# **Compensation Techniques Based on Reactive Power Conservation**

# Paolo TENTI, Paolo MATTAVELLI and Elisabetta TEDESCHI

University of Padova, ITALY

Summary: The paper introduces two definitions of instantaneous reactive power terms, which are conservative in every network and apply even under non-sinusoidal conditions. Both quantities coincide with usual reactive power for sinusoidal operation, however they assume a different meaning for distorted behavior. The first one is an energy-related term, and its unbiased integral accounts for the average energy stored in the network. The second has a differential nature and is suitable for dynamic analysis and control of harmonic and reactive power sources acting in the network. The combined use of both approaches allows cooperative operation of static and dynamic reactive and harmonic compensators, like STATCOMs and Active Power Filters.

#### 1. INTRODUCTION

Active power filters are flexible tools to provide widespread harmonic and reactive compensation, including supply of leading and lagging reactive power and compensation of current harmonics and unbalance generated by non-linear and time-varying loads. In high power applications, however, cost-effective solutions cannot use active filters only; rather, they may use hybrid systems made up of a combination of passive filters, thyristor-controlled VAR compensators and active power filters.

Design and control of active or hybrid filters have been deeply investigated in the past [1, 6, 9, 10, 11]. Most of active filter control strategies are based on current compensation techniques, e.g., non-active current compensation [6] and instantaneous power compensation [1]. Other approaches are based on supply current detection methods, where active filter currents are controlled by a feedback loop on supply currents, either using fast control or selective harmonic compensation [8]. Compensation strategies based only on voltage detection methods have also been proposed in order to reduce the interaction between active power filters and capacitive loads [2-3].

A common limitation of the above mentioned compensation techniques is that the compensator must be connected at the load terminals (load compensation) or at the point of common coupling (system compensation), while a synergistic action performed by compensating devices distributed in the network (distributed compensation) is very difficult to achieve. An example of coordinated control of multiple active filters based on a voltage detection method is reported in [7], showing that distributed compensation is indeed a challenging issue.

This paper proposes a compensation approach based on the definition of two instantaneous reactive power quantities, which are conservative and allow cooperative operation of more compensation units acting in the same network.

The first definition is based on integral variables and relates to the average energy stored in the network. It provides a powerful tool for concurrent analysis and control of distributed compensation units in a quasi-stationary time frame.

The second definition is based on differential variables and allows a dynamic control of the compensation units so as to eliminate non-active current components.

# 2. THEORETICAL BACKGROUND

Terms and properties referred hereafter have been introduced in a previous paper [12], although some quantities have been renamed.

# 2.1. Reactive power terms definition

The theory presented in [12] refers to the case of nonsinusoidal periodic operation with period T and angular

frequency  $\omega = \frac{2\pi}{T}$ . The main instantaneous quantities referred in the paper are:

*Instantaneous reactive power q*, defined by:

$$q = \frac{\widehat{u}i - u\widehat{i}}{2} \tag{1}$$

Differential reactive power r, defined by:

$$r = \frac{\check{u}i - u\check{i}}{2} \tag{2}$$

In (1), variable  $\hat{u}$  is called (unbiased) integral voltage and is given by:

$$\widehat{u}(t) = \omega \left( \psi \left( t \right) - \overline{\psi} \right) \tag{3}$$

$$\psi(t) = \int_0^t u(\tau)d\tau$$
 is integral of voltage  $u$ , and  $\bar{\psi} = \int_0^T \psi(\tau)d\tau$  is bias value of  $\psi$ .

$$\bar{\psi} = \int_{0}^{T} \psi(\tau) d\tau$$
 is bias value of  $\psi$ .

Similarly we define the (unbiased) integral current  $\hat{i}$ . Note that variables  $\tilde{u}$  and  $\tilde{i}$  are dimensionally homogeneous to a voltage and a current, respectively.

In (2), variables  $\check{u}$  and  $\check{i}$  are called *differential voltage* and *differential current*, respectively, and are given by:

$$\widetilde{u} = \frac{1}{\omega} \frac{du}{dt}, \quad \widetilde{i} = \frac{1}{\omega} \frac{di}{dt}$$
(4)

Note that  $\check{u}$  and  $\check{i}$  are dimensionally homogeneous to a voltage and a current, respectively.

It is easy to verify that under sinusoidal operation (i.e., for sinusoidal voltages and currents) variables q and r coincide, are constant and their value equals, at any time, reactive power Q.

