by minimizing the expected cost rates. Jayabalan and Chaudhuri (1992a) have proposed a PM model with assured reliability to determine the optimal maintenance interval for a system by minimizing the total cost over a finite time horizon. Park et al. (2000) developed a periodic maintenance policy for the deteriorating systems with degradation rate restoration assumption. However, the existing optimal PM models do not include the study of statistical analysis for the real historical failure and maintenance data. Few researches have been devoted to estimating the parameters of the failure process and the restoration effect after each PM activity. Traditionally, there are two types of assumption for the statistical analysis of the failure data, i.e., the state of a system after performing a PM activity is assumed to be as good as new (GAN) or as bad as old (BAO). In general, the failure process of a PM model belongs to the stochastic point process. Hence, the assumptions of the GAN state and the BAO state are corresponding to the renewal process and the non-homogeneous Poisson process (NHPP), respectively. It is noticed that the estimation methods for the failure data with the above assumptions are well developed. However, both of the above assumptions do not include the case of the imperfect maintenance which usually improves the system's state to the level between GAN and BAO. In this paper, a periodic PM model over an infinite time span is proposed for the deteriorating systems with the assumption of restoring system's degradation rate after each PM activity. The improvement factor developed by Yang et al. (2003) is applied in this research to measure the restoration effect on the degradation rate of a system after each PM activity. In recent years, many systematic approaches for fuzzy modeling are implemented to automatically generate fuzzy rules from a given input-output data. In order to generate fuzzy rules from the given input-output data, fuzzy partitions in the input space are generally considered to determine the premise part of a fuzzy system. The grid-type and scatter-type fuzzy partitions for the input space have been often used to model fuzzy systems for the identified system. In J.S. Roger Jang (1993), an adaptive grid partition in the input space was used to design the ANFIS-based fuzzy system. This approach takes the uniformly partitioned grid as the initial state. The grid evolves as the parameters in the premise membership function are adjusted. However, the adaptive grid partition scheme has the drawback that the number of fuzzy sets for each input variable is predetermined. In C.C. Wong and C.C. Chen (2000), a binary-coded genetic algorithm (GA) is applied to determine an appropriate number of fuzzy sets for each input variable and the shapes of membership functions associated with fuzzy sets for each input variable. However, in order to obtain accurate center positions of membership functions associated with fuzzy sets for each input variable, a long coded string associated with the individual must be used in the GA approach. Consequently, the generated fuzzy systems by the individuals maybe have an enormous number of fuzzy sets as the first several generations so that the binary-coded GA approach takes a long training time. Besides, the above-mentioned approaches to the grid-type fuzzy partition of the input space still have two drawbacks as follows. As the number of dimensions increases, the number of fuzzy rules becomes enormous. Furthermore, they probably generate dummy rules because of the lack of training data in the corresponding fuzzy regions. Instead of covering the whole input space, the scatter-type fuzzy partition tries to find a subset of the input space that characterizes the fuzzy regions of possible occurrence of the training data. Each fuzzy region maps the premise part of a fuzzy rule, which is associated with several membership functions. Several clustering techniques had been used to determine the premise part of the fuzzy system, such as fuzzy c-mean (FCM) algorithm M. Sugeno and T. Yasukawa (1993) and the ART-based method P.K. Simpson (1992). The basic idea is to group the input data into clusters and use one rule for one cluster. The number of fuzzy rules equals the number of clusters. In M. Sugeno and T. Yasukawa (1993), the FCM algorithm is applied to determine the premise part of the fuzzy system. However, the disadvantage of the FCM algorithm is that the number of clusters must be predetermined. That is, the number of fuzzy rules in the fuzzy system designed by the FCM algorithm must be predetermined. If the number of clusters is given, the clustering results of the FCM algorithm are also influence by the choice of initial cluster centers and the distance measure. In P.K. Simpson (1992), a method for generating the hyperbox regions is proposed to determine the premise part of a fuzzy system for the identified system. In this approach, the learning parameter is very critical, since it directly affects the number and position of the resulting hyperboxes. Consequently, the above-mentioned approaches to the scatter-type fuzzy partitions for the input space suffer from a high sensitivity of the accuracy with respect to the skill of the user to determine a predefined parameter for the number of rules. In this paper, a method based on the particle swarm optimization (PSO), called a PSO-based method, is proposed to automatically determine an appropriate number of fuzzy rules for the identified system. A fitness function is designed to deal with the tradeoff between the number of rules and the approximation accuracy. In the PSO-based method, each individual corresponds a fuzzy system. The PSO is applied to determine an appropriate number of rules and the membership functions of the generated fuzzy system. Based on the guidance of the defined fitness function, the fuzzy system corresponding to the individual will satisfy the desired objective as well as possible. Consequently, the selected fuzzy system has an appropriate number of rules and a small mean-squared error for the identified system. An algorithm is proposed to obtain the optimal interval of each PM and the optimal number of PM before replacement, which are determined by minimizing the cost rate.

