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Intrinsic Power: Some Observations

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Summary: Abstract This paper deals with the instantaneous powers called intrinsic powers. This form of nonactive power is always present when active power is present. The intrinsic power does not cause power loss in conductors or ferromagnetic components. The paper describes different kinds of intrinsic power and shows how it originates.

2. INTRODUCTION

The instantaneous powers can be separated in two groups: Active and nonactive powers. The active powers having non-zero mean value always carry energy in a unidirectional way. The nonactive powers have zero average power and their energy oscillates back and forth between sources, sources and loads or just between loads. In a single-phase system where a line is supplying a load with the sinusoidal voltage:

$$v = \hat{V}\sin(\omega t + \alpha) \qquad (1)$$

and the current:

$$i = \hat{I}\sin(\omega t + \beta) \qquad (2)$$

the instantaneous power has two terms:

$$p = p_p + p_q \qquad (3)$$

where:

$$p_p = P + p_i \qquad (4)$$

is the active instantaneous power:

$$P = VI\cos(\vartheta); \quad \vartheta = \alpha - \beta \qquad (5)$$

is the active power:

$$p_i = -P\cos(2\omega t + 2\alpha)$$

is the *intrinsic* power (incorrectly called intrinsical in [1]):

$$p_q = -Q\sin(2\omega t + 2\alpha) \qquad (6)$$

is the instantaneous reactive power with:

$$Q = VI\sin(\vartheta) \qquad (7)$$

the reactive power. The intrinsic power is an instantaneous nonactive power always present when the active power is present; the powers P and p_i are inseparable. If the supplying line has the resistance R, the power loss in line is:

$$\Delta P = RI^2 = \frac{R}{V^2}(P^2 + Q^2) \qquad (8)$$

It is clear from (8) that both p_p and p_q contribute to the power dissipated by R. The logical question to be asked is: If Q is the amplitude of p_q and P the amplitude of p_i, is the intrinsic power the component responsible for the power loss component $P^2(R/V^2)$?

The answer to this question may be revealed if we observe the time variation of the energy supplied to resistance R during measurement time . The amount of energy supplied is:

$$W = \int_0^\tau p_p dt = \int_0^\tau [P + p_i]dt = \int_0^\tau \left[P - P\cos(2\omega t + 2\alpha)\right]dt =$$

$$= P\tau + \frac{P}{2\omega}\left[\sin(2\omega\tau + 2\alpha) - \sin(2\alpha)\right]$$

The first term is ramping up at the rate of P while the second term oscillates between the extremes $[\pm 1 - \sin(2\alpha)](P/2\omega)$.

If the measuring time is an integer number of cycles, i.e. $\tau = NT = 2\pi N/\omega$, then the energy transferred to R is $W = NPT$, and this energy is being transmitted on the account of the Pt term only. A resistance supplied with dc under steady-state conditions dissipates a perfectly constant power $RI^2 = V^2/R$, hence as long as R and V do not vary in time no intrinsic power is present, nevertheless transfer of energy takes place.

One may find in literature proposals to single p_i as a consequential component that affects the power factor definition and value [2]. Such an attempt is not warranted. The reason for avoiding this approach stems from the interpretation of the rms current: If a resistance R dissipates the energy W, over the time duration τ, then the average power is $P = W/\tau$ and the rms current over the duration τ is:

$$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{W}{\tau R}}$$

From here is learned that for a given W dissipated by a resistance R during the time τ, there is only one value of rms current and this value is independent of the instantaneous power fluctuations. This claim is supported by the following condition: If the instantaneous power dissipated by R has the general expression:

$$p_p = P + F(t) \tag{9}$$

where:

$$\int_0^{\tau} F(t)dt = 0$$

Then the rms current $I = \sqrt{P/R}$ is independent of $F(t)$. Here represents the intrinsic power in a most broad form.

These observations may help conclude that in steady-state conditions the intrinsic power does not contribute to power loss in conductors or magnetic cores, nor will cause net energy transfer to loads.

