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Methods of Voltage Unbalance Estimation in Electric Power Networks

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Summary: **Methods of voltage unbalance estimation based on the theory of symmetrical components and the method of differential — angular coefficients of asymmetry have been discussed. It has been proposed the last one to estimate the asymmetry of square absolute voltage value, as useful for distorted processes.**

Key words:
voltage unbalance, symmetric component, differential coefficient, differential angle

1. INTRODUCTION

Polish access to European Union required plenty of work concerning adjustment of Polish Standards to International (IEC) and European (CENELEC) Standards. At present, measures concerning popularization unified requirements and their controlling methods are taken at different conferences and in different scientific magazines.

The subject of this paper is line voltage unbalance in three phase electric power networks. Current situation requires both further detailed research in this subject (or even changes of particular requirements) and conducting the popularization of power quality subject in wide range and on different levels [1]. Measures concerning popularization include familiarization both electric and other professions staff with requirements and measuring methods, hence existence of both scientific and popular publications.

2. SYMMETRIC COMPONENT METHOD

Basic method using in electrotechnics to analysis and estimation of current and voltage unbalance in three phase circuits is symmetric component method [2].

At the reference to line voltage – voltages $\underline{U}_{AB}$, $\underline{U}_{BC}$, $\underline{U}_{CA}$ are replaced with symmetric components:

— positive sequence:

$$\underline{U}_1 = \frac{\left(\underline{U}_{AB} + a\underline{U}_{BC} + a^2\underline{U}_{CA}\right)}{3} \tag{1}$$

— negative sequence:

$$\underline{U}_2 = \frac{\left(\underline{U}_{AB} + a^2\underline{U}_{BC} + a\underline{U}_{CA}\right)}{3} \tag{2}$$

(Zero components do not exist in line voltages).

In the case of negative sequence component $U_2 = 0$, there exists only circuit of positive sequence component U_1 and three phase circuit is symmetric, similarly it is possible to assume that U_1=0 and then there is only circuit of U_2. However if $U_1 \neq 0$ and $U_2 \neq 0$ then three phase circuit is unbalanced.

In overall case, line voltages:

$$\underline{U}_{AB} = \underline{U}_1 + \underline{U}_2 \tag{3}$$

$$\underline{U}_{BC} = a^2\underline{U}_1 + a\underline{U}_2 \tag{4}$$

$$\underline{U}_{CA} = a\underline{U}_1 + a^2\underline{U}_2 \tag{5}$$

($a = -1/2 + j\sqrt{3}/2$) have different absolute values and their phase angles are different from 120^0 (they do not create equilateral triangle).

In power electric network, the conjugate coefficient of line voltage unbalance of the form (6) is most often used to the assessment of voltage unbalance [1]:

$$\underline{k}_u = \frac{\underline{U}_2}{\underline{U}_1} = \frac{\underline{U}_{AB} + a^2\underline{U}_{BC} + a\underline{U}_{CA}}{\underline{U}_{AB} + a\underline{U}_{BC} + a^2\underline{U}_{CA}} = \frac{U_2}{U_1} e^{j\psi_u} \tag{6}$$

In practice, it is not necessary to take into account the angle ψ_u – phase angle between positive sequence and negative sequence components and estimation of unbalance is performed on the basis of the coefficient of line voltage unbalance defined in the following form:

$$k_u = \frac{\left|\underline{U}_{AB} + a^2\underline{U}_{BC} + a\underline{U}_{CA}\right|}{\left|\underline{U}_{AB} + a\underline{U}_{BC} + a^2\underline{U}_{CA}\right|} = \frac{U_2}{U_1} \tag{7}$$

