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Amplitude and Phase Modulation Effects of Waveform Distortion in Power Systems

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Summary: **The amplitude and phase modulation effects of waveform distortion in power systems are analyzed. Recalls on Amplitude Modulation (AM) and Phase Modulation (PM) are given with particular reference to spectral components. Then, simple inverse formulas are obtained to demonstrate that summations of one or more small tones frequencies to a given tone of interest can always be interpreted in terms of AM and PM. The usefulness of AM and PM representation, in particular in the presence of interharmonic tones, is demonstrated with reference to simple case-studies and practical applications.**

1. INTRODUCTION

Power system engineers are used to handling waveform distortion problems by means of Fourier expansion [1]. The presence of interharmonic components has introduced important and new effects in terms of variations in waveform periodicity. For this reason, an infinite frequency resolution should be adopted and a fixed resolution of 5 Hz is suggested by standards for industrial measurements [2]. A relatively small amount of attention is devoted to the spectral component phase angles in spite of their importance in determining the peak value, the number of zero crossing instants and the delay of the main zero crossing instants with respect to those in sinusoidal conditions, as well as the kind of modulations on the fundamental component.

Telecommunication engineers are used to handling information transmission problems by means of proper carrier tones that are modulated by the signal to be transmitted through a given channel in a chosen vector. Among an enormous variety of possible modulation techniques, analogical modulations were the first to be used and include Amplitude Modulation (AM) and Phase Modulation (PM) [3]. Particular attention is devoted to the phase angles of the signal's spectral components.

The basic literature demonstrates the perfect equivalence of AM and PM to the summation of sinusoidal tones of proper amplitudes and phase angles.

In power system analysis, there is a tone of prevailing interest, typically the fundamental component of voltage and current. It is interesting to study the modulating effects caused on this tone, considered as carrier tone, by one or more interacting tone of lower amplitudes at different frequencies with respect to it. The reason is related to the consequences that appear in terms of voltage fluctuations, flicker in lights, monitors, etc., malfunctions in power electronic devices for energy conversion, active filtering etc. and the inaccuracies introduced in the behavior of instrumentation that needs to be synchronized with the signals to be measured (PLL).

In this paper, it is demonstrated that the presence of spectral components in the power system waveforms can always be interpreted in terms of AM and PM of the fundamental component or of another tone of particular interest. This is obvious when the spectrum of the components exactly corresponds to that of a given kind of modulation; moreover, it is also demonstrated in the presence of a single tone of spectral components that do not correspond exactly to a single specific kind of modulation, that there is a combination of two or more modulations. The demonstration is obtained by deriving simple analytical formulas that from a rigorous mathematical point of view apply in the case of spectral components of very low values compared to those of carrier tones, which is the real case constituted by interharmonics.

On the other hand, the representation in terms of AM and PM is necessary to gain a deep insight into the origins and quantitative evaluations of very important interharmonic effects, such as Light Flicker, measurement instrument uncertainties, and the behavior of power electronic devices.

In what follows, AM and PM recalls are first given with particular reference to spectral analysis. Then, simple inverse formulas needed for the interpretation of signal distortion in terms of AM, PM, and their combination are given. Finally, the usefulness of AM and PM representation, in particular in the presence of interharmonic tones, is demonstrated with reference to simple theoretical case-studies and practical applications.

2. RECALLS ON MODULATIONS

Some recalls on AM and PM are here reported with reference to the simple case of sinusoidal carrier and sinusoidal modulating signals. Reference is made to the general representation of the modulated signal:

$$u(t) = U(t) \cdot cos\big(\Omega(t) \cdot t + \Phi(t)\big) \tag{1}$$

where $U(t)$ is the instantaneous amplitude, $\Phi(t)$ the instantaneous phase angle, and $\Omega(t)$ the instantaneous angular frequency. In what follows, the case of unitary magnitude and a null phase angle for the 50 Hz carrier is considered without loss of generality.

2.1. Amplitude Modulation

The analytical expression of an AM signal is:

$$u(t) = \left[1 + 2a\,cos\left(\Delta\omega t - \varphi_{i1}\right)\right] \cdot cos\left(\omega_1 t\right) \qquad (2)$$

where ω_1 is the carrier angular frequency, $\Delta\omega$ is the modulation angular frequency, $2a$ the modulation amplitude and φ_{i1} is the phase angle of the modulating signal.
By means of the Prosthaphaeresis formulas, (2) is converted into:

$$u(t) = cos\left(\omega_1 t\right) + a\,cos\left[\left(\omega_1 - \Delta\omega\right)t + \varphi_{i1}\right] + \\ + a\,cos\left[\left(\omega_1 + \Delta\omega\right)t - \varphi_{i1}\right] \qquad (3)$$

which is a summation of three sinusoidal tones.

The time waveform, frequency spectrum, and phase angles of an AM signal are shown in Figure 1. There are three spectral components and the phase angles of the two modulating components are opposite.