In addition, defining the internal product between generic variables x(t) and y(t) as:

$$\langle x, y \rangle = \frac{1}{T} \int_0^T x(t) y(t) dt$$

and the norm of variable x(t) as:

$$||x|| = \sqrt{\langle x, x \rangle} = \sqrt{\frac{1}{T} \int_0^T x^2(t) dt}$$

we can demonstrate that quantities defined according to (3) and (4) have the following properties:

$$y = \hat{x} \iff \tilde{y} = x$$

$$y = \tilde{x} \iff \hat{y} = x$$

$$\langle x, \hat{x} \rangle = \langle x, \tilde{x} \rangle = 0$$

$$\langle \hat{x}, \tilde{x} \rangle = -||x||^{2}$$

$$\langle x, \hat{y} \rangle = -\langle \hat{x}, y \rangle$$

$$\langle x, \tilde{y} \rangle = -\langle \tilde{x}, y \rangle$$

$$\langle \hat{x}, \tilde{y} \rangle = \langle \tilde{x}, \hat{y} \rangle = -\langle x, y \rangle$$
(5)

Moreover, expressing variable x(t) by its Fourier series:

$$x(t) = \sum_{k=1}^{\infty} x_k(t) = \sum_{k=1}^{\infty} \sqrt{2} X_k \sin(k\omega t + \alpha_k)$$

we have:

$$\hat{x}(t) = \sum_{k=1}^{\infty} \hat{x}_k(t) = -\sum_{k=1}^{\infty} \sqrt{2} \frac{X_k}{k} \cos(k\omega t + \alpha_k)$$

$$\widetilde{x}(t) = \sum_{k=1}^{\infty} \widetilde{x}_k(t) = \sum_{k=1}^{\infty} \sqrt{2kX_k} \cos(k\omega t + \alpha_k)$$
 (6)

where  $X_k$  is the rms value of the k-th harmonic term.

# 3.2. Conservation of reactive power

For any given network, the Tellegen's theorem ensures that for every set of branch voltages  $\underline{u}$  and branch currents  $\underline{i}$  consistent with the network we can write:

$$\sum_{\ell=1}^{L} u_{\ell} i_{\ell} = \underline{u} \cdot \underline{i} = 0$$

where L is number of branches in the network and the dot means scalar product.

Since voltage and current terms like those defined by (3) and (4) comply with Kirchhoff's Laws (KLC applies to current-like terms, while KLV applies to voltage-like terms), we can write:

$$\underline{u} \cdot \underline{i} = \underline{\hat{u}} \cdot \underline{i} = \underline{\check{u}} \cdot \underline{i} = 0$$

$$\underline{u} \cdot \underline{\hat{i}} = \underline{\hat{u}} \cdot \underline{\hat{i}} = \underline{\check{u}} \cdot \underline{\hat{i}} = 0$$

$$\underline{u} \cdot \check{i} = \underline{\hat{u}} \cdot \underline{\hat{i}} = \underline{\check{u}} \cdot \underline{\hat{i}} = 0$$

$$\underline{u} \cdot \check{i} = \underline{\hat{u}} \cdot \underline{\hat{i}} = \underline{\check{u}} \cdot \underline{\hat{i}} = 0$$
(7)

In other words, every power-like term is conservative in any given network. Consequently, reactive power terms q and r defined by (1) and (2) are conservative too.

This implies that the instantaneous and differential reactive power absorbed at the input ports of a network equal the sum of all corresponding reactive power terms absorbed by the network branches.

# 2.3 Basic theorem of compensation

Consider any port of a single-phase network, *u* and *i* being the corresponding voltage and current. The theorem claims that *if either instantaneous or differential reactive power absorbed at that port are identically zero, then port current and voltage are proportional.* 

In fact, assuming that differential reactive power r is zero we have:

$$r(t) = 0 \Rightarrow i \frac{du}{dt} = u \frac{di}{dt} \Rightarrow \frac{du}{u} = \frac{di}{i}$$

By integration, this equation gives:

$$ln(u) = ln(i) + K \implies ln(\frac{u}{i}) = K \implies \frac{u}{i} = e^{K}$$

Thus, port current is proportional to the port voltage. A similar demonstration holds for instantaneous reactive power q.

In case of a poly-phase network, the theorem applies to each phase independently.

# 2.4 Harmonic reactive power terms

Reactive power terms (1) and (2) can be integrated along period T, giving:

$$\overline{q} = \frac{1}{T} \int_{0}^{T} q dt = \frac{\langle \widehat{u}, i \rangle - \langle u, \widehat{i} \rangle}{2} = \langle \widehat{u}, i \rangle = -\langle u, \widehat{i} \rangle$$

$$\overline{r} = \frac{1}{T} \int_{0}^{T} r dt = \frac{\langle u, \widecheck{i} \rangle - \langle \widecheck{u}, i \rangle}{2} = \langle u, \widecheck{i} \rangle = -\langle \widecheck{u}, i \rangle$$
(8)