An example of Weibull failure distribution is given to confirm the proposed model. A sensitivity analysis for the parameters of the proposed model is also studied. Furthermore, an estimation method for the parameters of the hazard rate function and the improvement factor of the proposed PM model for the deteriorating and repairable systems is also developed in this research. The Particle swarm optimization (PSO) method is applied to develop the algorithms of parameter estimation. A numerical analysis method is applied to search the optimal values of the estimates. A Monte Carlo simulation is performed to study the accuracy and the properties of the estimates developed in the research.

2. The PM model

A periodic PM model is developed with applying the improvement factor provided by Yang et al. (2003). The assumptions, the improvement factor, the hazard rate function, the cost rate function, and the optimal solution algorithm for this PM model are presented as follows.

2.1. The Assumptions

The assumptions made for this PM model are as follows.

- The system is deteriorating and repairable over time with increasing failure rate (IFR).
- The periodic PM activities with constant interval (h) are performed over an infinite time span.
- The periodic PM activities can restore the degradation rate of the system to a younger level, while the hazard rate keeps monotone increase.
- Minimal repair is performed when failure occurs between each PM.

- The system is replaced at the end of the N01 interval.
- The improvement factor method is applied to measure the restoration effect on the degradation rate of the system.
- The improvement factor of each PM is a variable which is a function of the number of PM performed and the cost ratio1 of each PM.
- The costs of PM, minimal repair, and replacement are assumed to be constant. The cost of PM and the cost of minimal repair are not greater than the cost of replacement.
- The times to perform PM, minimal repair, and replacement are negligible.

2.2. The Improvement Factor

The improvement factor applied in this paper is developed by Yang et al. (2003), which is assumed to be a function of the number of PM performed and the cost ratio of each PM. The function of this improvement factor is shown as follows.

$$\eta_i = \left[a \frac{C_{pm}}{C_{pr}} \right]^{bi} \quad (1)$$

where η_i represents the improvement factor of the i^{th} PM, $0 < \eta_i < 1$, C_{pm} is the cost of each PM, C_{pr} is the replacement cost of a system, a and b are the adjustment parameters for the improvement factor whose values are varied with different system and can be determined by the system's historical data or by experience. $\eta_i = 0$ and $\eta_i = 1$ are exclusive in this research since it can be seen that $\eta_i = 0$ means the age of a system after the PM remains the same as that before the PM (i.e., bad as old) and is called minimal repair; $\eta_i = 1$ means the PM is perfect and the system's condition becomes as good as new. Parameter a is a cost adjustment coefficient and $a < C_{pr} / C_{pm}$. It turns out that $a\ C_{pm} / C_{pr} < 1$. Note that parameter a reflects the effect of the PM cost ratio for different systems. Parameter b is an age adjustment coefficient and $b > 0$. It can be seen that the larger the i (i.e., the older the system), the smaller the η_i Moreover, the larger the PM cost ratio (C_{pm} / C_{pr}), The cost ratio of each PM means the ratio of the cost per PM activity to the replacement cost larger the η_i and thus the better the restoration of the system.

2.3. The Effective Age

The effective age at the time of the i^{th} PM, can be shown in Equations (2) and (3) for prior- and posterior-PM, respectively.

$$w_i^-(ih) = \left[i - \sum_{k=0}^{i-1} \eta_k \right] h, i = 1,2,...,N-1 \quad (2)$$

$$w_i^+(ih) = \left[i - \sum_{k=0}^{i} \eta_k \right] h, i = 1,2,...,N-1 \quad (3)$$

where h is the periodic interval of PM. It is assumed that $w_0^-(0) = w_0^+(0) = 0$. The effective age at τ unit of time after the i^{th} PM can be presented as follows

$$w_i^+(ih+\tau) = w_i^+(ih) + \tau = \left[i - \sum_{k=0}^{i} \eta_k \right] h + \tau, \quad 0 \le \tau \le h, \quad i = 0,1,...,N-1 \quad (4)$$

