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A Poynting Vector Approach to the Study of Power Flow through a Transformer

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Summary: This work deals with the flow of the electromagnetic energy through an idealized single-phase transformer supplied with nonsinusoidal voltage and supplying a nonlinear load. The electromagnetic flux components of the energy that flows from primary to secondary as well as the flux of energy stored in and ejected from the dielectric space surrounding the windings are identified and quantified. The electromagnetic flux components are correlated to well known instantaneous powers. These powers are not clever mathematic expedients but correct expressions that mirror the actual physical phenomena and lead to a more realistic interpretation of apparent power and its resolution.

Key words:
power definitions, power quality, poynting vector, harmonics

1. INTRODUCTION

The ultimate goal of this study is to give a detailed picture of the flow of energy through a transformer and to pinpoint the distribution in space and time of different rates of flow, i.e. powers, defined by modern theories.

The transformer is an essential electrical device in the chain of components that facilitates the power flow of electric energy from the alternator to the end users.

The process of electromagnetic energy transfer from the primary to the secondary winding and the visualization of distribution in time and space of the flux of power can be implemented by means of Poynting Vector (PV) [1]. Such approach [2, 3, 4] enables in-depth understanding of the physical characteristics of different powers and ultimately leads to a realistic resolution of apparent power.

The PV approach gives clear information about the exact pattern of power flow and may trouble proponents of apparent power resolutions that are based on pure mathematical interpretations [5] that tend to oversight the actual electromagnetic phenomena.

2. BACKGROUND

The studied transformer has the circuit diagram depicted in Figure 1a. The primary winding is supplied by a nonsinusoidal voltage:

$$v_p = v_{p1} + v_{pH} \tag{1}$$

where:

$$v_{p1} = \hat{V}_{p1} \sin\left(\omega t + \theta_{p1}\right) \tag{2}$$

$$v_{pH} = \sum_{v \neq 1} \hat{V}_{pv} \sin\left(v\omega t + \alpha_{pv}\right) \tag{3}$$

The secondary voltage is:

$$v_s = v_{s1} + v_{sH} \tag{4}$$

where:

$$v_{s1} = \hat{V}_{s1} \sin\left(\omega t + \theta_{s1}\right) \tag{5}$$

$$v_{sH} = \sum_{v \neq 1} \hat{V}_{sv} \sin\left(v\omega t + \alpha_{sv}\right) \tag{6}$$

The secondary current being:

$$i_s = i_{s1} + i_{sH} \tag{7}$$

with:

$$i_{s1} = \hat{I}_{s1} \sin\left(\omega t\right) \tag{8}$$

$$i_{sH} = \sum_{v \neq 1} \hat{I}_{sv} \sin\left(v\omega t + \alpha_{sv} - \theta_{sv}\right) \tag{9}$$

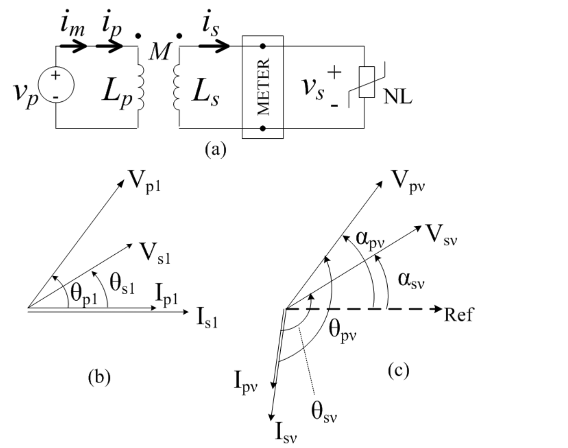

Fig. 1. Single-Phase transformer supplying a nonlinear load: (a) circuit; (b) phasor diagram: fundamental; (c) phasor diagram: v – order harmonic

The primary current has two components: the first, labeled i_m, is the magnetizing current that supports the magnetic flux, and it is detailed in Section 5. The second, labeled i_p, is due to the secondary current (Fig. 1b) and has two major components:

$$i_p = i_{p1} + i_{pH} \tag{10}$$

with:

$$i_{p1} = \hat{I}_{p1} \sin(\omega t) \tag{11}$$

$$i_{pH} = \sum_{v \neq 1} \hat{I}_{pv} \sin(v\omega t + \alpha_{pv} - \theta_{pv}) \tag{12}$$

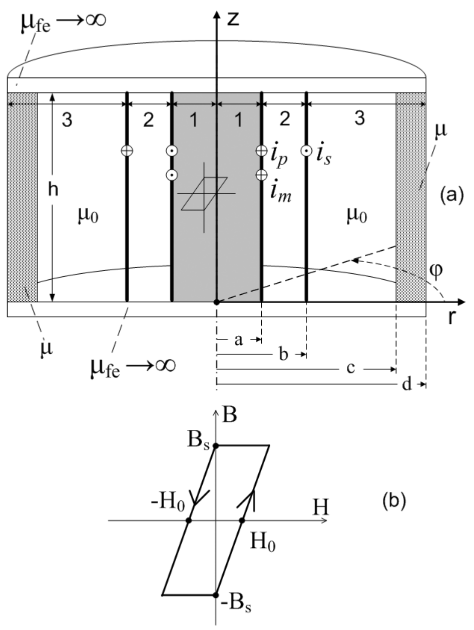

Fig. 2. The studied transformer: (a) geometry; (b) B/H characteristic at central core

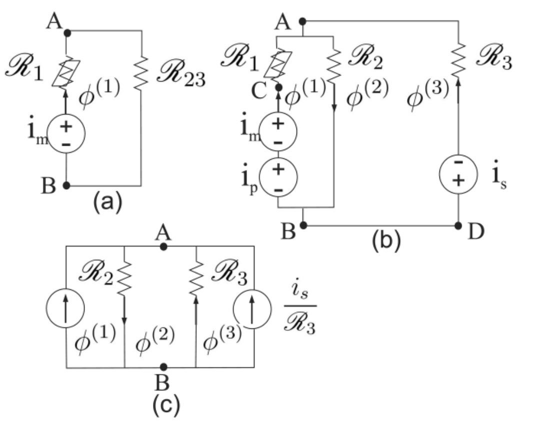

Fig. 3. Electric equivalents of magnetic circuits: (a) no-load condition; (b) loaded transformer; (c) loaded transformer (equivalent to circuit (b) where voltage sources are replaced by equivalent current sources)