Considering the harmonic terms we find:

$$\overline{q} = \sum_{k=1}^{\infty} \overline{q}_k = \sum_{k=1}^{\infty} \left\langle \widehat{u}_k, i_k \right\rangle = \frac{U_k I_k}{k} \sin \varphi_k$$

$$\overline{r} = \sum_{k=1}^{\infty} \overline{r_k} = \sum_{k=1}^{\infty} \left\langle u_k, \widetilde{i_k} \right\rangle = kU_k I_k \sin \varphi_k \tag{9}$$

where  $U_k$  and  $I_k$  are the rms values of the k-th harmonic voltage and current, respectively, and  $\varphi_k$  is the corresponding phase angle. The above equation shows that under sinusoidal conditions both  $\bar{q}$  and  $\bar{r}$  coincide with usual reactive power  $U_1I_1sin\varphi_1$ , while for distorted operation they have different expressions and no one coincides with Budeanu's harmonic reactive power:

$$Q_B = \sum_{k=1}^{\infty} Q_{Bk} = \sum_{k=1}^{\infty} U_k I_k \sin \varphi_k$$

However, unlike Budeanu's reactive power, both  $\bar{q}$  and  $\bar{r}$  are conservative quantities can be computed in the time domain and have a physical meaning.

In the following we will refer to average term  $\bar{q}$  as (generalized) reactive power Q.

# 2.5 Reactive power and stored energy

Let's now analyze the properties of reactive power Q in the case of elementary linear passive bipoles.

— For a *resistor* we have:

$$i = G \quad u \Leftrightarrow q = 0 \Rightarrow O = 0$$
 (10.a)

— For an *inductor* we have:

$$u = \omega \ L \ \check{i} \Rightarrow q_L = \omega \ L \frac{i^2 - \hat{i} \ \check{i}}{2} \Rightarrow Q_L = \omega \ L \|i\|^2$$

Considering that average energy in the inductor is:

$$W_L = \frac{1}{T} \int_0^T \frac{1}{2} L i^2 dt = \frac{1}{2} L ||i||^2$$

we have:

$$Q_L = 2\omega W_L \tag{10.b}$$

— For a *capacitor* we have:

$$i = \omega C \widecheck{u} \Rightarrow q_C = \omega C \frac{\widehat{u} - \widecheck{u}^2}{2} \Rightarrow Q_C = -\omega C \|u\|^2$$

Considering that average energy in the capacitor is:

$$W_C = \frac{1}{T} \int_0^T \frac{1}{2} C u^2 dt = \frac{1}{2} C \|u\|^2$$

we have:

$$Q_C = -2\omega W_C \tag{10.c}$$

From the above considerations it follows that under periodic operation, either sinusoidal or distorted, reactive power *Q* measured at the input port of a passive linear network is given by:

$$Q = 2\omega (W_L - W_C) \tag{11.a}$$

where  $W_L$  and  $W_C$  are respectively total average inductive energy and total average capacitive energy in the network, i.e.

$$W_{L} = \sum_{k=1}^{N_{L}} W_{Lk} = \sum_{k=1}^{N_{L}} \frac{1}{2} L_{k} I_{Lk}^{2}$$

$$W_{C} = \sum_{k=1}^{N_{C}} W_{Ck} = \sum_{k=1}^{N_{C}} \frac{1}{2} C_{k} U_{Ck}^{2}$$
(11.b)

 $L_k$  being all network inductors and  $I_{Lk}$  their rms currents. Similarly for capacitors  $C_k$ . Variable Q represents therefore the natural extension of the reactive power defined for sinusoidal conditions.

#### 3. CURRENT DECOMPOSITION

# 3.1 Active and reactive current terms

For any given network port, *active current*  $i_p$  is defined as the minimum current (i.e. the current with minimum norm) conveying active power P absorbed from the network at that port. It is given by:

$$i_p = \frac{\langle u, i \rangle}{\|u\|^2} u = \frac{P}{\|u\|^2} u = G_e u$$
 (12.a)

Symmetrically, we define reactive current  $i_q$  as the minimum current conveying reactive power Q absorbed from the network at that port. It is given by:

$$i_{q} = \frac{\left\langle \widehat{u}, i \right\rangle}{\left\| \widehat{u} \right\|^{2}} \widehat{u} = \frac{Q}{\left\| \widehat{u} \right\|^{2}} \widehat{u} = B_{e} \widehat{u}$$
 (12.b)

In the above equations,  $G_e$  is the equivalent port conductance and  $B_e$  the equivalent port susceptance.

# 3.2. Current decomposition

Based on the above definitions, we decompose the port current as:

$$i = i_p + i_a + i_v \tag{13.a}$$

where  $i_v$  is called *void current*, since it is not conveying active power nor reactive power. It is easily demonstrated that all current terms are orthogonal, thus:

$$\|i\|^2 = \|i_a\|^2 + \|i_a\|^2 + \|i_v\|^2$$
 (13.b)

The meaning of the void current can be explained by developing the port current in its Fourier series.