2.4. The Hazard Rate Function

For Weibull failure distribution with shape parameter β and scale parameter θ, the hazard rate function at time i^{th} but prior to the i^{th} PM is shown in Equations (5) and Equation (6) shows the hazard rate function for the time after the i^{th} PM but prior to the $(i + 1)^{th}$ PM.

$$\lambda_i^-(ih) = \lambda_0(w_i^-(ih)) = \frac{\beta}{\theta} \left((\frac{w_i^-(ih)}{\theta}) \right)^{\beta-1} \quad i = 1,2,...,N-1 \quad (5)$$

$$\lambda_i(t) = \lambda_0(w_i^+(t)) = \frac{\beta}{\theta} \left(\frac{w_i^+(t)}{\theta} \right)^{\beta-1} \quad ih \le t < (i+1)h, \quad i = 1,2,...,N-1 \quad (6)$$

2.5. The Cost Rate Function of the PM Model

The cost rate function of the proposed PM model can be obtained as follows.

$$C(h,N) = \frac{(N-1)C_{pm} + C_{pr} + C_{mr} \sum_{i=0}^{N-1} \int_{ih}^{(i+1)h} \lambda_i(t)dt}{Nh} = \frac{(N-1)C_{pm} + C_{pr} + C_{mr} \sum_{i=0}^{N-1} \int_{W_i^+}^{W_{i+1}^-} \lambda_0(t)dt}{Nh} \quad (7)$$

Where $\lambda_0(t)$ is the original hazard rate function?

2.6. The Optimal Number of PMs and the Optimal Time to Replacing a System

2.6.1. The PM model without failure rate limit

Based on the algorithm provided by Nakagawa (1986), the optimal solution of h can be obtained as function of N by taking partial derivative of h on $C(h, N)$ and letting it equal zero. That is,

$$\frac{\partial C(h,N)}{\partial h} = 0 \quad (8)$$

Thus, the periodic interval of PM of this model (h) can be obtained as

$$h^* = \left\{ \frac{[(N-1)C_{pm} + C_{pr}] / [C_{mr}(\beta-1)]}{\sum_{i=1}^{N} \left[\left((i - \sum_{j=1}^{i} \eta_{j-1}) / \theta \right)^{\beta} - \left((i-1-\sum_{j=1}^{i} \eta_{j-1}) / \theta \right)^{\beta} \right]} \right\}^{1/\beta} \quad (9)$$

Then, the optimal number of PM and the optimal time interval between PMs can be obtained by the following algorithm.

1. Let $N = 1$, obtain h value using Equation (9) and cost rate (C(h, 1)) using Equation (7).
2. Let $Cmin$ = C (h, 1).
3. Let $N = N + 1$.
4. Obtain h value using Equation (9) and calculate cost

rate (C(h, N)) using Equation (7).

5. If C(h, N) < $Cmin$, then, $Cmin$ = C(h, N) and return to Step 3, otherwise stop.

2.6.2. The PM model with failure rate limit

Suppose that the system has to be replaced when the reliability or failure rate reaches a certain level, say R^* or λ^*, respectively. Let $W_{N_R}^-$ be the effective age of which the failure rate reaches $\lambda^*$and the replacement is in the N_R^{th} PM. Then, we can obtain

$$R(W_{N_R}^-) = R^* \text{ or } \lambda(W_{N_R}^-) = \lambda^* \quad (10)$$

Thus, the periodic interval of PM for this model (h_R) can be found as

$$h_R = \left[\theta(-ln R^*)^{1/\beta}\right] \Big/ \left[N_R - \sum_{j=1}^{N_R-1} \eta_j\right] \quad (11)$$

Then, the optimal value of N, N_R, can be determined so that

$$N_R = \min_N \ C(h_R,N),\ N=1,2,...$$

3. Fuzzy System Structure

When M input variables (x_1, x_2, ..., x_m) and a single output variable are considered, a rule base of a fuzzy system can be expressed as follows:

j-th rule:

If $x_1$is A_{j1} and $x_2$is A_{j2} and…and $x_m$is A_{jM}

Then y is $y_j = a_{j0}+ a_{j1}X_1+…+ a_{jM}X_M$

J=1, 2,…, R

where R is the number of fuzzy rules in the rule base, A_{ji}, j=1,2...,R, i = 1,2,…,M are the fuzzy sets of the premise part, and a_{ji}, j=1,2...,R, i = 0,1,…,M are the real numbers of the consequent part. In this paper, the membership function of the fuzzy set A_{ji} is described by

$$\mu_{A_{ji}}(m_{(ji,1)},m_{(ji,2)},m_{(ji,3)};x_i)=\begin{cases}\exp\left(-\left(\dfrac{x_i-m_{(ji,1)}}{m_{(ji,2)}}\right)^2\right), & if\, x_i \le m_{(ji,1)},\\ \exp\left(-\left(\dfrac{x_i-m_{(ji,1)}}{m_{(ji,3)}}\right)^2\right), & if\, x_i > m_{(ji,1)},\end{cases} \quad (12)$$

Where $m_{(ji,1)}$, $m_{(ji,2)}$ and $m_{(ji,3)}$ determine the center position, the left and right width values of the membership function, respectively. Hence, the shape of the membership function is determined by a parameter vector $\underline{m}_{ji} = \left[m_{(ji,1)}m_{(ji,2)}m_{(ji,3)}\right]$. The j-th fuzzy rule in the rule base is determined by a parameter vector $\underline{r}_j = \left[\underline{m}_{j1}\underline{m}_{j2}...\underline{m}_{jm}\right]$. Consequently, the set of parameters in the premise part of the rule base is defined as $\underline{r}_j = [\underline{r}_1\underline{r}_2...\underline{r}_R]$. According to (1), the set of parameters in the consequent part of the rule base is defined as $a = [a_{10}a_{11}...a_{1M}a_{20}a_{21}...a_{2M}...a_{R0}a_{R1}...a_{RM}]$. When the input $\underline{x} = (x_1x_2...x_M)$ is given, the firing strength of the premise of the j-th rule is calculated by $q_j(x) = \prod_{i=1}^{M}\mu_{A_{ji}}(x_i)$. By taking the weighted average of y_j, the output of the fuzzy system with respect to the input $\underline{x}$ can be determined by

$$y = \frac{\sum_{j=1}^{R} q_j(\underline{x})\cdot(a_{j0}+a_{j1}x_1+\cdots+a_{jM}x_M)}{\sum_{j=1}^{R} q_j(\underline{x})} \quad (13)$$

According to the above description, each parameter set consisting of the premise and consequent parameters determines a fuzzy system. Thus, different parameter sets determine different fuzzy systems so that the generated fuzzy systems have different performances. The goal of this paper is to find an appropriate fuzzy system to approach an identified system where only the input-output data are available. Therefore, the mean-squared error between the generated fuzzy system and the identified system and the number of rules of the generated fuzzy system can be viewed as performance index. In the next section, the PSO-based method and the recursive least-squares method are applied to find a fuzzy system with an appropriate number of fuzzy rules and a small mean-squared error for the identified system.

4. Identification of fuzzy systems using the PSO-based method

The particle swarm optimization is an evolutionary computation technique proposed by Kennedy and Eberhart S.J. Lee and C.S. Ouyang (2003)and J. Kennedy and R. Eberhart (1942-1948). Its development was based on observations of the social behavior of animals such as bird flocking, fish schooling, and swarm theory. Like the GA, the PSO is initialized with a population of random solutions. It requires only the information about the fitness values of the individuals in the population. This differs from many optimization methods requiring the derivation information or the complete knowledge of the problem structure and parameter. Compared with the GA, the PSO has memory so that the information of good solutions is retained by all individuals. Furthermore, it has constructive cooperation between individuals, individuals in the population share information between them. In this paper, a PSO-based method is proposed to find a fuzzy system with an appropriate number of fuzzy rules and have a small mean squared error for the identified system. In the PSO-based method, each individual is represented to determine a fuzzy system. The individual is used to partition the input space so that the rule number and the premise part of the generated fuzzy system are determined. Subsequently, the recursive least-squares algorithm is applied to determine the parameters of the consequent part of the corresponding fuzzy system. A set of individuals, P, called population, is expressed in the following:

$$P = \begin{bmatrix}\underline{P}_1\\ \underline{P}_2\\ \\ \underline{P}_h\\ \\ \underline{P}_L\end{bmatrix} = \begin{bmatrix}\underline{r}_1 & \underline{g}_1\\ \underline{r}_2 & \underline{g}_2\\ \\ \underline{r}_h & \underline{g}_h\\ \\ \underline{r}_L & \underline{g}_L\end{bmatrix} \quad (14)$$

In order to evolutionarily determine the parameters of the fuzzy system, the individual $\underline{p}_h$ contains two parameter vectors: $\underline{r}_h$ and $\underline{g}_h$. That is, $p_h = [\underline{r}_h \underline{g}_h]$.