Determination of the coefficient k_u from formulas (6) and (7) requires knowledge of complex values of line voltage. It was argued a long time ago that equivalent result can be obtained using simple trigonometric relationships calculating for between-lines voltage triangle, then formulas (3–5) can be presented in following forms:

$$\underline{U}_{AB} = \underline{U}_1(1 + \underline{k}_u) \tag{8}$$

$$\underline{U}_{BC} = a^2\underline{U}_1(1 + a^2\underline{k}_u) \tag{9}$$

$$\underline{U}_{CA} = a\underline{U}_1(1+a\underline{k}_u) \quad (10)$$

and square absolute value of line voltages in forms:

$$U_{AB}^2 = U_1^2\left[1+k_u^2+2k_u\cos\psi_u\right] \quad (11)$$

$$U_{BC}^2 = U_1^2\left[1+k_u^2+2k_u\cos(\psi_u-120^0)\right] \quad (12)$$

$$U_{BC}^2 = U_1^2\left[1+k_u^2+2k_u\cos(\psi_u+120^0)\right] \quad (13)$$

Above relationships constituted the base of elaboration of several methods of unbalance voltage determination. In Poland, first who introduced such method was Rachwalski [3]. He proposed an approximated method basing on the assumption that positive sequence voltage component occurring in power network could be approximated, at small unbalance voltage, by the mean of absolute voltage values:

$$U_1 \approx \frac{1}{3}(U_{AB}+U_{BC}+U_{CA}) \quad (14)$$

Introducing auxiliary variables:

$$x_1 = U_{AB}/U_1; \quad x_2 = U_{BC}/U_1; \quad x_3 = U_{CA}/U_1 \quad (15)$$

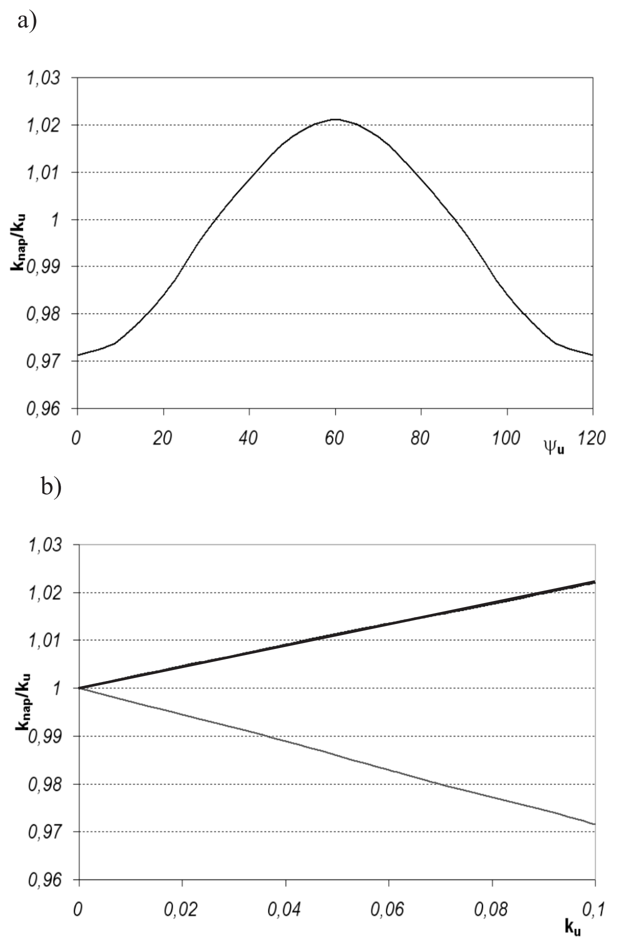

Fig. 1. Dependence of ratio k_{nap}/k_u on the angle ψ_u (a) and on the coefficient k_u (b)

he obtained simplified formula for the angle ψ_u [3]:

$$ctg\,\psi_u = \frac{\sqrt{3}}{3}\left[2\frac{x_1-x_2}{x_3-x_2}-1\right] \quad (16)$$

and for the coefficient of unbalance voltage:

$$k_u = \frac{(x_3-x_2)(x_3+x_2)}{2\sqrt{3}\sin\psi_u} \quad (17)$$

In commonly accepted version [4, 5], the coefficient of voltage unbalance is determined according to the following formula:

$$k_u = \sqrt{\frac{1-\sqrt{3-6\beta}}{1+\sqrt{3-6\beta}}} \quad (18)$$

where the auxiliary term β is calculated as:

$$\beta = \frac{U_{AB}^4+U_{BC}^4+U_{CA}^4}{(U_{AB}^2+U_{BC}^2+U_{CA}^2)^2} \quad (19)$$