Each harmonic phasor of i_p is in-phase, or 180° out of phase, with the corresponding secondary current harmonic phasor, Figure 1c. In this study it was assumed that the fundamental power flows from v_p to the nonlinear load. This translates in the following active power balance for the fundamental power:

$$V_{p1} I_{p1} \cos(\theta_{p1}) = V_{s1} I_{s1} \cos(\theta_{s1}) \tag{13}$$

It was also assumed that the nonlinear load converts a small part of the fundamental active power in harmonic active power: thus the harmonic power flows from the load to the voltage source v_p and:

$$V_{sv} I_{sv} \cos(\theta_{sv}) = V_{pv} I_{pv} \cos(\theta_{pv}) \tag{14}$$

3. THE STUDIED SYSTEM

The geometry of the studied system is presented in Figure 2a. A central cylindrical core with radius a and height h is made of ferromagnetic material with a nonlinear B/H characteristic, Figure 2b. The primary winding is modeled with the help of an equivalent, infinite thin, one turn foil with infinite conductivity, wrapped around the central core. The secondary winding is also modeled using a very thin foil with infinite conductivity, whose radius is b and height is h. The magnetic circuit is completed by a cylinder with inner radius c, outer radius d and height h, and by two end-disks with radius d. The external cylinder has a linear magnetizing characteristic, $B = \mu H$, while the disks are made of an ideal material with infinite magnetic permeability.

The flux in the central core (region 1, Fig. 2a) produced by the magnetizing current is:

$$\phi^{(1)} = \int v_p dt = -\sum_{v=1} \frac{\hat{V}_{pv}}{v\omega} \cos(v\omega t + \alpha_{pv}) \tag{15}$$

This flux is function only of the primary voltage v_p and returns through two regions:

— Region 2: $a < r < b$ $A_2 = \pi(b^2 - a^2)$
— Region 3: $b < r < d$ $A_3 = \pi(d^2 - b^2)$

An equivalent reluctance can be defined for each region:

$$\mathscr{R}_2 = \frac{1}{\mu_0} \frac{h}{A_2} \tag{16}$$

$$\mathscr{R}_3 = \frac{1}{\mu_3} \frac{h}{A_3} = \frac{\mathscr{R}_3' \mathscr{R}_3''}{\mathscr{R}_3' + \mathscr{R}_3''} \tag{17}$$

where:

$$\mathscr{R}_3' = \frac{h}{\mu_0 A_3'} \qquad \mathscr{R}_3'' = \frac{h}{\mu A_3''} \qquad \mu_3 = \frac{\mu A_3'' + \mu_0 A_3'}{A_3}$$

$$A_3' = \pi(c^2 - b^2), \qquad A_3'' = \pi(d^2 - c^2)$$

The equivalent electric circuits for no load and load conditions are shown in Figure 3. At no load, Figure 3a, the return reluctance is:

$$\mathscr{R}_{23} = \frac{\mathscr{R}_2 \mathscr{R}_3}{\mathscr{R}_2 + \mathscr{R}_3} \tag{18}$$

and the magnetomotive force developed between the nodes C and A is:

$$M_{CA} = i_m - \mathscr{R}_{23}\, \phi^{(1)} \tag{19}$$

For the loaded transformer, the amper-turns i_p and i_s are introduced as shown in Figure 3b. Since the flux $\phi^{(1)}$ has the same value and spectrum at no-load and loaded condition, it is possible to replace the branch ACB with an equivalent current source $\phi^{(1)}$; also it is possible to to replace the branch ADB with a current source is/R_3 in parallel to R_3 (Fig. 3c). From here, results:

$$M_{AB} = \mathscr{R}_{23}\left(\frac{i_s}{R_3} + \phi^{(1)}\right) \tag{20}$$

and, from Figure 3b:

$$M_{AB} = i_p + i_m - M_{CA} \tag{21}$$

Substitution of (19) and (20) in (21) gives:

$$i_p = \frac{\mathscr{R}_{23}}{\mathscr{R}_3} i_s = \frac{\mathscr{R}_2}{\mathscr{R}_2 + \mathscr{R}_3} i_s \tag{22}$$

The last expression confirms the previous claim that harmonics phasors $\bar{I}_{pv}$ and $\bar{I}_{sv}$ are in phase or 180° out of phase. From Figures 3b and 3c it results:

$$\phi^{(2)} = \frac{\mathscr{R}_3}{\mathscr{R}_2 + \mathscr{R}_3}\left(\phi^{(1)} + \frac{i_s}{\mathscr{R}_3}\right) \tag{23}$$

$$\phi^{(3)} = \frac{\mathscr{R}_2}{\mathscr{R}_2 + \mathscr{R}_3}\left(\phi^{(1)} + \frac{i_s}{\mathscr{R}_2}\right) \tag{24}$$

The secondary voltage is found applying Faraday's law:

$$v_s = \frac{d}{dt}\left(\phi^{(1)} - \phi^{(2)}\right) = \frac{d}{dt}\left[\phi^{(1)} - \frac{\mathscr{R}_3}{\mathscr{R}_2 + \mathscr{R}_3}\left(\phi^{(1)} + \frac{i_s}{\mathscr{R}_3}\right)\right]$$

keeping in mind that $v_p = d\phi^{(1)}/dt$, results:

$$v_s = \frac{\mathscr{R}_2}{\mathscr{R}_2 + \mathscr{R}_3} v_p - \frac{1}{\mathscr{R}_2 + \mathscr{R}_3}\frac{di_s}{dt} \tag{25}$$

4. THE FLOW OF POWERS

In this section the detailed distributions of the electric and magnetic fields inside the transformer are determined. This information in turn permits the calculation of the power density (W/m^2) and the direction of its flow.

