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MODEL MATEMATYCZNY WAHAŃ PRZESTRZENNYCH WAGONU PASAŻERSKIEGO

MATHEMATICAL MODEL OF SPATIAL FLUCTUATIONS OF PASSENGER WAGON

Dla komfortu podróżnych i dla zapewnienia trwałości części i zespołów taboru kolejowego niezbędnym jest wybór racjonalnych parametrów zawieszenia podwozia, tj. ustalić racjonalne stosunki pomiędzy dwoma elementami sztywności stopni zawieszenia i tłumienia drgań. Niniejszy artykuł jest przeznaczony dla stworzenia przestrzennego modelu matematycznego, gdy istnieje zawieszenia z dwoma stopniami i różnymi elementami sztywności. Opracowany system równań posiada 52 stopnie swobody. Przedstawiony model matematyczny może byś stosowany dla ustalenia rzeczywistych obciążeń dynamicznych wagonów pasażerskich i przez to do wyboru racjonalnych parametrów krótkich zawieszeń.

Sowa kluczowe: *wagon, zawieszenia, stopień, stopień swobody, wibracje, tłumienie, przesuw, sztywność*

To ensure passenger comfort as well as the durability of the components of rolling stock, it is necessary to determine the rational parameters of chassis suspension, i.e. rational relations between rigidity and vibration inhibition elements of the two stages of chassis suspension. The article describes a spatial mathematical model of a two-stage suspension that has different rigidity elements. The equation system created has 52 degrees of freedom. The model presented in this article can be employed to establish the real dynamic load of passenger wagons and to determine the rational parameters of resilient suspension.

Keywords: *wagon, suspension, stage, degree of freedom, vibration, inhibition, shift, rigidity.*

1. Introduction

When examining various dynamic processes, huge importance falls to vibration inhibition systems. For those systems to operate efficiently, it is necessary to determine their rational parameters. When talking about parameters of railway chassis, it is necessary to determine rational relations between rigidities of two stages of chassis suspensions, just as rational relations between inhibition parameters of these stages.

To solve the abovementioned tasks, the present article described a mathematical model for determination of spatial fluctuations of chassis.

2. Description of Structural Schemes

When making a mathematical model for determination of spatial vibrations of railway rolling stock chassis with two stages of suspension, we referred to a calculation scheme, close to carriages KVZ-CH11, Y-32 and 68-7007 [1, 3, 4]. In these carriages, vertical loads are transmitted through sliders of beam located above the resilient suspension. In case of structures of non-cradle carriages, they also transmit horizontal, longitudinal and transverse forces. In structures with a cradle, a pin is used for this purpose. To reduce shock load in pin node and sliders, it is planned to install resilient-viscous (rubber) elements there. Thus, in the structures in question, it is necessary to take into consideration the shifts of the beam above suspension in respect of the bodywork in longitudinal and transverse direction and when chassis is coiling.

The beam above chassis is linked to carriage frame by means of non-cradle structure, through a spring of increased resilience, which creates restorable moments for mutual shifts of beam and carriage in all the directions, and in cradle-type structures this function is performed by the cradle. We must also take into consideration that in case of mutual horizontal shifts in structures of above-suspension beam and carriage cradle, resilient leads operate as well.

In axle-box node, resilient-dissipative elements allow the carriage frame and axle-wheel pair shifting in respect of each other in all the direction. In some carriages, leads are mounted here.

3. Creation of Mathematical Model

Chassis calculation scheme with the said carriages constitutes a system consisting of 9 solid bodies (bodywork, 2 beams above suspension, 2 carriage frames, 4 pairs of axle-wheels) (Fig. 1).

The followings marks will be conferred (see Fig. 1): x, y, z – shifts of system bodies along road axis (x – jerks), across road axis (y – lateral shift) and along the vertical (z – jumps); θ, ϕ, ψ – angle shifts around the axes x (θ – lateral swing), y (ϕ – longitudinal swing) and z (ψ – soiling). Positive sliding shifts take place along respective axes, and positive angle shifts – counter clockwise, when looking from positive direction of respective axis (Fig. 1). Positive shifts for bodywork with centre of mass in point C are shown.