The instantaneous amplitude, the instantaneous phase angle, the instantaneous angular frequency, and the RMS over the effective signal period (100 ms) are respectively:

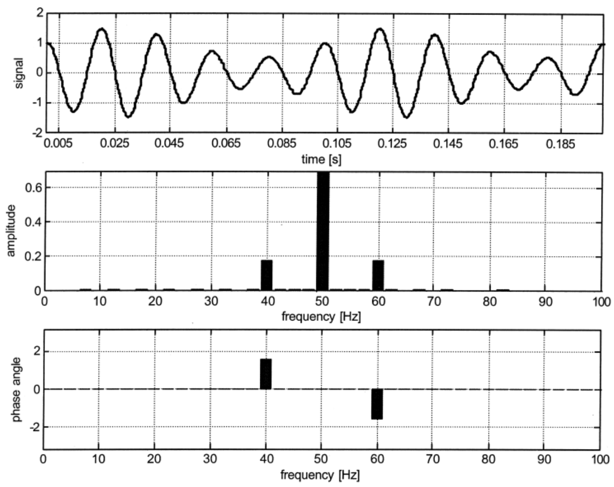

Fig. 1. Time waveform, frequency spectrum and phase angles of an AM signal

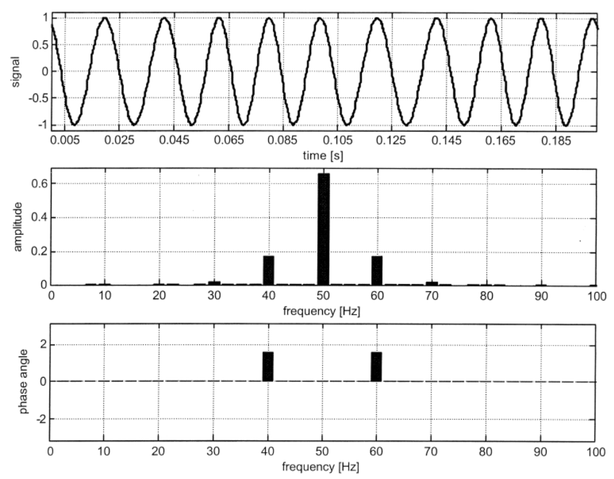

Fig. 2. Time waveform, frequency spectrum, and phase angles of the main components of a PM signal

$$\begin{aligned} U(t) &= 1 + 2a\,cos\left(\Delta\omega t - \varphi_{i1}\right) \\ \Phi(t) &= \text{const.} = 0 \\ \Omega(t) &= \text{const.} = \omega_1 \\ RMS &= \sqrt{\frac{1}{2}\left(1 + 2a^2\right)} \end{aligned} \qquad (4)$$

2.2. Phase Modulation

The analytical expression of a PM signal is:

$$u(t) = cos\left[\omega_1 t + \varphi_{MAX} \cdot cos\left(\Delta\omega t + \pi/2 - \varphi_{i1}\right)\right] \qquad (5)$$

where ω_1 and $\Delta\omega$ are as in (2) and φ_{MAX} is the index of modulation that is the absolute value of the maximum phase of the modulating signal and $\pi/2 - \varphi_{i1}$ is its phase angle.

By expanding the outer cosine and manipulating as shown in the Appendix for the case of $\varphi_{MAX} \le 1$, (5) is converted into:

$$\begin{aligned} u(t) &= \left(1 - \frac{\varphi_{MAX}^2}{4}\right) cos\left(\omega_1 t\right) + \\ &+ \frac{\varphi_{MAX}}{2}\left(1 - \frac{\varphi_{MAX}^2}{8}\right)\left\{cos\left[\left(\omega_1 - \Delta\omega\right)t + \varphi_{i1}\right] + cos\left[\left(\omega_1 + \Delta\omega\right)t + \pi - \varphi_{i1}\right]\right\} + \\ &+ \frac{\varphi_{MAX}^2}{8}\left(1 - \frac{\varphi_{MAX}^2}{12}\right)\left\{cos\left[\left(\omega_1 - 2\Delta\omega\right)t + 2\varphi_{i1}\right] + cos\left[\left(\omega_1 + 2\Delta\omega\right)t - 2\varphi_{i1}\right]\right\} + \ldots \end{aligned} \qquad (6)$$

which is a summation of infinite sinusoidal signals of proper amplitudes and phase angles.

The time waveform, the frequency spectrum, and the phase angles of the main components of a PM signal are shown in Figure 2. The spectrum is rich in components and the two main modulating component phase angles are each complementary to π.

The instantaneous amplitude, the instantaneous phase angle, the instantaneous angular frequency, and the RMS over the effective signal period (100 ms) are respectively:

$$\begin{aligned} U(t) &= 1 \\ \Phi(t) &= \varphi_{MAX} \cdot cos\left(\Delta\omega t + \pi/2 - \varphi_{i1}\right) \\ \Omega(t) &= \omega_1 - \Delta\omega \cdot \varphi_{MAX} \cdot sin\left(\Delta\omega t + \pi/2 - \varphi_{i1}\right) \\ RMS &= \sqrt{\frac{1}{2}\left[\left(1 - \frac{\varphi_{MAX}^2}{4}\right)^2 + \frac{\varphi_{MAX}^2}{2}\left(1 - \frac{\varphi_{MAX}^2}{8}\right)^2 + \frac{\varphi_{MAX}^4}{32}\left(1 - \frac{\varphi_{MAX}^2}{12}\right)^2 + ..\right]} = \sqrt{\frac{1}{2}}. \end{aligned} \qquad (7)$$

2.3. Considerations

It is worthwhile to note that (3) and (6) demonstrate the equivalence of AM and PM to the summation of sinusoidal tones, respectively. As a consequence, the considered modulations can always be handled in terms of spectral components and, in the following sections, it will be demonstrated that a summation of spectral components can always be interpreted in terms of AM and PM.

3. MODULATING EFFECTS ON A GIVEN TONE CAUSED BY TONES AT DIFFERENT FREQUENCIES

Once assumed that there is a tone of prevailing interest, typically the fundamental in power systems, it is interesting to study the modulating effects caused on it by different interacting tones of lower amplitudes at different frequencies.