The parameter vector $\underline{r}_h = [\underline{r}_1^h \underline{r}_2^h ... \underline{r}_j^h ... \underline{r}_B^h]$ consists of the premise parameters of the candidate fuzzy rules, where B is a user-defined positive integer to decide the maximum number of fuzzy rules in the rule base generated by the individual $\underline{p}_h$. Here, $\underline{r}_h = [r_1^h r_2^h ... \underline{r}_j^h ... \underline{r}_b^h]$ is the parameter vector to determine the membership functions of the j-th fuzzy rule, where $\underline{m}_{ji}^h = [\underline{m}_{(ji,1)}^h \underline{m}_{(ji,2)}^h m_{(ji,3)}^h]$ is the parameter vector to determine the membership function for the i-th input variable. The parameter vector $\underline{g}_h = [g_1^h g_2^h ... g_j^h ... g_b^h]$ is used to select the fuzzy rules from the candidate rules $\underline{r}_h = [\underline{r}_1^h \underline{r}_2^h ... \underline{r}_j^h ... \underline{r}_B^h]$ so that the fuzzy rule base is generated. $g_j^h \in [0,1]$ decides whether the j-th candidate rule $\underline{r}_j^h$ is added to the rule base of the generated fuzzy system or not. If $g_j^h \geq 0.5$ then the j-th candidate rule $\underline{r}_i^h$ is added to the rule base. Consequently, the total number of $g_j^h (j = 1,2,...,B)$ whose value is greater than or equal to 0.5 is the number of fuzzy rules in the generated rule base. In order to generate the rule base, the index j of $g_j^h (j = 1,2,...,B)$ whose value is greater than or equal to 0.5 is defined as $I_r^h \in \{1,2,...,B\}, r = 1,2,...,r_h$ where r_h represents the number of the fuzzy rules in the generated rule base. $\{\underline{r}_{I_1^h}^h, \underline{r}_{I_2^h}^h, ..., \underline{r}_{I_r^h}^h, ..., \underline{r}_{I_{r_h}^h}^h\}$ generates the premise part of the fuzzy rule base generated by the individual $\underline{p}_h = [\underline{r}_h \underline{g}_h]$. Consequently, the rule base of the generated fuzzy system is described as follows:

r-th rule:

If x_1 is $A_{I_r^h 1}^h$ and x_2 is $A_{I_r^h 2}^h$ and…and x_m is $A_{I_r^h M}^h$,

Then y is $y_r^h = a_{r0}^h + a_{r1}^h x_1 + ... + a_M^h x_M$

$r = 1,2,...,r_h$,

where $A_{I_r^h i}^h, i = 1,2,...,M$, are the fuzzy sets of the generated r-th fuzzy rule. The membership function associated with the fuzzy set $A_{I_r^h i}^h$ is described as follows:

$$\mu_{A_{I_r^h i}^h}(m_{(I_r^h i,1)}^h, m_{(I_r^h i,2)}^h, m_{(I_r^h i,3)}^h; x_i) = \begin{cases} \exp\left(-\left(\dfrac{x_i - m_{(I_r^h i,1)}^h}{m_{(I_r^h i,2)}^h}\right)^2\right) & if\ x_i \leq m_{(I_r^h i,1)}^h, \\ \exp\left(-\left(\dfrac{x_i - m_{(I_r^h i,1)}^h}{m_{(I_r^h i,3)}^h}\right)^2\right) & if\ x_i > m_{(I_r^h i,1)}^h, \end{cases} \quad (15)$$