Further bodywork shifts will be marked without index. Index i (i =*1,2* – carriage number) – carriage frame, index *i'i* – the beam above suspension, index *in* (m =1, 2 – number of axle-wheel pair in carriage) – axle-wheel pairs, *pmik* (k=1– left according to direction of movement, k=2 – right side of wagon) – in points of wagon to wheels contact. The calculation scheme takes into consideration recalculated road masses in points of wheels to wagon contacts, which shift in to directions – horizontal across road axis and according to the vertical.

Total number of shifts is equal to:

$$9 \cdot 6 + 2 \cdot 8 = 70$$

Let's review the system relations:

- Between bodywork and the beams above suspensions, mutual shifts are possible in horizontal, longitudinal and transverse direction, also when coiling, i.e. jumps, lateral and longitudinal swing of beams are determined from respective shifts of bodywork:

$$\left.\begin{aligned} z_{i'3} &= z + (-1)^i l\varphi, \\ \theta_{i'3} &= \theta, \\ \varphi_{i'3} &= \varphi, \end{aligned}\right\} \tag{1}$$

where l is half of chassis base;

- Longitudinal swing of axle-wheel pairs is expressed as their jerk (slips are determined when calculating forces of pseudo-slippage):

$$\varphi_{im} = \frac{x_{im}}{r} \tag{2}$$

where r is the radius of wheel rolling circle;

- Wheels move without breaking away from rail:

$$z_{pimk} = z_{im} + (-1)^k b_2\theta_{im} + \Delta r_{imk} - \eta_{i'imk} \tag{3}$$

where b_2 is half of the distance between wheel rolling circles in transverse direction; Δr_{imk} is change of wheel rolling radius in case of lateral axle-wheel shift; $\eta_{i'imk}$ is the ordinate of road's vertical inequalities.

Thus we make (introduce) 18 equations of relation. It means that the system has 70-18=52 degrees of freedom. Let's write down the summarised coordinates:

- Bodywork shifts:

$$q_1 = z,\ \ q_2 = \varphi,\ q_3 = \theta,\ \ q_4 = y,\ \ q_5 = \psi$$

- Shifts of carriage frames:

$$q_n = z_i\,(n = 6{,}7),\ \ q_n = \varphi_i\,(n = 8{,}9),\ q_n = \theta_i\,(n = 10{,}11),$$

$$q_n = y_i\,(n = 12{,}13),\ q_n = \psi_i\,(n = 14{,}15),$$

- Shifts of axle-wheel pairs:

$$q_n = z_{im}\left(n = \overline{16,\ 19}\right) \qquad q_n = \theta_{im}\left(n = \overline{20,\ 23}\right)$$

$$q_n = y_{im}\left(n = \overline{24,\ 27}\right) \qquad q_n = \psi_{im}\left(n = \overline{28,\ 31}\right)$$

- Shifts of beams above suspensions:

$$q_n = y_{i'3}\left(n = 32,\ 33\right) \qquad q_n = \psi_{i'3}\left(n = 34,\ 35\right)$$

- Rail impress (reaction) in the points of contacts with wheels:

$$q_n = y_{pimk}\left(n = \overline{36,\ 43}\right)$$

- Jerks of system bodies:

$$q_n = x_i\,(n = 44{,}45) \qquad q_n = x_{i'i}\left(n = 46,\ 47\right)$$

$$q_n = x_{im}\left(n = \overline{48,\ 51}\right) \qquad q_{52} = x$$

Static pressure:

- Wheel to rail:

$$D_{\text{ň}o} = \frac{m_{y'} g}{8}$$

where $m_{y'}$ is the weight of entire chassis; g is free fall acceleration;