2. CATEGORIES OF INTRINSIC POWER

When a linear resistance is supplied with the nonsinusoidal voltage and current:

$$v = \sum_h \hat{V} \sin(h\omega t + \alpha_h) = v_1 + v_H \tag{10}$$

$$i = \sum_h \hat{I} \sin(h\omega t + \alpha_h) = i_1 + i_H \tag{11}$$

where:

$$v_1 = \hat{V}_1 \sin(\omega t + \alpha_1) \quad \text{and} \quad v_H = \sum_{h\neq 1} \hat{V}_h \sin(h\omega t + \alpha_h)$$

$$i_1 = \hat{I}_1 \sin(\omega t + \alpha_1) \quad \text{and} \quad i_H = \sum_{h\neq 1} \hat{I}_h \sin(h\omega t + \alpha_h)$$

The total instantaneous power has the expressions:

$$p = vi = (v_1 + v_H)(i_1 + i_H) = R(i_1 + i_H)^2 =$$

$$= R\sum_h i_h^2 + 2R \sum_{\substack{m\neq n\\ m,n=1}} i_m i_n = (P_1 + p_{i1}) + \sum_{h\neq 1}(P_h + p_{ih}) + \sum_{\substack{m\neq n\\ m,n=1}} p_{\substack{ii\\mn}} \tag{12}$$

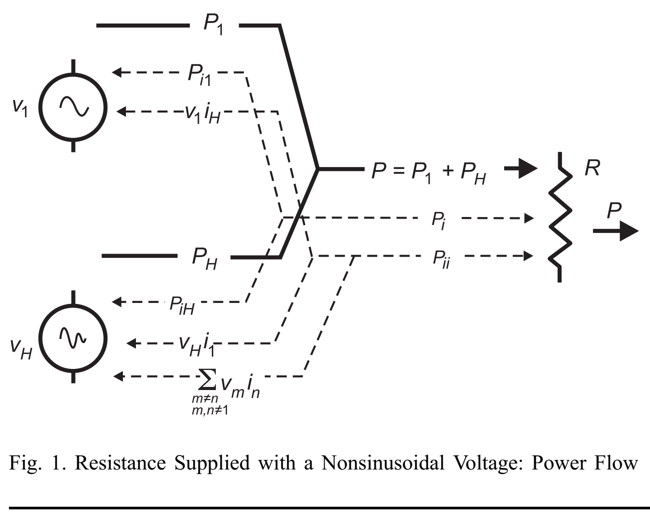

Fig. 1. Resistance Supplied with a Nonsinusoidal Voltage: Power Flow

and contains the fundamental active power $P_1 = RI_1^2$, the harmonic active powers $P_h = RI_h^2$ and the following intrinsic powers:

$$p_{i1} = -RI_1^2 \cos(2\omega t + 2\alpha_1) \tag{13}$$

is the *fundamental intrinsic power:*

$$p_{ih} = -RI_h^2 \cos(2h\omega t + 2\alpha_h) \tag{14}$$

is the *harmonic intrinsic power of order h* and:

$$p_{\substack{ii\\mn}} = RI_m I_n \big[\cos[(m-n)h\omega t + \alpha_m - \alpha_n] - \cos[(m+n)h\omega t + \alpha_m + \alpha_n]\big] \tag{15}$$

is a *harmonic intrinsic power of order* ***mn.***

Both, the h-order and the mn-order intrinsic powers exist simultaneously and can not be separated from the respective P_h. Both types of intrinsic powers oscillate having zero mean value and zero contribution to the power dissipated by the resistance R. The flow of these powers is illustrated in Figure 1, where the dashed lines show the flow of intrinsic powers and the solid lines the unidirectional flow of the active powers.

4. THE INTRINSIC POWER IN THREE-PHASE SYSTEMS

The active power of a polyphase system is also tied with an intrinsic power. The intrinsic power expression is a function of the actual location of the terminals where the power meter is connected.