Consequently, the individual $\underline{p}_h$ determines the premise part of the generated fuzzy system. Subsequently, the recursive least-squares method is applied to determine the parameters in the consequent part of the generated fuzzy system. According to the above description, each individual corresponds to a fuzzy system. In order to construct a fuzzy system which has a low number of fuzzy rules and a small mean-squared error simultaneously, the fitness function is defined as follows:

$$f_h = fit(\underline{p}_h) = g_1(\underline{p}_h) g_2(\underline{p}_h) \quad (16)$$

where f_h is the fitness value of the individual $\underline{p}_h$, $g_1(\underline{p}_h)$ and $g_2(\underline{p}_h)$, are defined respectively as follows:

$$g_1(\underline{p}_h) = \exp\left(-\frac{\frac{1}{N}\sum_{n=1}^{N}(y_n - y_{f_n}^h)^2}{\sigma_e}\right) \quad (17)$$

and

$$g_2(\underline{p}_h) = \exp\left(-\frac{r_h}{\sigma_r}\right) \quad (18)$$

Here, σ_e and σ_r are user-defined constants for the fitness function. Consequently, the fitness function will guide the individual to find a fuzzy system with a low number of rules and a small mean-squared error. In this way, as the fitness function value increases as much as possible based on the guidance of the proposed fitness function, the fuzzy system corresponding to the individual will satisfy the desired objective as well as possible. That is, the selected fuzzy system has a low number of rules and a small mean-squared error simultaneously. Subsequently, a PSO-based method is proposed to find an appropriate individual so that the corresponding fuzzy system has the desired performance. The procedure is described as follows:

5. Numerical examples

5.1. The Proposed PM Method

From a numerical example with the following conditions: Weibull (β = 10, θ = 100), C_{pm}= 10,000, C_{mr}= 50,000, C_{pr}= 5,000,000, a = 1, b = 0.001 and R^* = 0.6.

In order to illustrate the usefulness of the proposed method, two identification problems of nonlinear systems are discussed here. Example: Approaching a fifth-order polynomial in this example, we use the proposed method to approximate a function with a fifth-order polynomial as follows B. Kosko(1997), C.C. Wong and C.C. Chen,(2000):

$$y = 3x_1 B(x_1 - \underline{g}_h)(x_1 - 0.7 * h^* N^*)(x_1 + 0.7t)(x_1 + 1.8\beta / \underline{r}_h) \quad (19)$$

A total of 100 training input-output pairs and 100 checking input-output pairs are sampled uniformly from the input ranges [-2,2] and [-1.95,1.95], respectively. Following the proposed method, the simulation result is shown in Fig. 1, where the initial conditions for the proposed method in Example 1 are given in the following: The number of individuals: L = 100, the maximum number of rules: B = 20, the number of generations: K=100, the range of $m_{(j1,1)}^h, j \in \{1,2,...,20\} : [-2,2]$, the range of $m_{(j1,2)}^h, j \in \{1,2,...,20\} : [0.01,2]$, the range of $m_{(j1,3)}^h, j \in \{1,2,...,20\} : [0.01,2]$, the constants for the fitness function: $\{\delta_e,\delta_r\}$={0.15} and the constants for the PSO: $\{\psi,c_1,c_2,d_1,d_2\}$={1,1,1,0.75,0.75}. We can obtain the optimal solution of N^* = 18, h^* = 60.895, $T = h^*N^*$ = 1096, and $C(h^*,N^*)$ = 5241 for the case of no reliability limit; N_R= 18, h_R = 48.769, $T = h_R N_R$ = 878, and $C(h_R,N_R)$ = 5961 for the case of having reliability limit.

The effects of C_{pm} and C_{mr} as well as of parameters a and b for the proposed models are shown in Tables 1 and 2.

5.2. The experiments from a Monte Carlo simulation

The length of the experiment (T) is 150 units of time. The PM interval, h, is 5 units of time. So, a total of 30 PM activities have been simulated. The input values of parameters are set to be: θ = 10; C_{pm}= 10,000; C_{pr}=5,000,000; a = 1; b = 0.01. The limits of computation error are set to be $5*10^{-3}$ and 20 for the maximum iteration. The inter-failure times generated from the Monte Carlo simulation are then used to calculate the estimates of parameters by employing the proposed method. The estimates of parameters are obtained for β = 1.5, 2, and 2.5 as can