The flow in time and the spatial distribution of the powers are studied with the help of the PV, defined as: $\vec{\wp} = \vec{E} \times \vec{H}$ [VA/m^2]. The electric field is obtained from:

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t}\vec{B} \qquad\qquad \vec{B} = B\vec{u}_z$$

where $\vec{u}_z$ is unity versor in z-direction (see Fig. 2a). In a cylindrical coordinate system:

$$\frac{1}{r}\begin{vmatrix} \vec{u}_r & r\vec{u}_\varphi & \vec{u}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\ 0 & rE & 0 \end{vmatrix} = -\frac{\partial}{\partial t} B\vec{u}_z$$

and:

$$\frac{1}{r}\frac{\partial}{\partial t}(rE) = -\frac{\partial B}{\partial t} = -\frac{1}{\pi r^2}\frac{\partial \phi}{\partial t}$$

or:

$$\vec{E}(r) = -\frac{1}{2\pi r}\frac{d\phi}{dt}\vec{u}_\varphi \tag{26}$$

At $r = a$, $d\phi/dt = v_p$ and:

$$\vec{E}_a = \vec{E}_a^{(1)} = \vec{E}_a^{(2)} = \frac{v_p}{2\pi a}\left(-\vec{u}_\varphi\right) \tag{27}$$

At $r = b$:

$$\vec{E}_b = \vec{E}_b^{(2)} = \vec{E}_b^{(3)} = \frac{v_s}{2\pi b}\left(-\vec{u}_\varphi\right) \tag{28}$$

These results show that the electric field vector has concentric streamlines and rotates clockwise, i.e. opposite to the electric field impressed by v_p within the primary winding (note that the magnetizing current i_m flows counterclockwise, Figure 2a).
The magnetic fields are readily found. In region 2:

$$\vec{H}^{(2)} = -\frac{\phi^{(2)}}{\mu_0 A_2}\vec{u}_z = -\frac{\mathscr{R}_2}{h}\phi^{(2)}\vec{u}_z = \frac{\mathscr{R}_{23}}{h}\left(\phi^{(1)} + \frac{i_s}{\mathscr{R}_3}\right)\left(-\vec{u}_z\right) \tag{29}$$

In region 3:

$$\vec{H}^{(3)} = -\frac{\mathscr{R}_3}{h}\phi^{(3)}\vec{u}_z = \frac{\mathscr{R}_{23}}{h}\left(\phi^{(1)} + \frac{i_s}{\mathscr{R}_2}\right)\left(-\vec{u}_z\right) \tag{30}$$

The flow direction of the resulting PV is shown in Figure 4a. For the fundamental component the primary winding emits the electromagnetic wave that impinges energy to the system. Part of the energy, carried by $\vec{\wp}^{(1)}$, flows radially toward the center axis. This PV decreases linearly and carries the hysteresis losses as well as part of the energy that oscillates back and

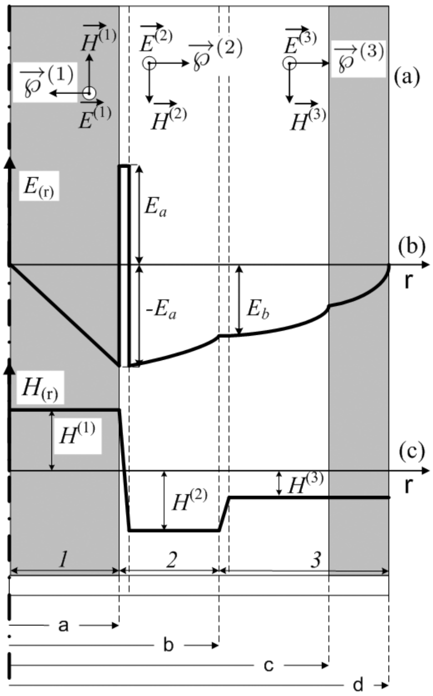

Fig. 4. Poynting vector (fundamental): (a) PV direction; (b) electric field distribution; (c) magnetic field

forth between the transformer and the voltage source v_p. The remaining *PV* flows in region 2, toward the secondary winding, and carries the active and the non-active power supplied to the non linear load as well as the non-active power due to the nonoscillating energy that is stored in and ejected from regions 2 and 3. In region 3 The *PV* transmits only non active powers In Figure 4b is shown the distribution of electric field versus radius. In Figure 4c is presented the magnetic field distribution [1)].

The expressions of *PV* are as follows:

The PV at $r = a$ in the region 2 is:

$$\vec{\wp}_a^{(2)} = \vec{E}_a \times \vec{H}^{(2)} = E_a H^{(2)} \left(-\vec{u}_\varphi\right) \times \left(-\vec{u}_z\right) \tag{31}$$

Substitution of (27) and (29), reminding (22):

$$\vec{\wp}_a^{(2)} = \frac{1}{2\pi a h}\left(\mathscr{R}_{23}\phi^{(1)} v_p + v_p i_p\right)\vec{u}_r \tag{32}$$

This expression represents the density of electromagnetic power exiting from the primary winding and flowing into region 2. The total flux that exits the winding through the surface $2\pi ah$ is:

$$p_a^{(2)} = \mathscr{R}_{23}\phi^{(1)} v_p + v_p i_p \tag{33}$$

The first term:

$$\mathscr{R}_{23}\phi^{(1)} v_p = \mathscr{R}_{23}\phi^{(1)} \frac{d\phi^{(1)}}{dt} \tag{34}$$

is purely non active since $\frac{1}{T}\int_0^T \phi^{(1)} \frac{d\phi^{(1)}}{dt} = 0$.

From (1) and (10) results that the second term of (33) has the components:

$$v_p i_p = v_{p1} i_{p1} + v_{p1} i_{pH} + v_{pH} i_{p1} + v_{pH} i_{pH} \tag{35}$$

Each of these components has a corresponding PV. The first component is:

$$\begin{aligned} v_{p1} i_{p1} &= V_{p1} I_{p1} cos(\theta_{p1})[1 - cos(2\omega t)] \\ &+ V_{p1} I_{p1} sin(\theta_{p1})[1 - sin(2\omega t)] \end{aligned} \tag{36}$$

In (36) one recognize the instantaneous fundamental power delivered to the system:

$$v_{p1} i_{p1} = P_{p1}[1 - cos(2\omega t)] + Q_{p1} sin(2\omega t) \tag{37}$$

where:

$$P_{p1} = V_{p1} I_{p1} cos(\theta_{p1}) \tag{38}$$

$$Q_{p1} = V_{p1} I_{p1} sin(\theta_{p1}) \tag{39}$$

are the fundamental active power and the fundamental reactive power respectively.