Assuming a m-phase system with the voltages:

$$v_1 = \hat{V}\sin(\omega t);\quad \hat{V} = \sqrt{2}V;\quad v_2 = \hat{V}\sin\left(\omega t - \frac{2\pi}{m}\right)$$

$$\dots\dots\dots \tag{16}$$

$$v_n = \hat{V}\sin[\omega t - \frac{2\pi}{m}(n-1)]$$

$$\dots\dots\dots$$

$$v_m = \hat{V}\sin[\omega t - \frac{2\pi}{m}(m-1)]$$

supplying a m-phase balanced unity power-factor load with the total active power $P = mVI$, where the line currents are purely positive-sequence currents with the rms value $I = V/R$. The instantaneous power measured between the terminal n and the neutral is:

$$p_n = \frac{P}{m} - \frac{P}{m}\cos[2\omega t + \frac{4\pi}{m}(n-1)] = \frac{P}{m} + p_{ni} \tag{17}$$

This instantaneous power has a distinct intrinsic power p_{ni}. The fact that the sum of all m intrinsic powers is nil, i.e. $\sum_{n=1}^{m} p_{ni} = 0$, misled in the past some researchers in believing that no oscillations of power take place in a perfectly balanced polyphase system. If the power is measured between the terminals k and j, using the voltage v_{kj} and the current i_k, the instantaneous power has the expression:

$$p_{kj} = v_{kj} i_k = 2\hat{V} \sin\left(\frac{j-k}{m}\pi\right)\cos\left(\omega t + \frac{2-j-k}{m}\pi\right)\cdot$$
$$\cdot \hat{I} \sin\left[\omega t - \frac{2\pi}{m}(k-1)\right] = \frac{P}{m-1} + p_{kj1} \quad (18)$$

where the intrinsic power is:

$$p_{kj} = 2VI \sin\left(\frac{j-k}{m}\pi\right) \sin\left[2\omega t + \frac{\pi}{m}(4-3k-j)\right] \quad (19)$$

For example, in the practical case of a three-phase system, R,S,T, using the two-wattmeter method one finds:

$$p_{RS} = \frac{P}{2} + \frac{P}{\sqrt{3}} \sin(2\omega t - \pi/3) \quad (20)$$

$$p_{TS} = \frac{P}{2} - \frac{P}{\sqrt{3}} \sin(2\omega t - \pi/3) \quad (21)$$

We observe that in this case the intrinsic power's amplitude does not equal the active power. This result confirms the fact that the intrinsic power does not control the value of power loss or energy transferred.

5. FREQUENCY MODULATED WAVEFORM

A frequency modulated voltage:

$$v = \hat{V} \sin[\omega t + m\cos(\Omega t)] \quad (22)$$

is periodical if the ratio ω/Ω is rational, and has components with the frequencies $\omega \pm k\Omega$, $\Omega < \omega$, k=0,1,2,... The power dissipated by a resistance R supplied with such a voltage is:

$$p = \frac{v^2}{R} = \frac{\hat{V}^2}{R}\left[\sin(\omega t)\cos[m\cos(\Omega t)] + \cos(\omega t)\sin[m\cos(\Omega t)]\right]^2 =$$
$$= \frac{V^2}{2R}\left[1 + \sin(2\omega t)\sin[2m\cos(\Omega t)] - \cos(2\omega t)\cos[2m\cos(\Omega t)]\right]$$

By using the identities:

$$\sin[x\cos(\phi)] = 2\sum_{k=1}(-1)^{k+1} J_{2k+1}(x)\cos[(2k+1)\phi]$$
$$\cos[x\cos(\phi)] = J_{0(x)} - 2\sum_{k=1}(-1)^k J_{2k}(x)\cos(2k\phi)$$

where $J_i(x)$ is a Bessel function of first kind and order i, one will obtain the following instantaneous power expression:

$$p = \frac{V^2}{R}\left\{\begin{array}{l} 1 + -J_0(2m)\cos(2\omega t) + 2\cos(2\omega t)\sum_{k=1}(-1)^k J_{2k}(2m)\cos(2k\Omega t) \\ +2\sin(2\omega t)\sum_{k=1}(-1)^{k+1} J_{2k+1}(2m)\cos[(2k+1)\Omega t] \end{array}\right\} \quad (23)$$

Equation (23) separates the active power V^2/R from the fundamental intrinsic power $-(V^2/R)J_0(2m)\cos(2\omega t)$, and the higher order intrinsic powers with frequencies $2(\omega \pm k\Omega)$ and $2[\omega \pm (2k+1)\Omega]$, k=1,2,3,....