In what follows, for the sake of clarity, we will first analyze the case of a couple of minor tones of equal amplitudes at symmetric frequencies, then a couple of minor tones of different amplitudes at symmetric frequencies, and finally two generic tones are analyzed.

Reference is, as in Section 2, to the case of a carrier tone of unitary magnitude and null phase angle at the fundamental power frequency, without loss of generality for the results.

3.1. Two superimposed tones of equal amplitudes at symmetric frequencies

A carrier tone with a couple of superimposed tones of equal amplitude and in symmetrical frequency positions with respect to it ($\omega_{i1} = \omega_1 - \Delta\omega$ and $\omega_{i2} = \omega_1 + \Delta\omega$) can be expressed as:

$$u(t) = cos(\omega_1 t) + a \cdot cos\left[(\omega_1 - \Delta\omega)t + \varphi_{i1}\right] + \\ + a \cdot cos\left[(\omega_1 + \Delta\omega)t + \varphi_{i2}\right] \quad (8)$$

with a the amplitude of both tones and φ_{i1} and φ_{i2} their phase angles.

Pure Amplitude Modulation

When the two tones are characterized by $\varphi_{i1} + \varphi_{i2} = 0$, becomes exactly equal to (3) once substituted $-\varphi_{i1}$ for φ_{i2}.

This demonstrates that these two tones produce a perfect AM of the fundamental tone.

The instantaneous amplitude, phase angle, angular frequency, and RMS are the same as reported in (4).

Prevailing Phase Modulation

When the two tones are characterized by $\varphi_{i1} + \varphi_{i2} = \pi$, (8) becomes:

$$u(t) = cos(\omega_1 t) + a \cdot cos\left((\omega_1 - \Delta\omega)t + \varphi_{i1}\right) + \\ + a \cdot cos\left((\omega_1 + \Delta\omega)t + \pi - \varphi_{i1}\right) \quad (9)$$

which, after some mathematical manipulations deriving from the general expression of the truncated Fourier expansion of a perfect phase modulated signal, shown in the Appendix, becomes:

$$u(t) \cong cos\left[\omega_1 t + 2a \cdot cos\left(\Delta\omega t + \pi/2 - \varphi_{i1}\right)\right] + \\ + a^2\left[1 + cos\left(2\Delta\omega t - 2\varphi_{i1}\right)\right] cos(\omega_1 t) \quad (10)$$

where the first term represents a perfect phase modulated signal of amplitude 1, modulation angular frequency $\Delta\omega$ and $\varphi_{MAX} = 2a$; the second term represents a modulated signal of amplitude a^2 at angular frequency ω_1, subject to a perfect AM of unitary amplitude and modulation angular frequency $2\Delta\omega$.

For a<<1, the first term is largely prevailing and (10) yields:

$$u(t) \cong cos\left[\omega_1 t + 2a \cdot cos\left(\Delta\omega t + \pi/2 - \varphi_{i1}\right)\right] \quad (11)$$

which is a perfect phase modulated signal.

The instantaneous amplitude, the instantaneous phase angle, and the instantaneous angular frequency of (11) are respectively:

$$U(t) = 1 \\ \Phi(t) = 2a \cdot cos\left(\Delta\omega t + \pi/2 - \varphi_{i1}\right) \\ \Omega(t) = \omega_1 - \Delta\omega \cdot 2a \cdot sin\left(\Delta\omega t + \pi/2 - \varphi_{i1}\right) \quad (12a)$$

It is worthwhile noting that the exact RMS value obtained directly from (9) is:

$$RMS = \sqrt{\frac{1}{2}\left(1 + 2a^2\right)} \quad (12b)$$

which for a<<1 becomes equal to 1/√2.

3.2. Two superimposed tones of different amplitudes at symmetric frequencies

The analytical expression of a carrier tone with a couple of superimposed tones of different amplitudes in symmetrical frequency positions with respect to it, is:

$$u(t) = cos(\omega_1 t) + a_1\, cos\left[(\omega_1 - \Delta\omega)t + \varphi_{i1}\right] + \\ + a_2\, cos\left[(\omega_1 + \Delta\omega)t + \varphi_{i2}\right] \quad (13)$$

A Single non Null Tone

When a_2=0 and $a_1 \neq 0$ or a_1=0 and $a_2 \neq 0$, a single non null tone is present and (13) becomes:

$$u(t) = cos(\omega_1 t) + a_1\, cos\left[(\omega_1 - \Delta\omega)t + \varphi_{i1}\right] \quad (14a)$$

or

$$u(t) = cos(\omega_1 t) + a_2\, cos\left[(\omega_1 + \Delta\omega)t + \varphi_{i2}\right] \quad (14b)$$

It is possible to demonstrate, as shown in the Appendix, that:

$$u(t) \cong \frac{1}{2}\left[1 + 2a_1\, cos\left(\Delta\omega t - \varphi_{i1}\right)\right] \cdot cos(\omega_1 t) + \\ + \frac{1}{2} cos\left[\omega_1 t + 2a_1 \cdot cos\left(\Delta\omega t + \pi/2 - \varphi_{i1}\right)\right] \quad (15a)$$

or

$$u(t) \cong \frac{1}{2}\left[1 + 2a_2\, cos\left(\Delta\omega t + \varphi_{i2}\right)\right] \cdot cos(\omega_1 t) + \\ + \frac{1}{2} cos\left[\omega_1 t + 2a_2 \cdot cos\left(\Delta\omega t - \pi/2 + \varphi_{i2}\right)\right] \quad (15b)$$

For both cases, the first term represents a modulated signal of amplitude $\frac{1}{2}$ at angular frequency ω_1, subject to a perfect AM of amplitude a_1 (or a_2) and modulation angular frequency $\Delta\omega$; the second term represents a perfect phase modulated signal of amplitude $\frac{1}{2}$, a modulation angular frequency equal to $\Delta\omega$ and an index of modulation equal to a_1 (or a_2).