After the substitution of (2) and (12) in the second term of (35) the expression is:

$$\begin{aligned} v_{p1} i_{pH} = \sum_{v \neq 1} D_{Ipv} \Big\{ & cos\left[(v-1)\omega t + \alpha_{pv} - \theta_{pv} - \theta_{p1}\right] \\ & - cos\left[(v+1)\omega t + \alpha_{pv} - \theta_{pv} - \theta_{p1}\right]\Big\} \end{aligned} \tag{40}$$

that is the current *distortion instantaneous power*, a nonactive power that has oscillating components with the amplitudes:

$$D_{Ipv} = V_{p1} I_{pv} \tag{41}$$

The third term of (35) is due to voltage distortion. Substitution of (3) and (11) gives:

$$\begin{aligned} v_{pH} i_{p1} = \sum_{v \neq 1} D_{Vpv} \Big\{ & cos\left[(v-1)\omega t + \alpha_{pv}\right] \\ & - cos\left[(v+1)\omega t + \alpha_{pv}\right]\Big\} \end{aligned} \tag{42}$$

that is the *instantaneous current distortion* power, a nonactive power that has oscillating components with the amplitude:

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1) The conductor are sketched with a finite thickness and the magnetic field transitions from region 1 to 2 and from 2 to 3 are shown. If the electric fields within the conductors (27) and (28) as well as the respective magnetic fields are used to compute the *PV* s inside the conductors, it will be found that *PV* s carry the power dissipated through joule and eddy current losses and also support the reactive power connected with the magnetic field inside the conductors. Due to limited space this phenomenon is not covered in this paper.

$$D_{Vpv} = V_{pv} I_{p1} \tag{43}$$

The last term of (35) is the *instantaneous voltage distortion* power. Substitution of (3) and (12) gives:

$$\begin{aligned} v_{pH} i_{pH} &= \sum_{v \neq 1} P_{pv} \left[1 - cos\left(2v\omega t + 2\alpha_{pv}\right) \right] + \sum_{v \neq 1} Q_{pv}\, sin\left(2v\omega t + 2\alpha_{pv}\right) \\ &+ \sum_{\substack{v \neq 1 \\ m \neq v}} D_{Hpvm} \left\{ cos\left[(v-m)\omega t + \alpha_{pv} - \alpha_{pm} + \theta_{pm} \right] \right. \\ &\left. - cos\left[(v+m)\omega t + \alpha_{pv} + \alpha_{pm} - \theta_{pm} \right] \right\} \end{aligned} \tag{44}$$

where:

$$P_{pv} = V_{pv} I_{pv} cos(\theta_{pv}) \tag{45}$$

is an *active harmonic power* of order v:

$$Q_{pv} = V_{pv} I_{pv} sin(\theta_{pv}) \tag{46}$$

is a *reactive power* of order v, and:

$$D_{Hpvm} = V_{pv} I_{pm} \qquad m \neq v \tag{47}$$

is the amplitude of a *harmonic instantaneous distortion power* with the power oscillations at the frequencies $v \pm m$, $v \neq 1, m \neq v$.

The electromagnetic flux of power that arrives at $r = b$ delivers the power density:

$$\vec{\wp}_b^{(2)} = \vec{E}_b \times \vec{H}^{(2)} = E_b H^{(2)} \left(-\vec{u}_\varphi\right) \times \left(-\vec{u}_z\right) \tag{48}$$

Substitution of (28) and (29) in (48) gives:

$$\vec{\wp}_b^{(2)} = \frac{\mathscr{R}_{23}}{2\pi bh} v_s \left(\phi^{(1)} + \frac{i_s}{\mathscr{R}_3} \right) \vec{u}_r \tag{49}$$

Substitution of (22) and (25) in (49) leads to a three terms expression for the flux of $\vec{\wp}_b^{(2)}$ through the surface $2\pi bh$, that is the instantaneous power:

$$p_b^{(2)} = \frac{\mathscr{R}_{23}\mathscr{R}_2}{\mathscr{R}_2 + \mathscr{R}_3} \phi^{(1)} v_p + v_s i_s \frac{\mathscr{R}_3}{\mathscr{R}_2 + \mathscr{R}_3} \left(v_s i_s + \phi^{(1)} \frac{di_p}{dt} \right) \tag{50}$$

The first term of (50) is a continuation of (34), indicating that the difference:

$$\mathscr{R}_{23} \phi^{(1)} v_p - \frac{\mathscr{R}_{23}\mathscr{R}_2}{\mathscr{R}_2 + \mathscr{R}_3} \phi^{(1)} v_p = \frac{\mathscr{R}_{23}\mathscr{R}_3}{\mathscr{R}_2 + \mathscr{R}_3} \phi^{(1)} v_p$$

is a non active power stored in and ejected out of region 2. The second term of (50) is the instantaneous power delivered to the nonlinear load. This term has similar component to the one described in (35), i.e.:

$$v_s i_s = v_{s1} i_{s1} + v_{s1} i_{sH} + v_{sH} i_{s1} + v_{sH} i_{sH} \tag{51}$$

where:

$$v_{s1} i_{s1} = P_{s1} \left[1 - cos(2\omega t)\right] + Q_{s1}\, sin(2\omega t) \tag{52}$$

with:

$$P_{s1} = V_{s1} I_{s1} cos(\theta_{s1}) \qquad Q_{s1} = V_{s1} I_{s1} sin(\theta_{s1}) \tag{53}$$

and:

$$\begin{aligned} v_{s1} i_{sH} &= \sum_{v \neq 1} D_{Isv} \left\{ cos\left[(v-1)\omega t + \alpha_{sv} - \theta_{sv} - \theta_{s1} \right] \right. \\ &\left. - cos\left[(v+1)\omega t + \alpha_{sv} - \theta_{sv} + \theta_{s1} \right] \right\} \end{aligned} \tag{54}$$

with:

$$D_{Isv} = V_{s1} I_{sv} \tag{55}$$

and:

$$\begin{aligned} v_{sH} i_{s1} &= \sum_{v \neq 1} D_{Vsv} \left\{ cos\left[(v-1)\omega t + \alpha_{sv} \right] \right. \\ &\left. - cos\left[(v+1)\omega t + \alpha_{sv} \right] \right\} \end{aligned} \tag{56}$$

with:

$$D_{Vsv} = V_{sv} I_{s1}$$

and:

$$\begin{aligned} v_{sH} i_{sH} &= \sum_{v \neq 1} P_{sv} \left[1 - cos\left(2v\omega t + 2\alpha_{sv}\right) \right] \\ &+ \sum_{v \neq 1} Q_{sv}\, sin\left(2v\omega t + 2\alpha_{sv}\right) \\ &+ \sum_{\substack{v \neq 1 \\ m \neq v}} D_{Hsvm} \left\{ cos\left(v-m\right)\omega t + \alpha_{sv} - \alpha_{sm} + \theta_{sm} \right. \\ &\left. - cos\left(v+m\right)\omega t + \alpha_{sv} + \alpha_{sm} - \theta_{sm} \right\} \end{aligned} \tag{57}$$

where:

$$P_{sv} = V_{sv} I_{sv} cos(\theta_{sv}) \qquad Q_{sv} = V_{sv} I_{sv} sin(\theta_{sv})$$

$$D_{Hsvm} = V_{sv} I_{sm} \qquad m \neq v$$

The last term of (50) is nonactive (see Appendix A.1) and arrives unchanged in region 3.