6. HOW INTRINSIC POWERS ARE PRODUCED

Any alternating voltage or current source, sinusoidal or nonsinusoidal, produces intrinsic power by the virtue of amplitude time variation. In this paragraph is studied the power flow in a dc to ac converter, Figure 2a. The inverter consists of an H-bridge with four ideal switches, *a, aa, b* and *bb*. The switches are sequentially turned on/off with a 50% duty cycle. The voltage across the resistance R is a perfect square wave:

$$v_R = v_{ac} = \frac{4V}{\pi}\sum_{k=1}\frac{\sin[(2k-1)\omega t]}{2k-1} \quad (24)$$

and the source V delivers a constant power:

$$p = P = VI = V^2/R$$

However, the power delivered to R is time varying:

$$p_R = \frac{v_R^2}{R} = \frac{16V^2}{\pi^2 R}\left\{\sum_{k=1}\frac{1-\cos[2(2k-1)\omega t]}{2(2k-1)^2}\right.$$
$$\left. + \sum_{\substack{m \neq n \\ m,n=1}}\frac{\sin[(2m-1)\omega t]\sin[(2n-1)\omega t]}{(2m-1)(2n-1)}\right\}$$

This instantaneous power has four terms:

$$p_R = P + p_{i1} + p_{iH} + p_{i_{mn}}$$

where:

$$P = \frac{8V^2}{\pi^2 R}\sum_{k=1}\frac{1}{(2k-1)^2} = \frac{V^2}{R} \quad (25)$$

is the active power:

$$p_{i1} = -\frac{8V^2}{\pi^2 R}\cos(2\omega t) \quad (26)$$

is the fundamental intrinsic power:

$$p_{iH} = \frac{-8V^2}{\pi^2 R}\sum_{k \neq 1}\frac{\cos[2(2k-1)\omega t]}{(2k-1)^2} \quad (27)$$

are the harmonic intrinsic powers and:

$$p_{i_{mn}} = \frac{16V^2}{\pi^2 R}\sum_{\substack{m \neq n \\ m,n=1}}\frac{\sin[(2m-1)\omega t]\sin[(2n-1)\omega t]}{(2m-1)(2n-1)} \quad (28)$$