With reference, for the sake of simplicity, only to the case where $a_2 = 0$ and $a_1 \neq 0$, the instantaneous amplitude, the instantaneous phase angle, and the instantaneous angular frequency are for the former, respectively:

$$U(t) = \frac{1}{2} + a_1 \cos(\Delta\omega t - \varphi_{i1})$$

$$\Phi(t) = 0 \tag{16a}$$

$$\Omega(t) = \omega_1$$

while for the latter:

$$U(t) = \frac{1}{2}$$

$$\Phi(t) = 2a_1 \cdot \cos(\Delta\omega t + \pi/2 - \varphi_{i1}) \tag{16b}$$

$$\Omega(t) = \omega_1 - \Delta\omega \cdot 2a_1 \cdot \sin(\Delta\omega t + \pi/2 - \varphi_{i1})$$

The RMS value of (14a) is:

$$RMS = \sqrt{\frac{1}{2}\left(1 + a_1^2\right)} \tag{16c}$$

Two non Null Tones

When in (13) $a_1 \neq a_2 \neq 0$, two non null tones are present and it is possible to demonstrate, as shown in the Appendix, that:

$$\begin{aligned} u(t) = {} & \frac{1}{4}\left[1 + 2a_1 \cos(\Delta\omega t - \varphi_{i1})\right]\cos(\omega_1 t) + \\ & + \frac{1}{4}\left[1 + 2a_2 \cos(\Delta\omega t + \varphi_{i2})\right]\cos(\omega_1 t) + \\ & + \frac{1}{4}\cos\left[\omega_1 t + 2a_1 \cdot \cos(\Delta\omega t + \pi/2 - \varphi_{i1})\right] + \\ & + \frac{1}{4}\cos\left[\omega_1 t + 2a_2 \cdot \cos(\Delta\omega t + \pi/2 - \varphi_{i2})\right] \end{aligned} \tag{17}$$

where the first and the second terms represent signals of amplitudes 0.25 at angular frequency ω_1 subject to perfect AM both of angular frequencies $\Delta\omega$ and amplitudes a_1 and a_2, respectively; the third and the fourth terms represent signals of amplitudes 0.25 subjected to PM, both with modulation angular frequencies $\Delta\omega$ and indexes of modulation a_1 and a_2, respectively.

Generalization for N Superimposed

The general case of a fundamental signal with N superimposed tones, is here considered:

$$u(t) = \cos(\omega_1 t) + \sum_{n=1}^{N} a_n \cos(\omega_{in} t + \varphi_{in}) \tag{18}$$

Once evidenced i) the effective pairs of tones having symmetric distances from the fundamental angular frequency N'_C and ii) considering single components as N''_C pairs with one of the terms equal to zero, $N_C = N'_C + N''_C$ pairs, with $N/2 \leq N_C \leq N$, can be referred to. So doing, it is possible to write:

$$\begin{aligned} u(t) = \sum_{n=1}^{N_C} \Bigg\{ & \frac{\cos(\omega_1 t)}{N_C} + \\ & + a_{1n} \cos\left[(\omega_1 - \Delta\omega_n)t + \varphi_{i1n}\right] + a_{2n} \cos\left[(\omega_1 + \Delta\omega_n)t + \varphi_{i2n}\right] \Bigg\} \end{aligned} \tag{19}$$

It is worth noting that the N superimposed components produce, in the most general case, N_C amplitude modulated signals and N_C phase modulated signals, all of amplitude $1/(2N_C)$.

Series of pairs of proper amplitudes and phase angles correspond to typical cases of complex modulations as, for instance, the square wave modulation.

4. CASE-STUDIES

The following case studies correspond to situations in which only a modulation approach can allow a close look at the phenomena and a correct assessment of their effects.

Obviously the modulation approach can be applied regardless of the frequencies of the carrier and modulating tones.

In what follows, reference is made to a carrier tone always constituted by a fundamental (1.0 pu, 50.00 Hz and $\varphi = 0$) and modulating tones constituted by harmonics (case-study 1) and interharmonics (case-studies 2 and 3). High values are assumed for the modulating signals due to the need for clarity in figure representation.

4.1. Case-study 1: single harmonic tone

A simple example consisting of a superimposed third harmonic (0.3 pu, 150.00 Hz, $\varphi_3 = \pi/2$) is considered.

Figure 3 shows the modulation approach analysis for this case, which can be treated as a carrier tone consisting of the fundamental and a modulating tone consisting of the harmonic tone (see Subsection 3.2.1).

Figure 3.a shows the resulting signal compared with the fundamental and Figure 3.b shows the amplitude modulated part of the total signal (see first term of (15.b)), while Figure 3c shows the phase modulated part (see second term of (15.b)).

It is worthwhile noting that the AM determines the increase (more generally, the modification) of the peak value of the distorted signal with respect to that of the fundamental, while the PM reflects the delay (more generally, the modification) of the zero crossing instants of the distorted signal, without affecting the periodicity.

In the general case of distortion caused only by harmonic components, it seems that the information added by the modulation representation is not of particular interest, because the signal periodicity does not change and the RMS values are easy to estimate and appear fixed to an observer synchronized with the fundamental power frequency.