In region 3 at $r = b$ we find:

$$\vec{\wp}_b^{(3)} = \vec{E}_b \times \vec{H}^{(3)} = E_b H^{(3)} \left(-\vec{u}_\varphi\right) \times \left(-\vec{u}_z\right) \tag{58}$$

Substitution of (28) and (29) in (58) gives:

$$\vec{\wp}_b^{(3)} = \frac{\mathscr{R}_{23}}{2\pi bh} v_s \left(\phi^{(1)} + \frac{i_s}{\mathscr{R}_2} \right) \vec{u}_r \tag{59}$$

Substitution of (22) and (25) in (59) gives the flux of $\vec{\wp}_b^{(3)}$ through the surface $2\pi bh$ that enters region 3:

$$p_b^{(3)} = \frac{\mathscr{R}_{23}\mathscr{R}_2}{\mathscr{R}_2+\mathscr{R}_3}\phi^{(1)}v_p - \frac{\mathscr{R}_3}{\mathscr{R}_2+\mathscr{R}_3}\left(v_s i_s + \phi^{(1)}\frac{di_p}{dt}\right) \quad (60)$$

We recognize two terms: they are identical to the first and third term of (50), respectively. One will observe that the instantaneous power $p_b^{(3)}$ is non active (Appendix A.1). Moreover, the total power stored/ejected in/from a segment of magnetic circuit with a reluctance $\mathscr{R}$ and a flux ϕ is:

$$p_i = \mathscr{R}_i\phi^{(1)}\frac{d\phi^{(1)}}{dt} \quad (61)$$

In Appendix A.2 it is proved that:

$$p_b^{(3)} = \mathscr{R}_3\phi^{(3)}\frac{d\phi^{(3)}}{dt} \quad (62)$$

This means that the instantaneous nonactive power at $r = b$ is the rate of flow of energy that enters/exits region 3. In the same Appendix the following relation is proved:

$$\mathscr{R}_2\phi^{(2)}\frac{d\phi^{(2)}}{dt} + \mathscr{R}_3\phi^{(3)}\frac{d\phi^{(3)}}{dt} = \mathscr{R}_{23}\phi^{(1)}v_p + v_p i_p + v_s i_s \quad (63)$$

This key expression shows clearly that the *PV* carries in regions 2 and 3 an input power $v_p i_p$ that contains active and nonactive powers. It also delivers a purely nonactive power $\mathscr{R}_{23}\phi^{(1)}v_p$ on account of the magnetizing current i_m. At $r = b$, the *PV* delivers the instantaneous power $v_s i_s$; at $r = d$, $p_d^{(3)} = 0$: this means that, as r increases in the range $b < r < d$, the *PV* gradually decreases.

5. THE NONLINEAR REGION

Due to its nonlinearity, region 1 is the most interesting part of this study. For this reason, it was left to the end. For the sake of clarity, the input voltage is assumed to have the expression:

$$v_p = \hat{V}_1 \sin(\omega t) + \hat{V}_3 \sin(3\omega t) + \hat{V}_5 \sin(5\omega t) \quad (64)$$

This voltage causes a magnetic flux:

$$\phi^{(1)} = -\frac{\hat{V}_1}{\omega}\cos(\omega t) - \frac{\hat{V}_3}{3\omega}\cos(3\omega t) - \frac{\hat{V}_5}{5\omega}\cos(5\omega t) \quad (65)$$

with a readily determined peak value:

$$\Phi_s = \frac{1}{\omega}\left(\hat{V}_1 + \frac{\hat{V}_3}{3} + \frac{\hat{V}_5}{5}\right) \quad (66)$$

According to Figure 2b the magnetic field $H^{(1)}$ has the following expressions: for the ascending branch (when $d\phi/dt > 0$, i.e. $0 < \omega t < \pi$):

$$H^{(1)}\uparrow = H_0 + \frac{H_0}{B_S}B = H_0\left(1 + \frac{\phi^{(1)}}{\Phi_s}\right)$$

For the descending flux ($\pi < \omega t < 2\pi$):

$$H^{(1)}\downarrow = H_0 + \frac{H_0}{B_S}B = H_0\left(-1 + \frac{\phi^{(1)}}{\Phi_s}\right)$$

Thus:

$$H^{(1)} = \frac{4H_0}{\pi}\sum_{v=1,3,5...}^{\infty}\frac{\sin(v\omega t)}{v} + \frac{H_0}{\Phi_s}\phi^{(1)} \quad (67)$$

The magnetizing current (see Fig. 3a) has two terms: the first is due to the MMF in region 1, the second is due to the MMF in regions 2 and 3:

$$i_m = H^{(1)}\quad h + \mathscr{R}_{23}\,\phi^{(1)} \quad (68)$$

Substitution of (65) and (67) in (68) gives:

$$i_m = \frac{4H_0 h}{\pi}\sum_{v=1,3,5...}^{\infty}\frac{\sin(v\omega t)}{v} - \left(\frac{H_0}{\Phi_s} + \mathscr{R}_{23}\right)\sum_{v=1,3}^{5}\frac{\hat{V}_v}{v}\cos(v\omega t) \quad (69)$$