Let  $\{K_i\}$  be the set of harmonics of current i,  $\{K_u\}$  the set of harmonics of voltage u and  $\{K\}$  the common set  $\{K_u \cap K_i\}$  of coexisting voltage and current harmonics, we can write:

$$i(t) = \sum_{k \in \{K\}} i_k(t) + \sum_{k \in \{K_i - K\}} i_k(t) = i_h + i_g$$

$$u(t) = \sum_{k \in \{K\}} u_k(t) + \sum_{k \in \{K_u - K\}} u_k(t) = u_h + u_g$$
(14)

where  $i_h$  and  $u_h$  include every *coexisting* voltage and current harmonics, while  $i_g$  and  $u_g$  collect all remaining terms, which are called *generated voltage* and *generated current*, respectively. These definitions are in agreement with those proposed by Czarnecki in his fundamental papers on power and current terms decomposition under non-sinusoidal conditions [4,5].

For each coexisting voltage and current harmonics, we can calculate the corresponding active and reactive currents according to (12):

$$i_{kp} = \frac{\left\langle u_{k}, i_{k} \right\rangle}{\left\| u_{k} \right\|^{2}} u_{k} = \frac{P_{k}}{U_{k}^{2}} u_{k} = G_{k} u_{k}$$

$$i_{kq} = \frac{\left\langle \widehat{u}_{k}, i_{k} \right\rangle}{\left\| \widehat{u}_{k} \right\|^{2}} \widehat{u}_{k} = \frac{k^{2} Q_{k}}{U_{k}^{2}} \widehat{u}_{k} = B_{k} \widehat{u}_{k}$$
(15.a)

where  $P_k$  and  $Q_k$  are the harmonic active and reactive power terms. Let  $\varphi_k$  be the phase displacement at the k-th harmonics, we have:

$$\begin{split} P_k &= U_k I_k \cos \varphi_k \quad with: \sum_{k \in K} P_k = P \\ Q_k &= \frac{U_k I_k \sin \varphi_k}{k} \quad with: \sum_{k \in K} Q_k = Q \end{split} \tag{15.b}$$

$$G_{k} = \frac{I_{k}}{U_{k}} \cos \varphi_{k}$$

$$B_{k} = \frac{kI_{k}}{U_{k}} \sin \varphi_{k}$$
(15.c)

Observing that for every  $k \in K$  we have  $i_k = i_{kp} + i_{kq}$ , port current i can be expressed as:

$$i = \sum_{k \in \{K\}} i_{kp} + \sum_{k \in \{K\}} i_{kq} + i_g = i_{hp} + i_{hq} + i_g$$
 (16)

where  $i_{hp}$  is (total) harmonic active current and  $i_{hq}$  is (total) harmonic reactive current. Note that  $i_{hp}$  and  $i_{hq}$  do not generally coincide with  $i_p$  and  $i_q$ . Since, by

definition,  $\|i_p\| \le \|i_{hp}\|$  and  $\|i_q\| \le \|i_{hq}\|$ , we define the scattering currents as:

$$i_{sp} = i_{hp} - i_p$$
 active scattering current  
 $i_{sa} = i_{ha} - i_a$  reactive scattering current (17)

Substituting these terms in (16), we have:

$$i = i_p + i_q + i_{sp} + i_{sq} + i_g$$
 (18.a)

All the above currents are orthogonal, thus:

$$\|i\|^2 = \|i_p\|^2 + \|i_q\|^2 + \|i_{sp}\|^2 + \|i_{sq}\|^2 + \|i_g\|^2$$
 (18.b)

By comparing (13.a) and (18.a) we see that:

$$i_{v} = i_{so} + i_{so} + i_{o} \tag{19}$$

showing that the void current includes active and reactive scattering currents and generated current.

The above notations hold both for single-phase and polyphase networks; in the latter case, however, scalar quantities are substituted by vector quantities.

## 4. COMPENSATION AIM AND METHODS

#### 4.1. Localized compensation

Generally, compensation is aimed at increasing the power factor  $\lambda$  at a given port of the grid. The power factor is defined as:

$$\lambda = \frac{P}{S} = \frac{\langle u, i \rangle}{\|u\| \|i\|} = \frac{\|i_p\|}{\|i\|}$$
 (20)

and approaches unity only if the absorbed current approaches active current  $i_p$ . This means that an ideal compensation system should compensate for all remaining current components.