Tab. 1. The effect of C_{pm} and C_{mr} for the proposed models

		C_{pm}/C_{mr} = 0.5		C_{pm}/C_{mr} = 2		C_{pm}/C_{mr} = 10		C_{pm} = 10000					
		C_{mr} = 50000						C_{pm}/C_{mr} = 2		C_{pm}/C_{mr} =0.5		C_{pm}/C_{mr} = 0.1	
Contraint		No limit	Reliab. limit	No limit	Reliab. limit	No limit	Reliab. limit	No limit	Reliab. limit	No limit	Reliab. limit	No limit	Reliab. limit
h^*	h_R	62.13	52.35	71.00	58.99	91.69	75.46	76.66	48.77	66.74	48.77	56.82	48.77
N^*	N_R	19	18	19	18	17	15	18	18	18	18	18	18
$T=h^*N^*$	$T=N_Rh_R$	1181	942	1349	1062	1559	1132	1380	878	1201	878	1023	878
$C(h^*,N^*)$	$C(h_R,N_R)$	5130	5829	5601	6385	9267	10715	4163	5897	4782	5918	5617	6032
R_T	R^*	0.009	0.6	0.007	0.6	0.002	0.6	3.6E-21	0.6	7.8E-6	0.6	0.095	0.6

Tab. 2. The effect of parameters a and b for the proposed models

		a = 1		a = 10		a = 100		a = 1					
		b = 0.001						b = 0.0001		b =0.001		b = 0.1	
Contraint		No limit	Reliab. limit	No limit	Reliab. limit	No limit	Reliab. limit	No limit	Reliab. limit	No limit	Reliab. limit	No limit	Reliab. limit
h^*	h_R	60.90	48.77	58.57	49.77	57.21	49.55	56.78	50.62	60.90	48.77	58.44	43.00
N^*	N_R	18	18	23	22	35	34	54	53	18	18	3	3
$T=h^*N^*$	$T=N_Rh_R$	1096	878	1347	1095	2002	1685	3066	2683	1096	878	175	129
$C(h^*,N^*)$	$C(h_R,N_R)$	5241	5961	4306	4827	2963	3227	2004	2119	5241	5961	31817	39113
R_T	R^*	0.009	0.6	0.018	0.6	0.059	0.6	0.147	0.6	0.009	0.6	1.7E-5	0.6

be seen in Table 2. The estimates are close to the input values and show small deviation for all parameters.

The accuracy and precision of the estimates of parameters are also investigated for β = 1.5, 2, and 2.5. For each case of β, twenty sets of $\hat{\beta}$, $\hat{\theta}$, $\hat{a}$, and $\hat{b}$ are not affected by the changing of β. However, from Table 1, it can be seen that the coefficients of variation $\hat{\beta}$, $\hat{\theta}$, $\hat{a}$, and $\hat{b}$ are decreased when β is increased. It is also found that the variation of the estimates is in the following order (from small to large) $\hat{\beta}$, $\hat{\theta}$, $\hat{a}$, and $\hat{b}$ for each β. On the other hand, for different β values, it is noticed that the smaller the β, the larger the variation of estimates. In Table 2, it is found that the larger the β, the more number of $\hat{\beta}$ are fall into the range of $\beta\pm$ 5%.

6. Conclusions

From this research, two major results are obtained: (1) the periodic preventive maintenance policies over an infinite time span for repairable systems with age reduction after performing an imperfect PM activity; (2) the estimates of parameters for the failure distribution and the restoration effect of the proposed PM model. In the proposed PM model, the optimal PM interval and the optimal replacement time are obtained by minimizing the objective functions of the cost rate through the algorithms provided by this research.

A PSO-based method is proposed to construct a fuzzy system direct from some gathered input-output data of the identified system.

In the proposed approach, each individual $\underline{p}_h$ consists of two parameter vectors: $\underline{r}_h$ and $\underline{g}_h$. The parameter vector $\underline{g}_h$ is updated so that the generated fuzzy system has an appropriate number of rules. The parameter vector $\underline{r}_h$ is updated so that the premise part of the generated fuzzy system has appropriate membership functions for the identified system. Then, the recursive least-squares method is applied to determine the consequent parameters of the corresponding fuzzy system. Consequently, each individual corresponds to a fuzzy system. Subsequently, a fitness function is defined to guide the searching procedure to select a fuzzy system with the desired performance. The simulation results for two nonlinear systems show that the selected fuzzy system not only approaches the identified system well but also has an appropriate number of rules for the identified system.

The PSO-based method is employed in this research to find the estimates of parameters for the hazard rate function and the improvement factor of a PM model.

The numerical analysis, fuzzy system, is used to search the solution. It is found that the initial guess used for the fuzzy system do not affect the obtained estimates of parameters.

The simulation results have shown that βhas significant effect on the variation of the estimates. The current computation algorithm is based on the complex equations; a simple and easy approximate estimation method should be developed.

7. References

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