Using the notation:

$$\mathscr{R}_1 = \frac{H_0 h}{\Phi_s} = \frac{H_0 h}{A_1 B_s} = \frac{h}{\mu_1 A_1} \qquad \mu_1 = \frac{B_s}{H_0}$$

and:

$$\mathscr{R}_e = \mathscr{R}_1 + \mathscr{R}_{23}$$

it is possible to rewrite eq. (69) as:

$$i_m = \frac{4H_0 h}{\pi}\sum_{v=1,3,5...}^{\infty}\frac{\sin(v\omega t)}{v} - \frac{R_e}{\omega}\sum_{v=1,3}^{5}\frac{\hat{V}_v}{v}\cos(v\omega t)$$

thus leading to a more convenient expression for $H(1)$, derived from (68):

$$H^{(1)} = \left(i_m - \mathscr{R}_{23}\phi^{(1)}\right)\frac{1}{h}$$
$$= \frac{4H_0}{\pi}\sum_{v=1,3,5...}^{\infty}\frac{\sin(v\omega t)}{v} - \frac{H_0}{\Phi_s\omega}\sum_{v=1,3}^{5}\frac{\hat{V}_v}{v}\cos(v\omega t) \quad (70)$$

The PV entering the central core has the expression:

$$\vec{\wp}_a^{(1)} = \vec{E}_a \times \vec{H}^{(1)} = E_a H^{(1)}(-\vec{u}_\varphi)\times(\vec{u}_z)$$
$$= \frac{v_p}{2\pi a}\left[\frac{4H_0}{\pi}\sum_{v=1,3,5...}^{\infty}\frac{\sin(v\omega t)}{v} - \frac{H_0}{\Phi_s\omega}\sum_{v=1,3}^{5}\frac{\hat{V}_v}{v}\cos(v\omega t)\right](-\vec{u}_r) \quad (71)$$

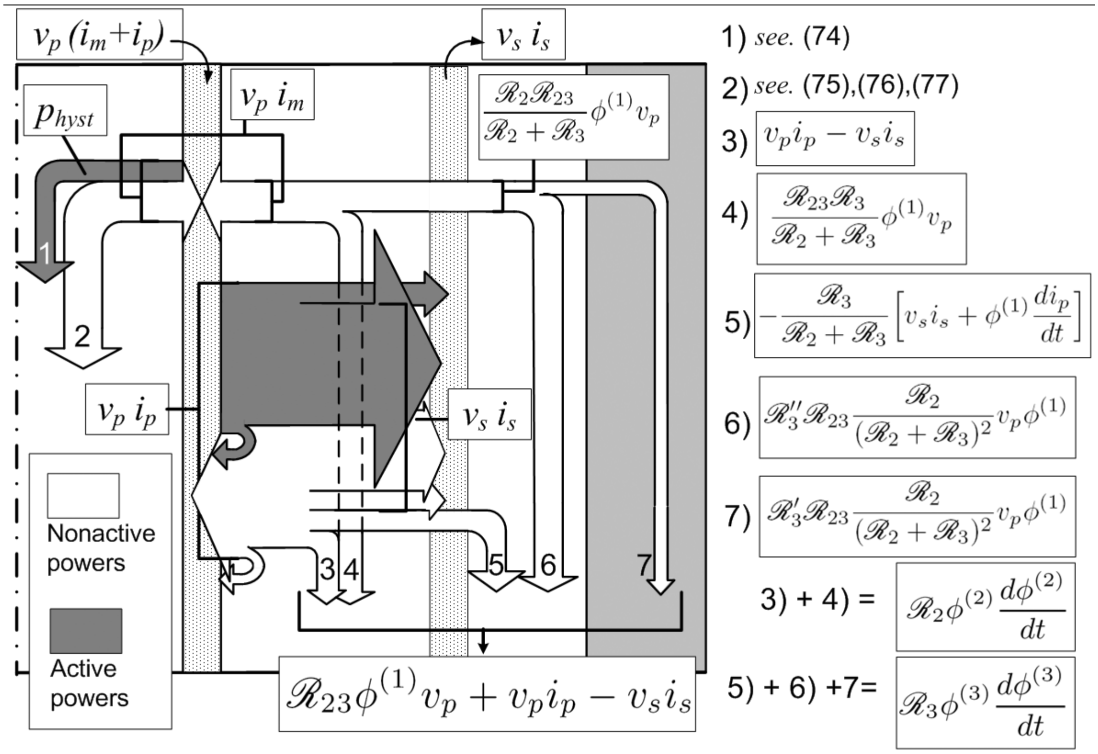

Fig. 5. The flow of PV components

it is radially oriented toward the axis of symmetry. At an arbitrary distance $r < a$, the PV is:

$$\vec{\wp}_r^{(1)} = \frac{1}{2\pi r}\frac{r^2}{a^2}\frac{d\phi}{dt}H^{(1)}\left(-\vec{u}_r\right) = \frac{r}{2\pi a^2}v_p H^{(1)}\left(-\vec{u}_r\right) \quad (72)$$

The power carried in region 1 by $\vec{\wp}_r^{(1)}$ is:

$$p_a^{(1)} = \left[\frac{4H_0}{\pi}\sum_{\nu=1,3,5...}^{\infty}\frac{sin(\nu\omega t)}{\nu} - \frac{H_0}{\Phi_s\omega}\sum_{\nu=1,3}^{5}\frac{\hat{V}_\nu}{\nu}cos(\nu\omega t)\right]\sum_{\nu=1,3}^{5}\hat{V}_\nu\, sin(\nu\omega t) \quad (73)$$

One will notice that this instantaneous power has all the components that were identified in the previous section. It has active instantaneous power produced by the in-phase harmonics:

$$p_{aA}^{(1)} = \frac{4H_0 h}{\pi}\sum_{\nu=1,3}^{5}\frac{\hat{V}_\nu}{\nu}sin^2(\nu\omega t) = \frac{4H_0 h}{\pi}\sum_{\nu=1,3}^{5}\frac{\hat{V}_\nu}{\nu}\left[1-cos(2\nu\omega t)\right] \quad (74)$$