This is the basis of the *localized compensation* method, which makes use of compensation systems (STATCOM and/or filters, active or passive) connected directly at the input port of the network. With this approach, the compensation system is driven to absorb a current  $i^c$  opposite to the unwanted load current components, i.e.:

$$i^C = -i_a - i_v \tag{21.a}$$

so that total current absorbed at the input port becomes:

$$i_i = i + i^C = i_p \tag{21.b}$$

# 5.2. Distributed compensation

A different approach applies in case of *distributed* compensation, where the compensation task is performed

by one or more units connected to various network ports, as shown in Figure 1.

In this case, compensation is possible only by making reference to conservative quantities, which add over the network thus determining total absorption at the input port.

In this perspective, the above theory provides a viable solution. In fact, according to the basic compensation theorem, unity power factor is obtained if the compensation system absorbs, as a whole, an instantaneous reactive power q (or a differential reactive power r) opposite to that absorbed by the non-compensated system. Since q and r are conservative quantities, the needed reactive power can be shared among different compensation units connected at various network ports.

Figure 1 illustrates the general principle of distributed compensation. Network  $\pi$ , absorbing instantaneous reactive power  $q^{\pi}$  at the input port, is compensated by a set of compensators  $C_1$ – $C_N$  so that:

$$q^{C} = \sum_{n=1}^{N} q_{n}^{C} = -q^{\pi}$$
 (22)

 $q_n^C$  being the instantaneous reactive power absorbed by the n-th unit of the compensating system.

Similar considerations apply to differential reactive power *r*. Note that, according to this principle, the distributed compensation system performs like a localized compensator, since reactive and void currents at the input port vanish.

In the following we will consider two compensation approaches, the first applicable to slow-varying networks (quasi-stationary compensation), the second applicable also in case of fast network dynamics (dynamic compensation).

# 5. QUASI-STATIONARY COMPENSATION

The compensation problem can be analyzed in the frequency domain, assuming slow network variations (*quasistationary operation*).

For this purpose, let's consider separately the current components which need to be compensated.

— Reactive current  $i_q$ . Reactive current  $i_q$  vanishes if total reactive power Q absorbed at the input port is zero. Thus, compensation of  $i_q$  requires that distributed compensators, as a whole, absorb a reactive power which is opposite to that of the non-compensated network, i.e:

$$\sum_{n=1}^{N} Q_n^C = -Q \iff i_q = 0$$
 (23)

— Active scattering current . From (15) and (17) we obtain:

$$i_{sp} = i_{hp} - i_p = \sum_{k \in \{K\}} (G_k - G_e) u_k = \sum_{k \in \{K\}} \frac{P_{sk}}{\|u_k\|^2} u_k$$
 (24.a)

where:

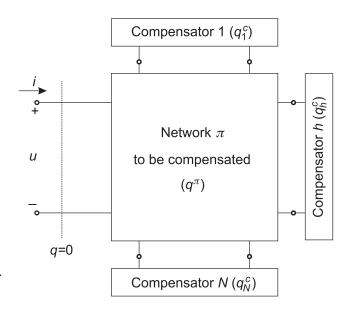


Fig. 1. Distributed compensation system

$$P_{sk} = P_k - \frac{\|u_k\|^2}{\|u\|^2} P$$
 (24.b)

The condition for  $i_{sp}$  to vanish is that each of its harmonic components is driven to zero, i.e., that the compensation system absorbs, for each harmonic, an active scattering power  $P_{sk}^C$  opposite to scattering active power  $P_{sk}$  absorbed by the non-compensated network:

$$P_{sk}^{C} = \sum_{n=1}^{N} P_{skn}^{C} = -P_{sk}$$
 (24.c)

Reactive scattering current i<sub>sq</sub>. From (15) and (17) we obtain:

$$i_{sq} = i_{hq} - i_q = \sum_{k \in \{K\}} (B_k - B_e) \hat{u}_k = \sum_{k \in \{K\}} \frac{Q_{sk}}{\|\hat{u}_k\|^2} \hat{u}_k$$
 (25.a)

where:

$$Q_{sk} = Q_k - \frac{\|\hat{u}_k\|^2}{\|\hat{u}\|^2} Q$$
 (25.b)

The condition for  $i_{sq}$  to vanish is therefore that each of its harmonic components is driven to zero, i.e., that the compensation system absorbs, for each harmonic, a reactive scattering power  $Q_{sk}^C$  opposite to scattering reactive power  $Q_{sk}$  absorbed by the non-compensated network:

$$Q_{sk}^{C} = \sum_{n=1}^{N} Q_{skn}^{C} = -Q_{sk}$$
 (25.c)

Generated current . Since this current is not involved with active nor reactive power terms, it cannot be compensated by controlling the power terms absorbed by distributed compensators. A dynamic compensation is required, as described in the following section.