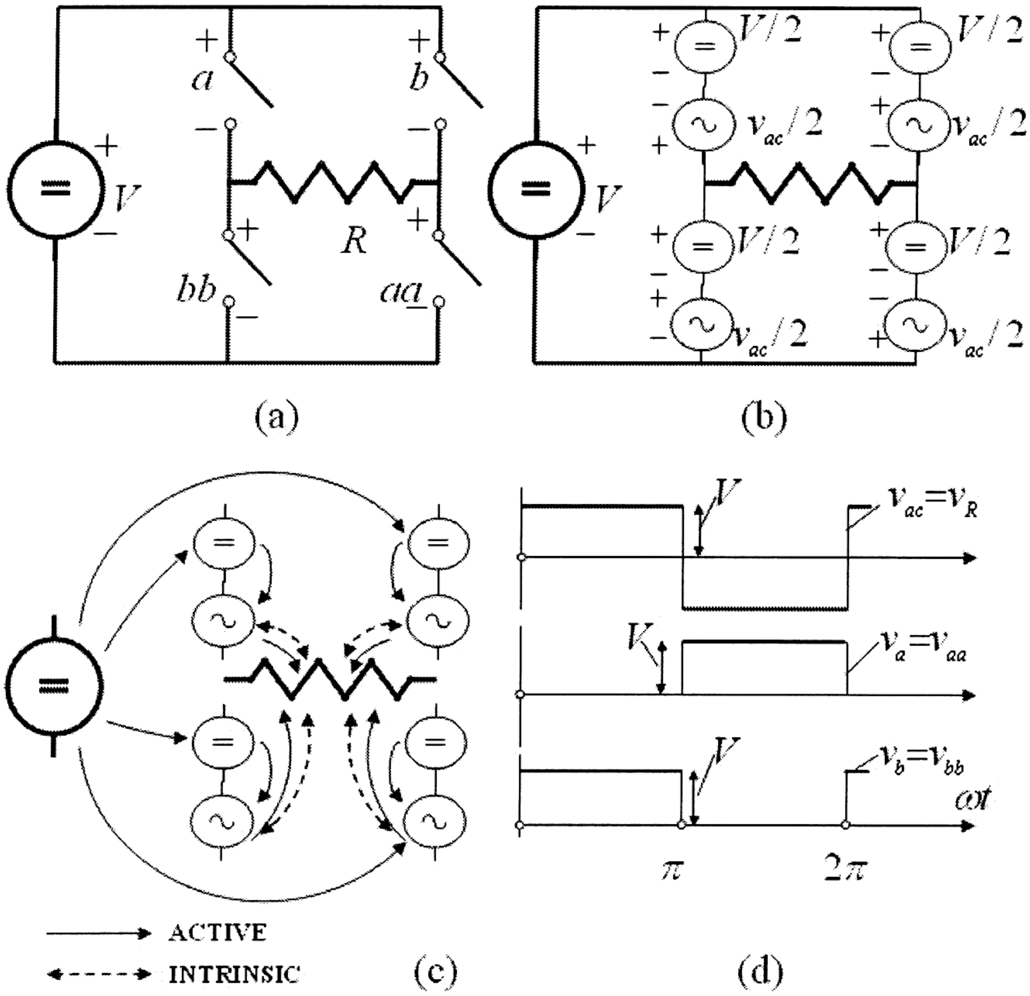

Figure 2. Power flow in a dc/dc converter: (a) Circuit schematic; (b) Equivalent circuit; (c) Power flow; (d) Waveforms: Load voltage v_R, switch voltage $v_a = v_{aa}$, and $v_b = v_{bb}$.

are the harmonic intrinsic powers of mn-order.

The equivalent circuit shown in Figure 2b is obtained by replacing each switch with an equivalent voltage (sketched in Figure 2d):

$$v_a = v_{aa} = \frac{V}{2} - \frac{v_{ac}}{2} = \frac{V}{2} - \frac{4V}{2\pi}\sum_{k=1} \frac{sin[(2k-1)\omega t]}{2k-1} \quad (29)$$

$$v_b = v_{bb} = \frac{V}{2} + \frac{v_{ac}}{2} = \frac{V}{2} + \frac{4V}{2\pi}\sum_{k=1} \frac{sin[(2k-1)\omega t]}{2k-1} \quad (30)$$

The correctness of the equivalent circuit is proved when one checks the voltage across the load R:

$$v_R = -v_a + v_b = -\frac{V}{2} + \frac{v_{ac}}{2} + \frac{V}{2} + \frac{v_{ac}}{2} = v_{ac} \quad (31)$$

and checks if the supply voltage is in perfect balance with the H-bridge input voltage:

$$v_a + v_{bb} = \frac{V}{2} - \frac{v_{ac}}{2} + \frac{V}{2} + \frac{v_{ac}}{2} = V \quad (32)$$

Since the switches are ideal the power dissipated by each switch is nil. For example $p_a = p_{aa} = v_a i_a = 0$, or:

$$\left(\frac{V}{2} - \frac{v_{ac}}{2}\right) i_a = 0 \quad (33)$$

This expression indicates that each switch acts like a converter; it absorbs dc power $V^2/4R$ and converts it in fundamental and harmonic power. The flow of powers is summarized in Figure 2c. The dc power supplied by the source V is not tied to any type of intrinsic power. The load is supplied by the four ac sources v_{ac}. Each one of them delivers a quarter of the instantaneous power. In this particular case, since no energy storage components are present, the entire instantaneous power consists only of active and intrinsic power.

7. CONCLUSIONS

The intrinsic power is an instantaneous power with the following characteristics:

1. Always present when active power is present, except for direct current systems with zero voltage and current ripple.
2. Under steady-state conditions, the intrinsic power does not cause net transfer of energy to the load, nor is causing power loss in active materials, over N-cycles duration.
3. Does not affect the apparent power value.

When powers are studied, it is necessary to recognize the intrinsic power as an inconsequential power and not to confuse it with other types of nonactive powers [2].

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