Formulas (18) and (19) are recommended in the part 4 of IEC and Polish Standard [5] and in the part 2 of this Standard [6], the formula of the form (20) determining approximated value of the coefficient of voltage unbalance k_{nap} is also recommended [6]:

$$k_{nap} = \sqrt{\frac{6(U_{AB}^2+U_{BC}^2+U_{CA}^2)}{(U_{AB}+U_{BC}+U_{CA})^2}-2} \quad (20)$$

Figure 1 shows the relationship between precise and approximated values of the coefficient k_{nap}/k_u as a function of the angle ψ_u, for $\psi_u = 0...120^o$ and $k_u = 0.1$ — Figure 1a, and as a function of the coefficient k_u, for $k_u = 0...0.1$ and $\psi_u = 0^o, 120^o$ and 240^o (bottom line) and $\psi_u = 60^o, 180^o$ and 300^o (upper line) – Figure 1.b (at assumed values of the coefficient k_u and the angle ψ_u, first values of $U_{AB}^2, U_{BC}^2, U_{CA}^2$ were calculated from (11, 12 and 13) and next the approximated value of the coefficient k_{nap} was determined from (20)). Depending on the angle ψ_u, the ratio k_{nap}/k_u oscillates between bottom and upper lines, for example at $k_u = 0.02$, the range of variability is approximately equal to 1 ± 0.005. It can be notice that in application including power network, formula (18) and (20) are equivalent. In authors opinion these formulas should occur together in Standards, for example formula (18) along with (19) as recommended and formula (20) as alternative. However the placement of different formulas in different parts of the Standard seems an improper approach.

3. DIFFERENTIAL–ANGULAR COEFFICIENT METHOD

One of advantages of symmetric component method is the ability to use it together with Białek's method — differential-angular coefficient method [7, 8].

It allows not only deeper current and voltage analysis of three-phase unbalance circuit but also ensures its further extension on other physical quantities such as power and impedance.

For example, phase power of electrothermal receiver P_A, P_B, P_C — Figure 2, can be expressed with the use of mean power P, differential module W_P and differential angle α_P of power unbalance.

$$P_A = I_A^2 r_A = P + W_p \cos\alpha_p \quad (21)$$

$$P_B = I_B^2 r_B = P + W_p \cos(\alpha_p - 120^0) \quad (22)$$

$$P_C = I_C^2 r_C = P + W_p \cos(\alpha_p + 120^0) \quad (23)$$

where:

$$P = \frac{1}{3}(P_A + P_B + P_C) \quad (24)$$

$$W_p = \frac{\sqrt{2}}{3}\sqrt{(P_A - P_B)^2 + (P_B - P_C)^2 + (P_C - P_A)^2} \quad (25)$$

$$\alpha_p = arc\ tg\left(\frac{P_B - P_C}{\sqrt{3}(P_A - P)}\right) + \left[1 - sign(P_A - P)\right] \cdot 90^0 \quad (26)$$

Proposed differential-angular coefficient would be of the form:

$$\underline{k}_p = k_p e^{j\alpha_p} = \frac{W_p}{P} e^{j\alpha_p} \quad (27)$$

and would consist of the terms – differential coefficient $k_p = W_p/P$ and differential angle α_P.

Similarly, for the resistance and reactance of supplying circuit, phase quantities are replaced by R, W_R and α_R and X, W_X and α_X. The assessment of impedance unbalance of supplying circuit of the mean value:

$$\underline{Z} = Ze^{j\varphi_z} = R + jX \quad (28)$$

has been carried out with the help of proposed differential-angular coefficient [8]:

$$\underline{k}_z = k_z e^{j\delta} = \frac{D}{Z} e^{j\delta} \quad (29)$$

differential module D and differential angle δ (for consecutive phase sequence A, B, C) were calculated from formulas:

$$D\cos\delta = W_R \cos\alpha_R - W_X \sin\alpha_X \quad (30)$$

$$D\sin\delta = W_R \sin\alpha_R + W_X \cos\alpha_X \quad (31)$$

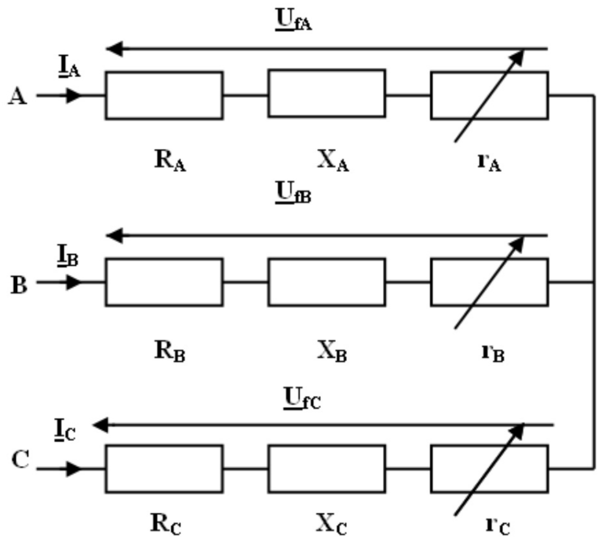

Fig. 2 Simplified equivalent circuit diagram of the arc furnace (arcs are replaced by linear resistances)

Occurring current unbalance has been determined with the use of following formula:

$$\underline{k}_i = k_i e^{j\psi_i} = \frac{\underline{I}_2}{\underline{I}_1} = -\frac{De^{j\delta} + W_r e^{j\alpha_r}}{2(Ze^{j\varphi_z} + r)} \quad (32)$$

and power asymmetry from:

$$\underline{k}_p = k_p e^{j\alpha_p} = \frac{I_1^2}{P}\left[2rk_i e^{j\psi_i} + (1 + k_i^2)W_r e^{j\alpha_r} + k_i W_r e^{-j(\psi_i + \alpha_r)}\right] \quad (33)$$

at the mean power value equal to:

$$P = I_1^2\left[(1 + k_i^2)r + k_i W_r \cos(\psi_i - \alpha_r)\right] \quad (34)$$

Simultaneous application of these two methods allows for the determination in the simple manner conditions ensuring: maximization of power and its symmetry [10], choosing transformer phase ratio with respect to phase current balance of carbide furnace [11] or currents fed from supplying network resulting from the limitation of supplying voltage unbalance [12].

4. NOMOGRAPHY METHODS

Thinking of practical applications, many nomograms ensuring higher or lower accuracy have been worked out. Because of possibility of calculation of low values of unbalance coefficients in the range of 0,1–2% , nomograms developed in Institute of Electrodynamics (Ukrainian Science Academy) [1,13] turned out to be useful in the assessment of voltage unbalance of power networks. Method used in the development of these nomograms is based on three absolute values of line voltage U_{AB}, U_{BC}, U_{CA}, obtained from measurements:

— on the basis of these three values, ratios of two of them to third one are calculated:

$$x = U_{BC} / U_{AB} \quad \text{and} \quad y = U_{CA} / U_{AB} \tag{35}$$

and absolute value of positive:

$$U_1 = \frac{U_{AB}}{3} \sqrt{ \left[1 + x \cos\left(\arccos \frac{1+x^2-y^2}{2x} - \frac{\pi}{3} \right) + y \cos\left(\frac{\pi}{3} + \arccos \frac{1+y^2-x^2}{2y} \right) \right]^2 + \left[x \sin\left(\arccos \frac{1+x^2-y^2}{2x} - \frac{\pi}{3} \right) + y \sin\left(\frac{\pi}{3} - \arccos \frac{1+y^2-x^2}{2y} \right) \right]^2 } \tag{36}$$

and negative sequence component:

$$U_2 = \frac{U_{AB}}{3} \sqrt{ \left[1 + x \cos\left(\frac{\pi}{3} + \arccos \frac{1+x^2-y^2}{2x} \right) + y \cos\left(\frac{\pi}{3} + \arccos \frac{1+y^2-x^2}{2y} \right) \right]^2 + \left[x \sin\left(\frac{\pi}{3} + \arccos \frac{1+x^2-y^2}{2x} \right) - y \sin\left(\frac{\pi}{3} + \arccos \frac{1+y^2-x^2}{2y} \right) \right]^2 } \tag{37}$$

are also calculated.