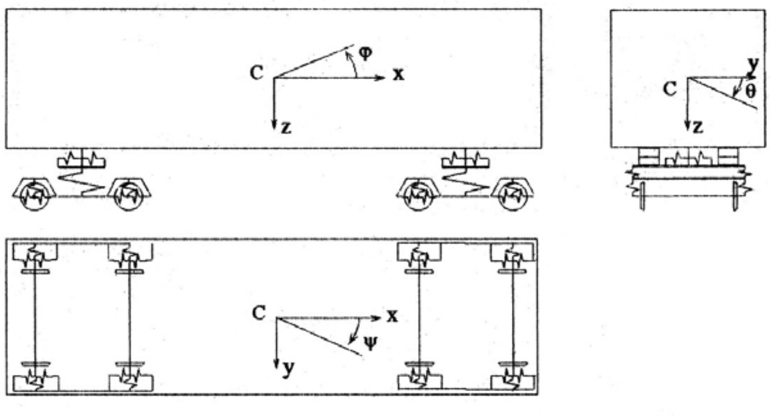

Fig. 1. Calculation Scheme for Chassis with Two-Stage Suspension

- To resilient elements of central suspension:

$$D_{\tilde{n}\cdot o\ddot{o}} = \frac{(m+2m_{i'})g}{4}$$

where m is bodywork weight; $m_{i'}$ m_i is the weight of the beam above suspension;

- To resilient elements of axle-box suspension:

$$D_{\tilde{n}\cdot oa'} = \frac{(m+2m_{i'}+2m_{\cdot o})g}{8}$$

where $m_{\cdot o}$ is weight of carriage frame.

When determining inertia parameters it is assumed that the carriage weight:

$$m_{\cdot o\dot{a}\ddot{e}} = m_{\cdot o} + m_{i'} + 2m_{\dot{e}}$$

and the chassis weight:

$$m_{y'} = m + 2m_{o\dot{a}\ddot{e}}$$

Total static bend of resilient suspension is determined from the following formula:

$$f = f_{\ddot{o}} + f_{a'}$$

where $f_{\ddot{o}}$ and $f_{a'}$ are static bends of first and second stages of resilient suspension:

$$f_{\ddot{o}} = \frac{P_{\tilde{n}\cdot o\ddot{o}}}{k_{\ddot{o}}} \qquad f_{a'} = \frac{P_{\tilde{n}\cdot oa'}}{k_{a'}}$$

where $k_{\ddot{o}}$ is recalculated rigidity of central suspension; $k_{a'}$ is rigidity of axle-box suspension.

Let's review mutual shifts of all the bodies of the system. Markings of geometrical parameters included into shift expressions are described in detail in [1].

Mutual shifts between bodywork and the beam located above suspension:

- In pin zone, in longitudinal and transverse horizontal directions:

$$\left.\begin{aligned} &\Delta_{\varnothing\,\ddot{o}i} = x + h_{\varnothing}\varphi - x_{i'} \\ &\Delta_{\varnothing\,yi} = y - h_{\varnothing}\theta - (-1)^i l\psi - y_{i'i;} \end{aligned}\right\} \qquad (4)$$

- Between sliders in longitudinal and horizontal transverse directions:

$$\left.\begin{aligned} &\Delta_{cxik} = x + h_c\varphi - (-1)^k b_c\psi - x_{i^{'3}} + (-1)^k b_c\psi_{i^{'3}}, \\ &\Delta_{cyik} = y - h_c\theta - (-1)^i l\psi - y_{i^{'3}}. \end{aligned}\right\} \qquad (5)$$

Respective forces are calculated from the following formulas:

$$\left.\begin{aligned} &S_{\varnothing\,xi} = k_{\varnothing\ \ddot{o}}\,\Delta_{\varnothing\,xi} + \beta_{\varnothing\,\ddot{o}}\dot{\Delta}_{\varnothing\,xi}, \\ &S_{\varnothing\,o'i} = k_{\varnothing\ o'}\,\Delta_{\varnothing\,o'i} + \beta_{\varnothing\,o'}\dot{\Delta}_{\varnothing\,o'i}, \\ &S_{\tilde{n}\ddot{o}ik} = k_{cx}\Delta_{\tilde{n}\ddot{o}ik} + \beta_{cx}\dot{\Delta}_{\tilde{n}\ddot{o}ik}, \\ &S_{\tilde{n}yik} = k_{cy}\Delta_{\tilde{n}yik} + \beta_{cx}\dot{\Delta}_{\tilde{n}yik}. \end{aligned}\right\} \qquad (6)$$