It has reactive instantaneous powers produced by the harmonics in quadrature:

$$p_{aQ}^{(1)} = -\frac{R_1}{\omega}\sum_{\nu=1,3}^{5}\frac{\hat{V}_\nu}{\nu}sin(2\nu\omega t) \quad (75)$$

and the two instantaneous distortion powers, due to the harmonics of current and voltage respectively:

$$p_{aDI}^{(1)} = \hat{V}_1\, sin(\omega t)\cdot \left[\frac{4}{\pi}\sum_{\nu=1,3,5...}^{\infty}\frac{sin(\nu\omega t)}{\nu} - \frac{1}{\Phi_s\omega}\sum_{\nu=1,3}^{5}\frac{\hat{V}_\nu}{\nu}cos(\omega t)\right]H_0 h \quad (76)$$

$$p_{aDV}^{(1)} = \sum_{\nu=1,3}^{5}\hat{V}_\nu\, sin(\nu\omega t)\cdot \left[\frac{4}{\pi}sin(\omega t) - \frac{1}{\Phi_s\omega}\sum_{\nu=1,3,5...}^{\infty} - \frac{1}{\Phi_s\omega}\hat{V}_1\, cos(\omega t)\right]H_0 h \quad (77)$$

The real power dissipated in the central core is:

$$P = \frac{1}{T}\int_0^T p_{aA}^{(1)}dt = \frac{4H_0 h}{\pi}\frac{\omega}{2}\frac{1}{\omega}\sum_{\nu=1.3}^{5}\vec{V}_\nu = \frac{2H_0 h\omega}{\pi}\Phi_s \quad (78)$$

this power supports the hysteresis losses (Fig. 2b):

$$P_{Hyst} = \left(Area_{B(H)}\right)\left(CoreVolume\right)\cdot\frac{\omega}{2\pi} = = \left(4B_s H_0\right)\left(A_1 h\right)\frac{\omega}{2\pi} = \frac{2H_0 h\omega}{\pi}A_1 B_s = \frac{2H_0 h\omega}{\pi}\Phi_s \quad (79)$$

Figure 5 shows the flow of PV components related to the instantaneous powers $v_p i_p$, $v_p i_m$, $v_s i_s$ [1]).

1) In Figure 5 the conductors are represented with a finite thickness, as a consequence the active power dissipated throughjoule and eddy current losses and reactive power exchanged with the field inside the conductor are shown.

5.1. The power flow

Starting from the observation that the instantaneous powers flow follows exactly the *PV* flux lines, the different components of the instantaneous power are defined. The magnetizing current i_m produces the flux $\phi^{(1)}$ that does not depend on the transformer load. The current i_m sustains both the field in the central core and the flux in the remaining pathes of the lossless magnetic circuit. In every section of the magnetic circuit, the instantaneous power (61) is non active: it has no mean value and does not transfer net energy from source to load or viceversa. Such a power oscillates between the magnetic circuit and the voltage or current source. Usually the oscillations take place with frequencies $2v\omega$ ($v = 1, 3, 5...$) and $(v \pm m)\omega$ (v = 1, 3, 5; m = 1, 3, 5; $m \neq v$). In certain situations v and m could be non integer numbers. The physical mechanism behind such oscillations is tied to capability of inductances and capacitances to store electromagnetic energy. The oscillations of this energy are characterized by five distinct types of instantaneous powers:

1. Fundamental reactive power [var]:
   $p_{Q1} = V_1 I_1\, sin(\theta_1)\, sin(2\omega t)$,
2. Harmonic reactive power [var]:
   $p_{Qh} = V_v\, I_v\, sin(\theta_v)\, sin(2v\omega t + 2\alpha_v)$,
3. Current distortion power [var]:
   $p_{DIv} = V_1\, I_v\, \{cos[(v-1)\omega t + \gamma_v] - cos[(v+1)\omega t + \delta_v]\}$,
4. Voltage distortion power [var] ($v \neq 1$):
   $p_{DVv} = V_v\, I_1 \{cos[(v-1)\omega t + \varepsilon_v] - cos[(v+1)\omega t + \chi_v]\}$,
5. Harmonic distortion power [var] ($m \neq v$, $m \neq 1$):
   $p_{Hv} = V_v\, I_m\, \{cos[(v-m)\omega t + \gamma_{vm}] - cos[(v+m)\omega t + \chi_{vm}]\}$,

The meter connected to the load (Fig. 1a) should record the active and nonactive power quantities:

1. Fundamental active power [W]:
   $P_{s1} = V_{s1}\, I_{s1}\, cos(\theta_{S1})$,
2. Total harmonic active power [W]:
   $P_{sh} = V_{sv}\, I_{sv}\, cos(\theta_{Sv})$,
3. Fundamental reactive power [var]:
   $Q_{s1} = V_{s1}\, I_{s1}\, sin(\theta_{S1})$,
4. Current distortion power [var]:
   $D_{sI} = V_{s1}\sqrt{\sum_{v\neq 1} I_{sv}^2} = V_{s1} I_{sH}$,
5. Voltage distortion power [var]:
   $D_{sV} = I_{s1}\sqrt{\sum_{v\neq 1} V_{sv}^2} = I_{s1} V_{sH}$,
6. Harmonic apparent power [var]:
   $S_{sH} = \sqrt{\sum_{v\neq 1} V_{sv}^2 \sum_{v\neq 1} I_{sv}^2} = V_{sH} I_{sH}$,

The expression of the active power supplied by the voltage source contains the losses in the windings:

$$\Delta P = r_p I_p^2 + r_s I_s^2 = \left(\frac{\mathscr{R}_2}{\mathscr{R}_2 + \mathscr{R}_S} r_p + r_s \right) I_s^2 = r_e I_s^2 =$$

$$= r_e \frac{V_s^2 I_s^2}{V_s^2} = r_e \frac{\left(V_{s1}^2 + V_h^2\right)\left(I_{s1}^2 + I_h^2\right)}{V_s^2}$$

or:

$$\Delta P = \frac{r_e}{V_s^2}\left(P_{s1}^2 + Q_{s1}^2 + D_{sI}^2 + D_{sV}^2 + S_{sH}^2\right) = \frac{r_e}{V_s^2}\left(S_s^2\right)$$

with:

$$S_{sH}^2 = P_{sH}^2 + D_{sH}^2$$

Finally, the total active power injected in the primary winding is (The term P_{ec} represents the losses due to eddy currents, that in this paper are not detailed):

$$P_p = P_{s1} + P_{sH} + \Delta P + P_{hyst} + P_{ec}$$

Usually, $P_{sH} \leq 0$. Correctly monitored, this harmonic power helps to determine if the load generates or absorbs harmonics.

A useful component is the *non-fundamental apparent power*:

$$S_{sN} = \sqrt{D_{sI}^2 + D_{sV}^2 + S_{sH}^2}$$

It quantifies the overall contributions of harmonic pollution to the line losses and helps to extimate the size (kVA) of the equipment needed to compensate the harmonic pollution.