The value of unbalance coefficient will be determined from the formula:

$$k_u = U_2 / U_1 \tag{38}$$

Basing on [13], it is possible to estimate this method error, whose value does not exceed 0,05%. For determination of the angle ψ_u simple formula has been proposed:

$$\psi_u = arc\ tg \frac{\sqrt{3}(x^2 - y^2)}{2 - x^2 - y^2} + \pi \cdot n \tag{39}$$

(n depends on the sign of numerator and denominator of formula (39)).

Exemplary nomogram is shown in Figure 3.

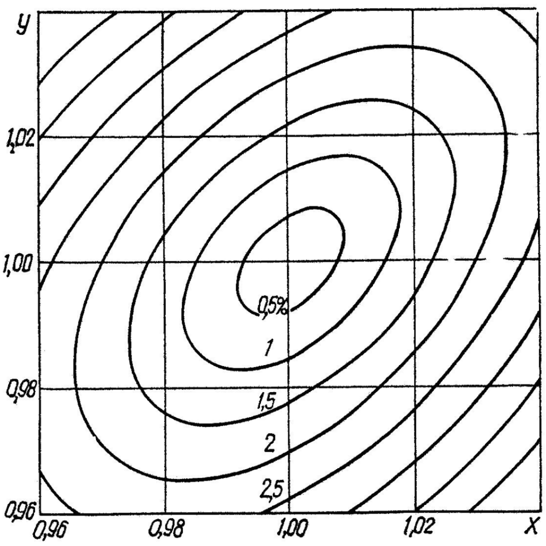

Fig. 3. Nomogram for the estimation of the coefficient of unbalance voltage k_u (in %%), for $x = 1 \pm 0{,}04$ and $y = 1 \pm 0{,}04$ [13]

Taking into consideration the need of determination of negative sequence component voltage with the higher accuracy, the special tables given in the book [13] have been elaborated with the use of computer. Thanks to it, it is possible to read k_u with the accuracy of ten positions after the comma for the range of $x = 0.9...1$ and $y = 0.9...1$ with the step of 0.001 and for extended range with the step of 0.002 or 0.005.

New proposition has been presented by Jagieła and Gała [14], in which authors argued that it is possible to determine unbalance coefficient k_{uf} with respect to phase voltage taking into account both coefficients of amplitude inconsistency $k_{u1} = U_B / U_A$, $k_{u2} = U_C / U_A$, and coefficients of phase angle inconsistency $\xi_{u1} = \varphi_{uB} - \varphi_{uA}$; $\xi_{u2} = \varphi_{uC} - \varphi_{uA}$. It allows to treat the coefficient k_{uf} as a function of four variables and depending on the choice of any two variables from the set $\{k_{u1}, k_{u2}, \xi_{u1}, \xi_{u1}\}$ as independent, it is possible to determine nomograms showing for example the influence of amplitude or phase angle inconsistency on the coefficient of voltage unbalance. Thanks to it, the complex assessment of coefficient changes is possible.

5. DISCUSSION PROPOSAL

Present methods of the assessment of voltage and current unbalance based on symmetric component method are correct for sinusoidal waveforms, however they should not be used for distorted waveforms. In this case the analysis of harmonic components with the help of Fast Fourier Transform algorithm is recommended [6, 14].

Wąsowski's works conducted together with Białek and independently were concentrated on the power asymmetry in arc and arc-resistance furnaces, therefore on active power (generated by all harmonic components). Authors propose as an additional method, the adaptation of differential-angular coefficient method using in these works to the assessment of voltage and current unbalance of three-phase circuits. Particularly for distorted waveforms, authors propose the application of this method to the assessment of asymmetry of square current and voltage rms.