#### 6. DYNAMIC COMPENSATION

The basic theorem of compensation claims that purely active currents are absorbed by a network taking zero differential reactive power at the input port, i.e.:

$$r = 0 \Leftrightarrow i = G \quad u$$

This means that, in order to avoid non-active current absorption, the compensation system should absorb, as a whole, a differential reactive power opposite to that absorbed by the remaining network.

In the general case depicted in Figure 1, each compensation unit (active filter  $C_n$ ) should therefore be driven to provide a suitable portion  $r_n^C$  of total reactive power  $r^\pi$  absorbed by the non-compensated network. Sharing of the compensation duty among different units can be tentatively done in proportion of their rated power. The sharing algorithm deserves however a deeper investigation, since network response to compensators action is influenced by network dynamics too.

For this purpose, the active filter controller must transform its reactive power reference  $r_n^*$  into a suitable current reference  $i_n^*$ . This requires solution of the differential equation:

$$u_n \frac{di_n^*}{dt} - i_n \frac{du_n}{dt} = 2\omega r_n^*$$
 (26)

where  $u_n$  is the voltage feeding unit  $C_n$ . Solution of (26) in the continuous time domain is not possible for whichever behaviour of function  $r_n^*(t)$ . Instead, (26) can be solved in the discrete time domain, giving:

$$i_n^*(t_{k+1}) = i_n(t_k) + \frac{2 \omega T_s r_n^*(t_{k+1}) + \left[u_n(t_k) - u_n(t_{k-1})\right] i_n(t_k)}{u_n(t_k)}$$
(27)

where  $T_s$  is sampling interval and  $t_k$  the k-th sampling time.

Equation (27) shows that computation of current reference can diverge when supply voltage approaches zero; this problem, however, can be solved by modifying the computation algorithm around voltage zeroing.

Another problem may occur because the differential reactive power is insensitive to active current components. This means that the current reference given by (27) may include an active current term which must be identified and eliminated in order to avoid active power absorption by the active filter. This may introduce some control delay, affecting the dynamic response.

Lastly, a perfect compensation would require an estimation of the differential reactive power absorbed by parasitic inductances of network connections.

Based on the above considerations, a digital control technique has been developed which provides feed-forward

control of the active filter currents, resulting in precise tracking of power reference  $r_n^*$ . Accordingly, cooperative operation of the distributed compensation units  $C_1 - C_N$  I can be achieved, so that total differential reactive power absorbed at the input port becomes zero and non-active currents vanish.

An application example is given hereafter.

#### 7. APPLICATION EXAMPLES

# 7.1. Example of quasi-stationary compensation

As an example of quasi-stationary compensation of a distributed compensation system [13], the simple test case of Figure 2 was simulated, with a network including a distribution line with one distorting load and two active filters A and B. The per unit line parameters are:

$$Z_{s1}$$
=0.01+j 0.01 pu,  $Z_{sA}$ = 0.03+j 0.03 pu

$$Z_{SB}$$
=0.01+j 0.01 pu,  $Z_{LA}$ = $Z_{LB}$ =10 pu

For the purpose of compensation, only two harmonic components (5<sup>th</sup> and 7<sup>th</sup>) have been considered. Moreover, assuming the same ratings for both active filters, the compensation power is equally split between them.

Figure 3 shows voltage u and current i at the input port (infinite power bus) in absence of compensation, while Figure

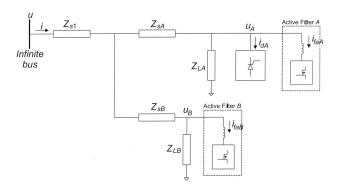


Fig. 2. Application example with distributed compensation system

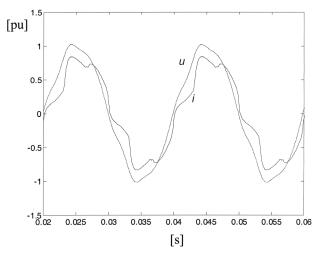


Fig. 3. Voltage u and current i at the infinite power bus port with compensators turned off

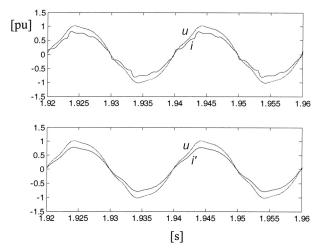


Fig. 4. Voltage u and currents i and i' at the infinite power bus port with compensators turned on

4 gives the same waveforms when active filters A and B are turned on. We see that the distributed compensation system is capable to ensure a line current i nearly proportional to line voltage u, despite the harmonic components above the  $7^{\text{th}}$  are not compensated.