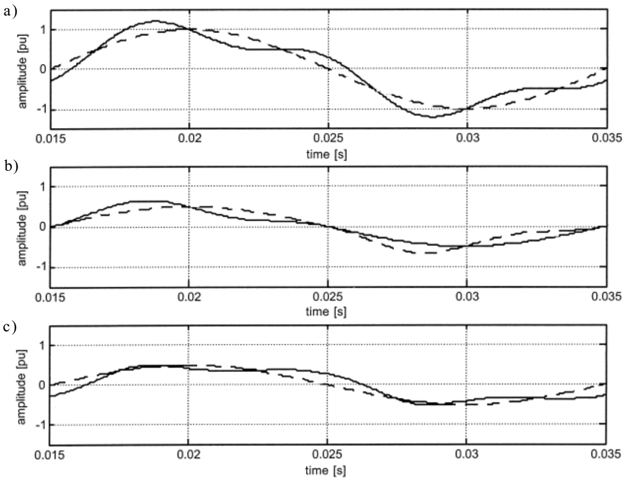

Fig. 3. Case-study 1: 1.0 pu fundamental with superimposed 0.3 pu 3rd harmonic: modulated signal (—), carrier signal (- - -), a) whole signal, b) AM term, c) PM term

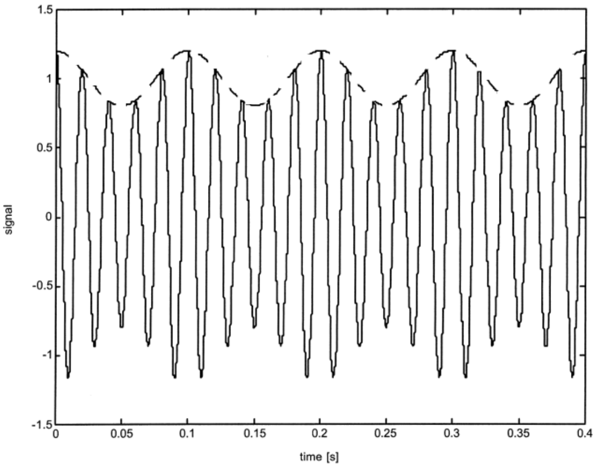

Fig. 4. Case-study 2: 1.0 pu fundamental with two superimposed interharmonics causing AM (0.2pu @ 10 Hz): modulated signal (—), instantaneous amplitude $U(t)$ (- - -)

Nevertheless, it is important to note that information about harmonic phase angles, usually not considered in the Fourier expansion representation, plays a determinant role in both the modulation effects, which are peak amplitude modification and zero crossing delay.

4.2. Case-study 2: two interharmonic tones causing amplitude modulation

A simple example will be considered consisting of two superimposed interharmonics at frequencies of 40 Hz and 60 Hz respectively (0.1 pu, $\varphi_{i1} = \varphi_{i2} = 0$). AM effects on the signal's instantaneous amplitude are shown in Figure 4.

It is interesting to analyze the RMS of the modulated signal because the modulating frequency of 10 Hz is one of the frequencies most sensitive to light flicker. Starting from an accurate Fourier expansion of the signal over a period of 100 ms, the RMS value is constant because the signal is periodic and assumes the value 0.714 pu.

Incandescent lamps act as quadratic demodulators and due to their thermal inertia are not able to follow the light variations caused by the fundamental voltage, while they follow 10 Hz variations of voltage RMS [4]. In practice, they behave approximately as low-pass filters with a time constant of about 20 ms (depending on the nominal power), so they are sensitive to RMS values evaluated over a 20 ms sliding window. Applying the RMS definition to (2) or to (3) over a general window T, the following expression is obtained:

$$
\sqrt{\begin{aligned}
&\frac{1}{2}\Big\{\left(1+2a^2\right)+\frac{2}{T}\frac{1+2a^2}{2\omega_1}sin(\frac{T}{2}2\omega_1)cos[(t+\frac{T}{2})2\omega_1]+\\
&+\frac{2}{T}\frac{a^2}{2\omega_1}sin(\frac{T}{2}2(\omega_1+\Delta\omega)cos[(t+\frac{T}{2})2(\omega_1+\Delta\omega)]+\\
&+\frac{2}{T}\frac{a^2}{2\omega_1}sin(\frac{T}{2}2(\omega_1-\Delta\omega)cos[(t+\frac{T}{2})2(\omega_1-\Delta\omega)]+\\
&+\frac{2}{T}\frac{2a}{2\omega_1+\Delta\omega}sin\left[\frac{T}{2}(2\omega_1+\Delta\omega)\right]cos\left[(t+\frac{T}{2})(2\omega_1+\Delta\omega)\right]+\\
&+\frac{2}{T}\frac{4a}{\Delta\omega}sin\left[\frac{T}{2}(\Delta\omega)\right]cos\left[(t+\frac{T}{2})(\Delta\omega)\right]+\\
&+\frac{2}{T}\frac{2a}{2\omega_1-\Delta\omega}sin\left[\frac{T}{2}(2\omega_1-\Delta\omega)\right]cos\left[(t+\frac{T}{2})(2\omega_1-\Delta\omega)\right]+\\
&+\frac{2}{T}\frac{2a^2}{2\Delta\omega}sin\left[\frac{T}{2}(2\Delta\omega)\right]cos\left[(t+\frac{T}{2})(2\Delta\omega)\right]\Big\}
\end{aligned}} \quad (20)
$$