6. CONCLUSION

The PV computation enables the tracking of all the types of instantaneous powers, their origin, flow direction and time variation. As a result of this thorough analysis, it was possible to conclude what are the components that should be monitored for the quantification of demand of power, energy consumption or harmonics pollution caused or sank by the observed load. The authors do no advocate the use of PV to directly measure powers. The PV is an excellent tool that enables one *to look* inside different electrical devices such as an energy converter and observe the patterns of radiated energy and the time variation of energy flowing from sources to loads, among loads and within loads.

APPENDIX A.1

Substitution of (1, 4, 10) in the last term of (50) and (60) gives:

$$v_s i_s + \phi \frac{di_p}{dt} =$$

$$= \sum_v \hat{V}_{sv}\, sin\left(v\omega t + \alpha_{sv}\right) \sum_v \hat{I}_{sv}\, sin\left(v\omega t + \alpha_{sv} - \theta_{sv}\right)$$

$$- \sum_v \frac{\hat{V}_{sv}}{v\omega} cos\left(v\omega t + \alpha_{pv}\right) \sum_v v\omega \hat{I}_p\, cos\left(v\omega t + \alpha_{pv} - \theta_{pv}\right)$$

The only terms that may yield real power result from the products of terms with the same frequency, i.e.:

$$\sum_v \hat{V}_{sv}\hat{I}_{sv}\, sin\left(v\omega t + \alpha_{sv}\right) sin\left(v\omega t - \alpha_{sv} + \theta_{sv}\right)$$
$$- \sum_v \hat{V}_{pv}\hat{I}_{pv}\, cos\left(v\omega t + \alpha_{pv}\right) cos\left(v\omega t + \alpha_{pv} - \theta_{pv}\right)$$
$$= \sum_v V_{sv} I_{sv}\left[cos\left(\theta_{sv}\right) - cos\left(2v\omega t + 2\alpha_{sv} - \theta_{sv}\right)\right]$$
$$- \sum_v V_{sv} I_{sv}\left[cos\left(\theta_{pv}\right) - cos\left(2v\omega t + 2\alpha_{pv} - \theta_{pv}\right)\right]$$

Since:

$$V_{sv}I_{sv}\,cos\left(\theta_{sv}\right)=V_{sv}I_{sv}\,cos\left(\theta_{pv}\right)$$

results that the studied term $v_s i_s+\phi^{(1)}\dfrac{di_p}{dt}$ is a nonactive instantaneous power, i.e. has zero mean value.

APPENDIX A.2

A N turns inductance with a reluctance $\mathcal{R}$ and a current i has the instantaneous power:

$$p=Li\frac{di}{dt}=\frac{N^2}{\mathcal{R}}i\frac{di}{dt}$$

Since the magnetic flux is $\phi = Ni\ \mathcal{R}$, (61) is obtained. Substitution of (24) in (61), for region 3:

$$\mathcal{R}_3\phi^{(3)}\frac{d\phi^{(3)}}{dt}=\mathcal{R}_3\left(\frac{\mathcal{R}_2}{\mathcal{R}_2+\mathcal{R}_3}\right)^2\left(\phi^{(1)}-\frac{i_s}{\mathcal{R}_2}\right)\cdot\left(v_p-\frac{1}{\mathcal{R}_2}\frac{di_s}{dt}\right)=$$
$$=\frac{\mathcal{R}_{23}\mathcal{R}_2}{\mathcal{R}_2+\mathcal{R}_3}\phi^{(1)}v_p-\frac{\mathcal{R}_2\mathcal{R}_3}{\left(\mathcal{R}_2+\mathcal{R}_3\right)^2}\phi^{(1)}\frac{di_s}{dt}-\frac{\mathcal{R}_2\mathcal{R}_3}{\left(\mathcal{R}_2+\mathcal{R}_3\right)^2}v_p i_s+ \qquad \text{(A1)}$$
$$+\frac{\mathcal{R}_3}{\left(\mathcal{R}_2+\mathcal{R}_3\right)^2}i_s\frac{di_s}{dt}$$

Using (25) and (22) we find that (A.1) equals (60) and (62) is proved. Applying the same, in region 2 we find:

$$\mathcal{R}_2\phi^{(2)}\frac{d\phi^{(2)}}{dt}=R_2\left(\frac{\mathcal{R}_2}{\mathcal{R}_2+\mathcal{R}_3}\right)^2\left(\phi^{(1)}+\frac{i_s}{\mathcal{R}_3}\right)\cdot\left(v_p+\frac{1}{\mathcal{R}_3}\frac{di_s}{dt}\right)=$$
$$=\frac{\mathcal{R}_{23}\mathcal{R}_3}{\mathcal{R}_2+\mathcal{R}_3}\phi^{(1)}v_p+\frac{\mathcal{R}_2\mathcal{R}_3}{\left(\mathcal{R}_2+\mathcal{R}_3\right)^2}\phi^{(1)}\frac{di_s}{dt}+\frac{\mathcal{R}_2\mathcal{R}_3}{\left(\mathcal{R}_2+\mathcal{R}_3\right)^2}v_p i_s+$$
$$+\frac{\mathcal{R}_2}{\left(\mathcal{R}_2+\mathcal{R}_3\right)^2}i_s\frac{di_s}{dt}$$

The total instantaneous power stored/ejected in/from regions 2 and 3 is:

$$\mathcal{R}_2\phi^{(2)}\frac{d\phi^{(2)}}{dt}+\mathcal{R}_3\phi^{(3)}\frac{d\phi^{(3)}}{dt}=\mathcal{R}_{23}\phi^{(1)}v_p+\frac{i_s}{\mathcal{R}_2+\mathcal{R}_3}\frac{di_s}{dt} \qquad \text{(A2)}$$

From (25):

$$\frac{\mathcal{R}_2}{\mathcal{R}_2+\mathcal{R}_3}v_p i_s-v_s i_s=\frac{1}{\mathcal{R}_2+\mathcal{R}_3}\frac{di_s}{dt}$$

or:

$$v_p i_p-v_s i_s=\frac{1}{\mathcal{R}_2+\mathcal{R}_3}\frac{di_s}{dt}$$

The last expression substituted in (A.2) proofs that total instantaneous power stored/ejected in/from regions 2 and 3 is expressed by eq. (63).

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