Mutual shifts between carriage frame and the beam above suspension in all the directions (deformations of resilient sets of suspension):

$$\left.\begin{aligned} &\Delta_{\ddot{o}\ddot{o}ik} = x_{i^{''3}} - (-1)^k b\psi_{i^{'3}} - x_i + (-1)^k b\psi_i, \\ &\Delta_{\ddot{o}yik} = y_{i^{'3}} - y_i, \\ &\Delta_{\ddot{o}zik} = z + (-1)^i l\varphi + (-1)^k b\theta - z_i - (-1)^k b\theta_i, \\ &\Delta_{\ddot{o}\psi ik} = \psi_{i^{'3}} - \psi_i. \end{aligned}\right\} \qquad (7)$$

Respective resilient forces will be as follows:

$$\left.\begin{aligned} &S_{\ddot{o}xik} = k_{\ddot{o}x}\Delta_{\ddot{o}xik}, \\ &S_{\ddot{o}yik} = k_{\ddot{o}y}\Delta_{\ddot{o}yik}, \\ &S_{\ddot{o}zik} = k_{\ddot{o}z}\Delta_{\ddot{o}zik}, \\ &S_{\ddot{o}\psi ik} = k_{\ddot{o}\psi}\Delta_{\ddot{o}\psi ik}. \end{aligned}\right\} \qquad (8)$$

Mutual shifts between carriage frame and the beam above suspension in vibration amortisation zone:

$$\left.\begin{aligned} &\Delta^{\ddot{a}}_{\ddot{o}yik} = y_{i'i-y_i}, \\ &\Delta^{\ddot{a}}_{\ddot{o}zik} = z + (-1)^i l\varphi + (-1)^k b_{\ddot{a}i}\theta - z_i - (-1)^k b_{\ddot{a}o}\theta_i \end{aligned}\right\} \qquad (9)$$

In force hydro-absorbers:

$$\left.\begin{aligned} &S^{\ddot{a}}_{\ddot{o}o'ik} = \beta_{\ddot{o}o'}\dot{\Delta}_{\ddot{o}o'ik}, \\ &S^{\ddot{a}}_{\ddot{o}zik} = \beta_{\ddot{o}z}\dot{\Delta}_{\ddot{o}zik}, \end{aligned}\right\} \qquad (10)$$

where $\beta_{\ddot{o}o'} = \beta_{\ddot{o}}\cos^2\alpha_{\ddot{a}}$, $\beta_{\ddot{o}z} = \beta_{\ddot{o}}\sin^2\alpha_{\ddot{a}}$, inclination angle of hydro-absorber towards horizontal plane:

$$tg\alpha_{\ddot{a}} = \frac{h_{\dot{e}o} - h_{\ddot{a}\dot{e}} - h_{\ddot{a}o}}{b_{\ddot{a}o} - b_{\ddot{a}i'}}$$

Let's write down mutual shifts and forces arising in leads of the central suspension:

$$\left.\begin{aligned} &\Delta^{\ddot{i}}_{\ddot{o}xik} = \tilde{o}_{i^{'3}} - (-1)^k b_{\ddot{i}\ddot{o}}\psi_{i^{'3}} - \tilde{o}_i + (-1)^k b_{\ddot{i}\ddot{o}}\psi_3, \\ &\Delta^{\ddot{i}}_{\ddot{o}yik} = y_{i^{'3}} - y_i, \\ &\Delta^{\ddot{i}}_{\ddot{o}zik} = z + (-1)^i l\varphi + (-1)^k b_{\ddot{i}\ddot{o}}\theta - z_i - (-1)^k b_{\ddot{i}\ddot{o}}\theta_i \end{aligned}\right\} \qquad (11)$$