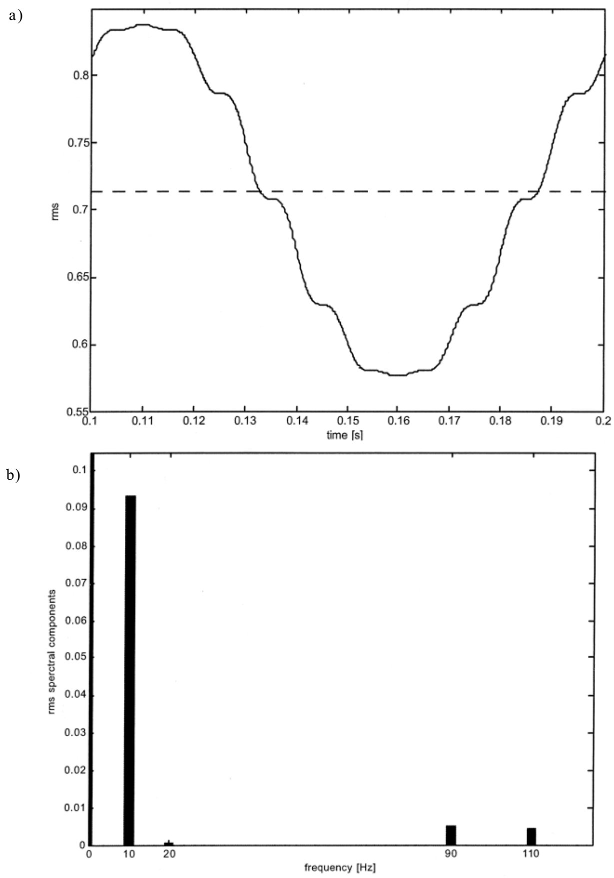

Fig. 5. Case-study 2: a) RMS value calculated over a 20 ms shifting window versus the time (—) and RMS value calculated from Fourier expansion over 100 ms (-.-.-); b) spectrum of curve (—)

The formula with $T = 20.00$ ms, gives the RMS reported in Figure 5. It is constituted by a DC component, a sinusoidal component at 10 Hz, and other components that are harmonics of 10 Hz.

It is worthwhile to note that the component at 10 Hz of the RMS clearly shows the presence of conditions of high Light Flicker PU values. The constant value for RMS obtained by

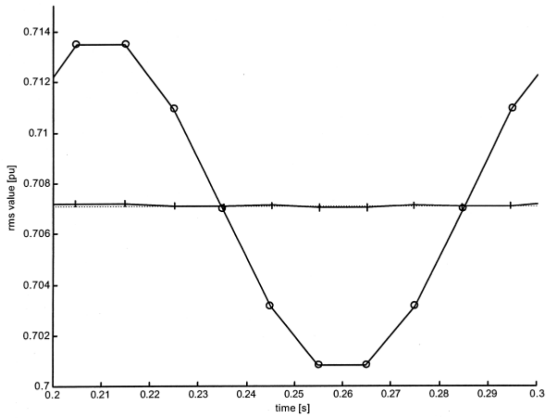

Fig. 6. Case-study 3: 1.0 pu fundamental with two superimposed interharmonics causing PM (0.1 rad @ 10 Hz): (+) RMS values of the modulated signal in the presence of an ideal PLL; (o) RMS value evaluated with reference to the period of the fundamental power frequency carrier 20.00 ms.

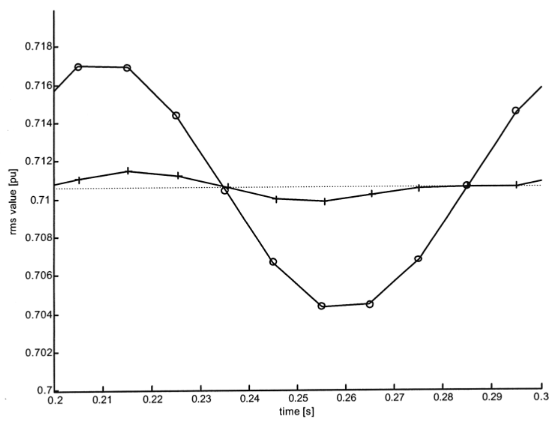

Fig. 7. Case-study 3: same signal of Figure 6 with a third harmonic (0.1 pu) synchronized with the fundamental carrier signal: (+) RMS values of the whole signal in the presence of an ideal PLL; (o) RMS value evaluated with reference to the period of the fundamental power frequency carrier 20.00 ms; (:) true RMS value evaluated over 100.00 ms time window.

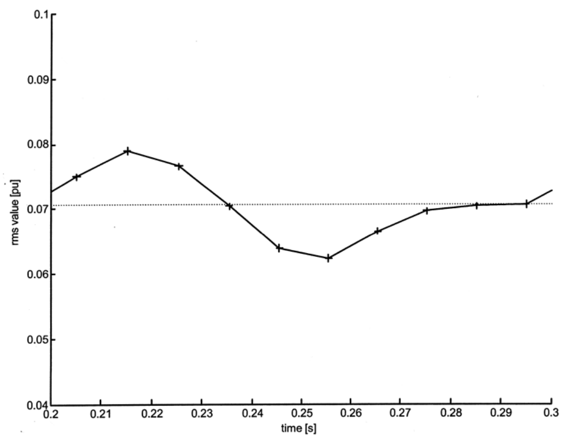

Fig. 8. Case-study 3: Third harmonic (0.1pu) synchronized with the fundamental carrier signal: (+) RMS values of the third harmonic in the presence of an ideal PLL; (o) RMS value evaluated with reference to the period of the fundamental power frequency carrier 20.00 ms; (:)true RMS value evaluated over 100.00 ms time window.

the Fourier series expansion over $T = 100.00$ ms (Fig. 5a) completely masks the presence of light flicker conditions.