In order to check the validity of the proposed solution, the fundamental,  $5^{\text{th}}$  and  $7^{\text{th}}$  harmonic components of line current i have been extracted and reported in current i'. Figure 4 shows that, in fact, current i' is proportional to voltage u. The distorted load current  $i_A$  and the active filter currents  $i_{faA}$  and  $i_{faB}$  are also given in Figure 5.

# 7.2. Example of dynamic compensation

As an example of dynamic compensation based on differential reactive power r, a thyristor rectifier has been considered, fed by a distorted voltage source with a Total Harmonic Distortion (THD) equal to 5%. Figure 6 shows voltage u and load current i (in p.u.) before and after compensation.

Figure 7 shows the differential reactive power generated by the compensator compared with the opposite of the differential reactive power measured at load terminals. Note that the compensation errors are negligible, even around zerocrossing of the line voltage.

In order to highlight the dynamic properties of the compensation algorithm, the load current has been doubled at 100ms. Figure 8 shows the voltage and current waveforms before and after compensation. It is worth noting that the system dynamics is only associated to the active current component, the compensated current being always in phase with the supply voltage.

#### 8. CONCLUSIONS

The paper has introduced two definitions of instantaneous reactive power terms, which are conservative in every network and provide a theoretical basis for compensation strategies of active and hybrid power filters.

The first definition refers to energy-related quantities and is applicable to the control of slow compensation units,

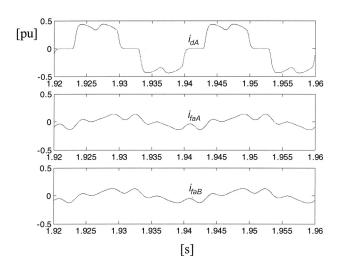
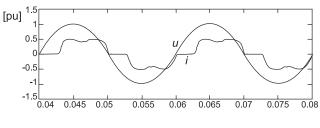


Fig. 5. Distorted load current  $i_{dA}$  and active filter currents  $i_{faA}$  and  $i_{faB}$  providing harmonic compensation



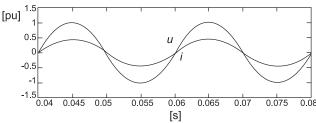


Fig. 6. Load voltage u and current i without compensation (top) and with compensation (bottom)

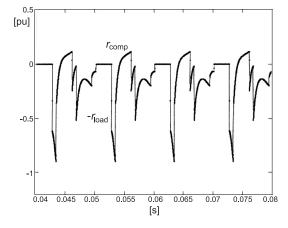
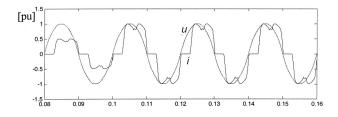


Fig. 7. Compensation differential reactive power vs. the opposite of load differential reactive power

including thyristor-controlled VAR compensators and hybrid power filters.

The second definition is based on differential quantities and is suitable for dynamic control of fast compensators, like active power filters.



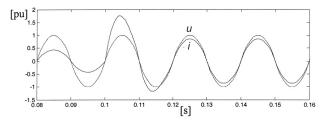


Fig. 8. Voltage u and current i at load terminals, before (top) and after (bottom) compensation, in presence of a load current step in t = 0.1 s

The combined use of both approaches allows coordinated control of distributed reactive and harmonic compensators.