Respective forces will be as follows:

$$\left.\begin{aligned} &S^{\ddot{i}}_{\ddot{o}\ddot{o}ik} = k^{\ddot{i}}_{\ddot{o}\ddot{o}}\Delta^{\ddot{i}}_{\ddot{o}\ddot{o}ik} + \beta^{\ddot{i}}_{\ddot{o}\ddot{o}}\dot{\Delta}^{\ddot{i}}_{\ddot{o}\ddot{o}ik}, \\ &S^{\ddot{i}}_{\ddot{o}o'ik} = k^{\ddot{i}}_{\ddot{o}o'}\Delta^{\ddot{i}}_{\ddot{o}o'ik} + \beta^{\ddot{i}}_{\ddot{o}o'}\dot{\Delta}^{\ddot{i}}_{\ddot{o}o'ik}, \\ &S^{\ddot{i}}_{\ddot{o}zik} = k^{\ddot{i}}_{\ddot{o}z}\Delta^{\ddot{i}}_{\ddot{o}zik} + \beta^{\ddot{i}}_{\ddot{o}z}\dot{\Delta}^{\ddot{i}}_{\ddot{o}zik}. \end{aligned}\right\} \qquad (12)$$

In case of beam shift restriction in respect of carriage frame in longitudinal and transverse horizontal directions, in the central suspension, after elimination of respective spaces $\Delta_{\ddot{o}\ddot{o}0}, \Delta_{\ddot{o}o'0}$ forces emerge, where rigidity and energy dissipation in the structure itself are already assessed. These forces arise during movement of the top beam in respect carriage in the sliders zone:

$$\left.\begin{aligned} &\Delta^{\dot{e}}_{\ddot{o}xik} = x_{i'i} - (-1)^k b_c\psi_{i'i} - x_i + (-1)^k b_{\tilde{n}}\psi_i, \\ &\Delta^{\dot{e}}_{\ddot{o}yik} = y_{i'i} - y_i, \end{aligned}\right\} \qquad (13)$$

exceeding, under absolute value, the $\Delta_{\ddot{o}\ddot{o}0}, \Delta_{\ddot{o}o'0}$.

These forces may be expressed as follows:

$$S^{\dot{e}}_{\ddot{o}xik} = \begin{cases} 0, \\ k^{\dot{e}}_{\ddot{o}\ddot{o}}\left(\Delta^{\dot{e}}_{\ddot{o}xik} - (-1)^k \Delta_{\ddot{o}x0}\right)\beta^{\dot{e}}_{\ddot{o}\ddot{o}}\dot{\Delta}^{\dot{e}}_{\ddot{o}xik}, \end{cases}$$

$$S^{\dot{e}}_{\ddot{o}yik} = \begin{cases} 0, \\ k^{\dot{e}}_{\ddot{o}y}\left(\Delta^{\dot{e}}_{\ddot{o}yik} - (-1)^k \Delta_{\ddot{o}y0}\right)\beta^{\dot{e}}_{\ddot{o}y}\dot{\Delta}^{\dot{e}}_{\ddot{o}yik}, \end{cases}$$

$$\left.\begin{array}{l} \dot{a}\tilde{n}\ddot{e}`e\left|\Delta^{\hat{e}}_{\tilde{o}xik}\right| \le \Delta_{\tilde{o}x0}; \\ \dot{a}\tilde{n}\ddot{e}`e\left|\Delta^{\hat{e}}_{\tilde{o}xik}\right| > \Delta_{\tilde{o}x0}; \\ \dot{a}\tilde{n}\ddot{e}`e\left|\Delta^{\hat{e}}_{\tilde{o}yik}\right| \le \Delta_{\tilde{o}y0}; \\ \dot{a}\tilde{n}\ddot{e}`e\left|\Delta^{\hat{e}}_{\tilde{o}yik}\right| > \Delta_{\tilde{o}y0}. \end{array}\right\} \quad (14)$$

Let's write down the mutual shifts between carriage frame and axle-wheel pair:

$$\left.\begin{array}{l} \Delta_{a'.ximk} = x_i - (-1)^k b_1\psi_i - x_{im} + (-1)^k b_1\psi_{im}, \\ \Delta_{a'.yimk} = y_i - (-1)^m l_1\psi_i - y_{im}, \\ \Delta_{a'.zimk} = z_i + (-1)^m l_1\varphi_i - x_{im} + (-1)^k b_1\theta_i - z_{im} - (-1)^k b_1\theta_{im}, \\ \Delta_{a'.\psi imk} = \psi_i - \psi_{im}. \end{array}\right\} \quad (15)$$