4.3. Case-study 3: two interharmonic tones causing phase modulation

A simple example will be considered consisting of two superimposed interharmonics at frequencies of 40 and 60 Hz respectively (0.1 pu, $\varphi_{i1} = 0$, $\varphi_{i2} = \pi/2$); furthermore, in the second stage there is also a third harmonic (0.1 pu @ 150.00 Hz).

It is interesting to analyze the effects of the desynchronization with respect to the fundamental period caused by the PM of the fundamental component. Two extreme reference conditions are considered for the sake of simplicity to evaluate RMS values: a) the presence of an ideal PLL that exactly follows the instantaneous variation of phase angle (and of the frequency) of the modulated signal; b) the presence of a perfect synchronization with the frequency of the carrier signal.

Figure 6 shows the RMS values of the modulated signal (fundamental + interharmonics) in both conditions a) and b).

It is worthwhile to note that the tuning action caused by the PLL gives the possibility of an accurate evaluation of the RMS instant by instant. On the other hand, RMS evaluation tuned with the frequency of the carrier component is affected by inaccuracies that depend on the period considered and that assume null values only around instants in which there is a sort of compensation, which are met two times along a Fourier period of 100.00 ms. The results are characterized by oscillations around the exact value and so their mean over a long interval of observation gives compensation effects (smoothing).

Figure 7 shows the RMS values of the modulated signal (fundamental + interharmonics) with the presence also of the third harmonic (0.1 pu @ 150.00 Hz). RMS values are evaluated once again in both conditions a) and b).

It is worthwhile to note that as the PLL tunes the instantaneous frequency of the modulated fundamental, so the third harmonic appears desynchronized. This causes the presence of inaccuracies in the results compared to the exact RMS value that is obtained with reference to the 100.00 ms period. For RMS evaluation tuned with the frequency of the carrier component, the same considerations applied to Figure 6 apply also in this case.

Figure 8 shows the same curves of Figure 7 with reference to the RMS value of the third harmonic only, and evidences the inaccuracies introduced by the PLL synchronization with the PM fundamental.

5. PRACTICAL APPLICATIONS

The usefulness of the modulation representation remains to be demonstrated, since the Fourier representation is the most popular among power system engineers.

Practical applications are:

- Light Flicker assessment;
- determination of limits for low frequency (0–100 Hz) interharmonics;
- development of robust techniques for harmonic and interharmonic measurement;

— active filter design;
— power electronic conversion apparatus design;
— measurement of true RMS values of currents and voltages and of powers and energies.

The applications mentioned for the first three points are reported in [4–5].

6. CONCLUSIONS

Amplitude and Phase Modulation effects of waveform distortion in power systems have been analyzed. Recalls on AM and PM have been given with particular reference to spectral components. Then, simple inverse formulas were obtained to demonstrate that summations of one or more small tones to a given tone of interest can always be interpreted in terms of AM and PM. The usefulness of AM and PM representation, in particular in the presence of interharmonic tones, has been demonstrated with reference to simple case-studies and practical applications.

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APPENDIX A

A.1. Phase modulation

The analytical expression of a phase modulated signal is:

$$u(t) = cos\left[\omega_1 t + \varphi_{MAX} \cdot cos\left(\Delta\omega t + \pi/2 - \varphi_{i1}\right)\right] \quad (A.1)$$

by expanding the outer cosine and manipulating, it becomes:

$$\begin{aligned} u(t) &= J_0(\varphi_{MAX}) cos(\omega_1 t) + \\ &+ \sum_{h=1}^{\infty} J_h(\varphi_{MAX}) \left\{ cos\left[(\omega_1 - h\Delta\omega)t + h\varphi_{i1}\right] + \right. \\ &\left. + cos\left[(\omega_1 + h\Delta\omega)t + h(\pi - \varphi_{i1})\right] \right\} \end{aligned} \quad (A.2)$$

with:

$$\begin{aligned} J_h(\varphi_{MAX}) &= \frac{\varphi_{MAX}^h}{2^h h!}\left(1 - \frac{\varphi_{MAX}^2}{2 \cdot (2h+2)} + \frac{\varphi_{MAX}^4}{2 \cdot 4(2h+2)(2h+4)} - \ldots\right) = \\ &= \sum_{k=0}^{\infty} \frac{(-1)^k (\varphi_{MAX}/2)^{h+2k}}{k!\,\Gamma(h+k+1)} \end{aligned} \quad (A.3)$$

the Bessel function of the first kind of order h and Γ denotes the Gamma function [2].

The expression of the spectrum is:

$$\begin{aligned} u(t) &= cos\left[\omega_1 t + \varphi_{MAX} cos\left(\Delta\omega t + \pi/2 - \varphi_{i1}\right)\right] = +\left(1 - \frac{\varphi_{MAX}^2}{4}\right) cos(\omega_1 t) + \\ &+ \frac{\varphi_{MAX}}{2}\left(1 - \frac{\varphi_{MAX}^2}{8}\right)\left\{cos\left[(\omega_1 - \Delta\omega)t + \varphi_{i1}\right] + cos\left[(\omega_1 + \Delta\omega)t + \pi - \varphi_{i1}\right]\right\} + \ldots + \\ &+ \frac{\varphi_{MAX}^h}{2^h h!}\left(1 - \frac{\varphi_{MAX}^2}{2(2h+2)}\right)\left\{cos\left[(\omega_1 - h\Delta\omega)t + h\varphi_{i1}\right] + cos\left[(\omega_1 + h\Delta\omega)t - h\varphi_{i1}\right]\right\} + \ldots. \end{aligned} \quad (A.4)$$