#### REFERENCES

- 1. Akagi H. and Nabae A.: Control Strategy of Active Power Filters Using Multiple Voltage Source PWM Converters. IEEE Trans. on Ind. App., Vol. IA-22, 3, May/June 1986, pp.460– 465
- Akagi H.: Control strategy and site selection of a shunt active filter for damping of harmonic propagation in power distribution systems. IEEE Trans. Power Delivery, Vol. 12, pp. 354–363, Jan. 1997
- 3. Akagi H., Fujita H. and Wada K.: A shunt active filter based on voltage detection for harmonic termination of a radial power distribution line. IEEE Trans on Industry Applications., Vol. 35, pp. 638-645, May/June 1999.
- 4. Czarnecki L.S.: Orthogonal Decomposition of the Currents in a 3-Phase Nonlinear Asymmetrical Circuit with a Nonsinusoidal Voltage Source. IEEE Trans. on Instrumentation and Measurements, Vol. IM-37, 1, pp.30-34, March 1988.
- Czarnecki L.S.: Scattered and reactive current, voltage, and Power in Circuits with nonsinusoidal waveforms and their compensation. IEEE Trans. on Instrumentation and Measurements, Vol. IM-40 (1991), 3, pp. 563-567.
- 6. Ferrero A.: Definitions of Electrical Quantities Commonly Used in Non-Sinusoidal Conditions. European Trans. on Electrical Power Engineering (ETEP), Vol. 8, 4, 1998, pp. 235–240.
- 7. Jintakosonwit P., Fujita H., Akagi H., Ogasawara S.: Implementation and Performance of Cooperative Control of Shunt Active Filters for Harmonic Damping Throughout a Power Distribution System. IEEE Trans on Industry Applications., Vol. 39, 2, pp. 556-563, March/April 2003.
- 8. Mattavelli P.: A Closed-loop selective harmonic compensation for Active Filters. IEEE Transactions on Industry Applications, Vol. 37, 1, Jan/Feb 2001, pp. 81–89.
- Merhej S. J., Nichols W.H.: Harmonic Filtering for the Offshore Industry. IEEE Trans. on Industry Applications, Vol. 30, 3, May/June 1994, pp. 533-542.
- Peng F.Z.: Application Issues of Active Power Filters. IEEE Industry Applications Magazine, Vol. 4, 5, September/October 1998, pp. 21-30.
- Peng F.Z., Akagi H. and Nabae A.: A New Approach to harmonic Compensation in Power Systems A Combined system of shunt passive and series active filters. IEEE Trans on Industry Applications, Vol. 26, 6, 1990, pp. 983-990.
- 12. Tenti P., Mattavelli P.: A Time-Domain Approach to Power Terms Definition under non-sinusoidal conditions. Six International Workshop on Power Definition and Measurements under Nonsinusoidal Conditions, Milan (Italy), October 2003.
- 13. Tenti P., Tedeschi E., Mattavelli P.: Optimization of Hybrid Filters for Distributed Harmonic and Reactive Compensation. IEEE International Conferenze on Power Electronics and Drive Systems (PEDS 2005), Kuala Lumpur (Malaysia), December, 2005.



#### Paolo Tenti (Fellow IEEE)

is professor of Power Electronics and head of the Department of Information Engineering at the University of Padova, Italy. His main interests are industrial and power electronics and electromagnetic compatibility. In these areas he holds national and international patents and published more than 100 scientific and technical papers. His research focuses on application of modern control methods to power

electronics and EMC analysis of electronic equipment. In 1996 Paolo Tenti was elected Vice-President of the IEEE Industry Applications Society (IAS), in 1997 he served as IAS President. In 2000 he chaired the IEEE World Conference on Industrial Applications of Electrical Energy (Rome). For the years 2000-2001 he was appointed IEEE-IAS Distinguished Lecturer on "Electromagnetic compatibility in industrial equipment". Paolo Tenti is a Fellow of the IEEE. He is also President of CREIVen, an industrial research consortium for tecnological advancements in industrial electronics, with special emphasis on electromagnetic compatibility.

Address:

Department of Information Engineering (DEI), University of Padova, Via Gradenigo 6/B, 35131 Padova, Italy, e-mail tenti@dei.unipd.it,

tel.: +39-049-8277503, fax: +39-049-8277699.



#### Paolo Mattavelli

received the Dr. Ing. Degree (with honors) and the Ph. D. degree, both in electrical engineering, from the University of Padova (Italy) in 1992 and in 1995, respectively. From 1995 to 2001, he was a researcher at the University of Padova. In 2001 he joined the Department of Electrical, Mechanical and Management Engineering (DIEGM) of the University of Udine, where he was an Associate Professor of

Electronics from 2002 to 2005. From 2001–2005 he was leading the Power Electronics Laboratory of the DIEGM at the University of Udine, which he founded in 2001. Since 2005 he has been with the DTG at the University of Padova with the same duties. His major fields of interest include analysis, modeling and control of power converters, digital control techniques for power electronic circuits, and power quality issues. Paolo Mattavelli is a member of IEEE Power Electronics, IEEE Industry Applications, IEEE Industrial Electronics Societies and the Italian Association of Electrical and Electronic Engineers (AEI). He also serves as an Associate Editor for IEEE Transactions on Power Electronics and Member-at-Large of PELS Adcom for terms 2004–2006.

Address:

Department of Technology and Management of Industrial Systems (DTG),

University of Padova,

Stradella S. Nicola, 3, 36100 Vicenza, Italy, e-mail: paolo.mattavelli@unipd.it,

tel.: +39-0444-998819, fax: +39-0444-998888.



#### Elisabetta Tedeschi

was born in Noventa Vicentina, Italy, in 1980. She received the Dr. Ing. Degree (with honors) in electrical engineering from the University of Padova, Italy in 2005. She is currently pursuing the PhD degree in Mechatronics and Industrial Systems at the University of Padova in Vicenza. Her research interests are in the field of digital control of power converters and in power quality issues.

Address:

Department of Technology and Management of Industrial Systems (DTG),

University of Padova,

Stradella S. Nicola, 3, 36100 Vicenza, Italy,

e-mail: elisabetta.tedeschi@unipd.it,

tel.: +39-0444-998855, fax: +39-0444-998888.