In axle-box suspensions of some carriages, absorbers of dry friction vibrations are installed, having rubber elements that simulate the viscous friction. Therefore, in general case, the forces acting in axle-box suspension will be:

$$\left.\begin{array}{l} S_{a'ximk} = k_{a'\ddot{o}}\Delta_{a'ximk} + \beta_{a'\ddot{o}}\dot{\Delta}_{a'ximk} + F_{a'\ddot{o}}sign\left(\dot{\Delta}_{a'ximk}\right), \\ S_{a'yimk} = k_{a'y}\Delta_{a'yimk} + \beta_{a'y}\dot{\Delta}_{a'yimk} + F_{a'y}sign\left(\dot{\Delta}_{a'yimk}\right), \\ S_{a'zimk} = k_{a'z}\Delta_{a'zimk} + \beta_{a'z}\dot{\Delta}_{a'ximk} + F_{a'z}sign\left(\dot{\Delta}_{a'zimk}\right), \\ S_{a'\psi imk} = k_{a'\psi}\Delta_{a'\psi imk} + \beta_{a'\psi}\dot{\Delta}_{a'\psi imk} + F_{a'\psi}sign\left(\dot{\Delta}_{a'\psi imk}\right), \end{array}\right\} \quad (16)$$

where $F_{a'}$ is amplitude values of dry friction forces:

$$F_{a'\ddot{o}} = f_{a'\ddot{o}} \cdot P_{\tilde{n}oa'} \qquad F_{a'o'} = f_{a'o'} \cdot P_{\tilde{n}oa'}$$
$$F_{a'z} = f_{a'z} \cdot P_{\tilde{n}oa'} \qquad F_{a'\psi} = f_{a'\psi} \cdot P_{\tilde{n}oa'}$$

Mutual shifts and forces emerging in leads of axle-box stage:

$$\left.\begin{array}{l} \Delta^{\ddot{i}}_{a'ximk} = x_i - (-1)^k b_{\ddot{i}a'}\psi_i - x_{im} + (-1)^k b_{\ddot{i}a'}\psi_{im}, \\ \Delta^{\ddot{i}}_{a'o'imk} = y_i - (-1)^m l_1\psi_i - y_{im}, \\ \Delta_{a'zimk} = z_i + (-1)^m l_1\varphi_i + (-1)^k b_{\ddot{i}a'}\theta_i - z_{im} - (-1)^k b_{\ddot{i}a'}\theta_{im}. \end{array}\right\} \quad (17)$$

$$\left.\begin{array}{l} S^{\ddot{i}}_{a'ximk} = k^{\ddot{i}}_{a'x}\Delta^{\ddot{i}}_{a'ximk} + \beta^{\ddot{i}}_{a'x}\dot{\Delta}^{\ddot{i}}_{a'ximk}, \\ S^{\ddot{i}}_{a'yimk} = k^{\ddot{i}}_{a'y}\Delta^{\ddot{i}}_{a'yimk} + \beta^{\ddot{i}}_{a'y}\dot{\Delta}^{\ddot{i}}_{a'yimk}, \\ S^{\ddot{i}}_{a'zimk} = k^{\ddot{i}}_{a'z}\Delta^{\ddot{i}}_{a'zimk} + \beta^{\ddot{i}}_{a'z}\dot{\Delta}^{\ddot{i}}_{a'ximk}. \end{array}\right\} \quad (18)$$

The leads and fasteners, installed in axle-box suspension, limit the shifts of carriage frame in respect of axle-wheel pair on horizontal plane in longitudinal and transverse directions. Let's mark as $\Delta_{a'\ddot{o}0}$, $\Delta_{a'o'0}$ the respective spaces, in the limit of which the resilient sets of axle-box suspensions operate. After elimination of these spaces, it is necessary to assess the coefficients of rigidity and viscous friction of the structure. Shifts and forces arising after elimination of these spaces will be as follows:

$$\begin{array}{l} \Delta^{\hat{e}}_{a'ximk} = \Delta_{a'ximk}, \\ \Delta^{\hat{e}}_{a'yimk} = \Delta_{a'yimk}, \end{array} \quad (19)$$