A.2. Prevailing Phase modulation

Once assuming $\varphi_{MAX} = \arctan(2a) \approx 2a$, for very small values of a, (A.4) can be rewritten as:

$$\begin{aligned} u(t) &\cong \left(1 - \frac{4a^2}{4}\right) cos(\omega_1 t) + \\ &+ a\left(1 - \frac{1}{2}a^2\right)\left\{cos\left[(\omega_1 - \Delta\omega)t + \varphi_{i1}\right] + cos\left[(\omega_1 + \Delta\omega)t + \pi - \varphi_{i1}\right]\right\} + \\ &+ \frac{4}{8}a^2\left(1 - \frac{4}{12}a^2\right)\left\{cos\left[(\omega_1 - 2\Delta\omega)t + 2\varphi_{i1}\right] + cos\left[(\omega_1 + 2\Delta\omega)t - 2\varphi_{i1}\right]\right\} \end{aligned} \quad (A.5)$$

neglecting the terms for $h > 2$. After some trivial algebra (A.5), after discarding the terms a^3 and a^4, it becomes:

$$\begin{aligned} u(t) &\cong cos(\omega_1 t) + a\left\{cos\left[(\omega_1 - \Delta\omega)t + \varphi_{i1}\right] + cos\left[(\omega_1 + \Delta\omega)t + \pi - \varphi_{i1}\right]\right\} - \\ &- a^2\left\{cos(\omega_1 t) - \frac{4}{8}\left\{cos\left[(\omega_1 - 2\Delta\omega)t + 2\varphi_{i1}\right] + cos\left[(\omega_1 + 2\Delta\omega)t - 2\varphi_{i1}\right]\right\}\right\} \end{aligned} \quad (A.6)$$

Rearranging, it is:

$$u(t) \cong cos\left[\omega_1 t + 2a \cdot cos\left(\Delta\omega t + \pi/2 - \varphi_{i1}\right)\right] + \\ + a^2\left[1 - cos\left(2\Delta\omega t - 2\varphi_{i1}\right)\right] cos\left(\omega_1 t\right) \tag{A.7}$$

For a<<1, the first term is prevailing and (A.7) yields:

$$u(t) \cong cos\left[\omega_1 t + 2a \cdot cos\left(\Delta\omega t + \pi/2 - \varphi_{i1}\right)\right] \tag{A.8}$$

which is a perfect PM.

A.3. Single interharmonic tone

The analytical expression:

$$u(t) = cos\left(\omega_1 t\right) + a\, cos\left[\left(\omega_1 - \Delta\omega\right)t + \varphi_{i1}\right] \tag{A.9}$$

by summing and subtracting the quantity:

$$\frac{a}{2} cos\left[\left(\omega_1 + \Delta\omega\right)t - \varphi_{i1}\right] \tag{A.10}$$

and rearranging, becomes:

$$u(t) = \frac{1}{2} cos\left(\omega_1 t\right) + \frac{a}{2}\left\{cos\left[\left(\omega_1 - \Delta\omega\right)t + \varphi_{i1}\right] + cos\left[\left(\omega_1 + \Delta\omega\right)t - \varphi_{i1}\right]\right\} + \\ + \frac{1}{2} cos\left(\omega_1 t\right) + \frac{a}{2}\left\{cos\left[\left(\omega_1 - \Delta\omega\right)t + \varphi_{i1}\right] + cos\left[\left(\omega_1 + \Delta\omega\right)t + \pi - \varphi_{i1}\right]\right\} \tag{A.11}$$

Considering (2) and (A.8), it is:

$$u(t) \cong \frac{1}{2}\left[1 + 2a\, cos\left(\Delta\omega t - \varphi_{i1}\right)\right] cos\left(\omega_1 t\right) + \\ + \frac{1}{2} cos\left[\omega_1 t + 2a\, cos\left(\Delta\omega t + \pi/2 - \varphi_{i1}\right)\right] \tag{A.12}$$

A.4. Two symmetric components

The analytical expression considered:

$$u(t) = cos\left(\omega_1 t\right) + a_1\, cos\left[\left(\omega_1 - \Delta\omega\right)t + \varphi_{i1}\right] + \\ + a_2\, cos\left[\left(\omega_1 + \Delta\omega\right)t + \varphi_{i2}\right] \tag{A.13}$$

can be rewritten as:

$$u(t) = \frac{1}{2} cos\left(\omega_1 t\right) + a_1\, cos\left[\left(\omega_1 - \Delta\omega\right)t + \varphi_{i1}\right] + \\ + \frac{1}{2} cos\left(\omega_1 t\right) + a_2\, cos\left[\left(\omega_1 + \Delta\omega\right)t + \varphi_{i2}\right] \tag{A.14}$$

which corresponds to the summation of two terms as (A.9). By using (A.12), it is:

$$u(t) = \frac{1}{4}\left[1 + 2a_1\, cos\left(\Delta\omega t - \varphi_{i1}\right)\right] cos\left(\omega_1 t\right) + \\ + \frac{1}{4}\left[1 + 2a_2\, cos\left(\Delta\omega t + \varphi_{i2}\right)\right] cos\left(\omega_1 t\right) + \\ + \frac{1}{4} cos\left[\omega_1 t + 2a_1 \cdot cos\left(\Delta\omega t + \pi/2 - \varphi_{i1}\right)\right] + \\ + \frac{1}{4} cos\left[\omega_1 t + 2a_2 \cdot cos\left(\Delta\omega t + \pi/2 - \varphi_{i2}\right)\right] \tag{A.15}$$