$$S^{\hat{e}}_{a'ximk} = \begin{cases} 0, \\ k^{\hat{e}}_{a'\ddot{o}}\left[\Delta^{\hat{e}}_{a'ximk} - (-1)^k \Delta_{a'x0}\right]\beta^{\hat{e}}_{a'\ddot{o}}\dot{\Delta}^{\hat{e}}_{a'\ddot{o}imk}, \end{cases}$$

$$S^{\hat{e}}_{ayimk} = \begin{cases} 0, \\ k^{\hat{e}}_{\ddot{o}y}\left[\Delta^{\hat{e}}_{a'yimk} - (-1)^k \Delta_{a'y0}\right]\beta^{\hat{e}}_{a'y}\dot{\Delta}^{\hat{e}}_{a'yimk}, \end{cases}$$

$$\left.\begin{array}{l} \dot{a}\tilde{n}\ddot{e}`e\left|\Delta^{\hat{e}}_{a'ximk}\right| \le \Delta_{a'x0}; \\ \dot{a}\tilde{n}\ddot{e}`e\left|\Delta^{\hat{e}}_{a'ximk}\right| > \Delta_{a'x0}; \\ \dot{a}\tilde{n}\ddot{e}`e\left|\Delta^{\hat{e}}_{a'yimk}\right| \le \Delta_{a'o'0}; \\ \dot{a}\tilde{n}\ddot{e}`e\left|\Delta^{\hat{e}}_{a'yimk}\right| > \Delta_{a'o'0}. \end{array}\right\} \quad (20)$$

Forces, acting the axle-wheel pair in the contact zone, are usually determined using the Carter theory [3].

For making differential equations of fluctuation system, we will use the sort 2 Lagrange equation:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_n}\right) = Q_n \quad (21)$$

where O' is kinetic energy of the system; q_n is summarised coordinates; Q_n is respective summarised forces.

General expression of kinetic energy:

$$T = \frac{1}{2}\Big[a_z\dot{z}^2 + a_\varphi\dot{\varphi}^2 + a_\theta\dot{\theta}^2 + a_y\dot{y}^2 + a_\psi\dot{\psi}^2 + a_x\dot{x}^2 + \sum_{i=1}^{2}\left(a_{zi}\dot{z}_i^2 + a_{\varphi i}\dot{\varphi}_i^2 + a_{\theta i}\dot{\theta}_i^2 + a_{yi}\dot{y}_i^2 + a_{\psi i}\dot{\psi}_i^2 + a_{xi}\dot{x}_i^2 + a_{y'i}\dot{y}_{i'i}^2 + a_{\psi'i}\dot{\psi}_{i'i}^2 + a_{xi'i}\dot{x}_{i'i}^2\right) + \sum_{i-1}^{2}\sum_{m-1}^{2}\left(a_{zim}\dot{z}_{im}^2 + a_{\theta im}\dot{\theta}_{im}^2 + a_{yim}\dot{y}_{im}^2 + a_{\psi im}\dot{\psi}_{im}^2 + a_{xim}\dot{x}_{im}^2\right) + \sum_{i=1}^{2}\sum_{m=1}^{2}\sum_{k=1}^{2}a_{ypimk}\,\dot{y}_{pimk}^2\Big] \quad (22)$$

Let's make differential equations of system fluctuations. We will insert the expression of kinetic energy (22) into sort 2 Lagrange equation (21) and obtain the following system of differential equation:

$$a_n\ddot{q}_n = Q_n \qquad \left(n = \overline{1,52}\right) \quad (23)$$

For integration of this system of differential equations, a program is made.

Analogous mathematical models are made also for chassis of other rolling stock [3, 4]. With the help of mathematical model presented here, broad theoretic research was done in the field of passenger wagon spatial fluctuations. The research was made in speed range from 20 to 200 km/h for various road sections. These calculations greatly contributed to creation of new-class passenger wagons in Kriukovo factory [5].

4. Conclusion

We have presented mathematical model for description of spatial vibrations in four-axle passenger wagon with various types of carriages having two-stage resilient suspension. This model can be successfully employed to examine dynamic loads of passenger wagons and, at the same time, it allows determining rational parameters of both resilient stages of carriages.

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