For example for line voltages of absolute values U_{AB}, U_{BC}, U_{CA}, it would consist in the expression of square absolute values U_{AB}^2; U_{BC}^2; U_{CA}^2 by their mean value U_M^2, differential module value W_{U^2} and differential angle α_{U^2}:

$$U_{AB}^2 = U_M^2 + W_{U^2} \cos \alpha_{U^2} \tag{40}$$

$$U_{BC}^2 = U_M^2 + W_{U^2} \cos(\alpha_{U^2} - 120^0) \tag{41}$$

$$U_{CA}^2 = U_M^2 + W_{U^2} \cos(\alpha_{U^2} + 120^0) \tag{42}$$

mean value and differential parameters would be calculated from formulas:

$$U_M^2 = \frac{1}{3}(U_{AB}^2 + U_{BC}^2 + U_{CA}^2) \tag{43}$$

$$W_{U^2}\cos\alpha_{U^2}=U_{AB}^2-U_M^2 \tag{44}$$

$$W_{U^2}\sin\alpha_{U^2}=\frac{1}{\sqrt{3}}(U_{BC}^2-U_{CA}^2) \tag{45}$$

or:

$$W_{U^2}=\frac{\sqrt{2}}{3}\sqrt{(U_{AB}^2-U_{BC}^2)^2+(U_{BC}^2-U_{CA}^2)^2+(U_{CA}^2-U_{AB}^2)^2} \tag{46}$$

$$\alpha_{U^2}=arc\ tg\frac{U_{BC}^2-U_{CA}^2}{\sqrt{3}(U_{AB}^2-U_M^2)}+\left[1-sign(U_{AB}^2-U_M^2)\right]\cdot 90^0 \tag{47}$$

The differential-angular coefficient could be used to assessment of asymmetry of square line voltages:

$$\underline{k}_{U^2}=k_{U^2}e^{j^{\alpha_{U^2}}} \tag{48}$$

consisting of two terms: asymmetry differential coefficient k_{U^2}:

$$k_{U^2}=\frac{W_{U^2}}{U_M^2}=\sqrt{2}\frac{\sqrt{(U_{AB}^2-U_{BC}^2)^2+(U_{BC}^2-U_{CA}^2)^2+(U_{CA}^2-U_{AB}^2)^2}}{(U_{AB}^2+U_{BC}^2+U_{CA}^2)} \tag{49}$$

and differential angle α_{U^2}.

In the case of sinusoidal waveforms, the attention should be paid on the occurrence of simple relationship between parameters used to the assessment of asymmetry in two presented in the paper methods:

— between mean square voltage and the sum of square positive and negative — sequence voltage components:

$$U_M^2=U_1^2+U_2^2=U_1^2(1+k_u^2) \tag{50}$$

— between differential coefficient of square voltage asymmetry and the coefficient of voltage unbalance:

$$k_{U^2}=\frac{2k^2{}_u}{1+k_u^2} \tag{51}$$

and:

$$k_u=\frac{\sqrt{1-k_{U^2}^2}-1}{k_{U^2}};\quad k_{U^2}\neq 0 \tag{52}$$

(in case when $k_{U^2}=0$, also $k_u=0$).

In case of little voltage unbalance it can assume in approximation:

$$k_u\approx 0{,}5k_{U^2} \tag{53}$$

— between differential angle of square voltage asymmetry and phase shift angle between negative and positive sequence voltage components:

$$\alpha_{U^2}=\psi_u \tag{54}$$

As the example of usefulness of proposed method, we consider the case of the assessment of power unbalance of hypothetical resistance furnace consisting of three different heating elements delta connecting $R_{AB}\neq R_{BC}\neq R_{CA}$, connected to the network of unequal absolute values of line voltages $U_{AB}\neq U_{BC}\neq U_{CA}$. Phase heating power are expressed by following formulas:

$$P_{AB}=U_{AB}^2/R_{AB}=U_{AB}^2G_{AB} \tag{55}$$

$$P_{BC}=U_{BC}^2/R_{BC}=U_{BC}^2G_{BC} \tag{56}$$

$$P_{CA}=U_{CA}^2/R_{CA}=U_{CA}^2G_{CA} \tag{57}$$

Determining mean value U_M^2, differential coefficient k_{U^2} and asymmetry differential angle α_{U^2} for square line voltage and for heating elements mean value of equivalent phase conductance G, their differential coefficient k_G and differential angle α_G, mean heating power can be calculated with the use of following formula:

$$P=U_M^2G\left[1+\frac{1}{2}k_{U^2}k_G\cos(\alpha_{U^2}-\alpha_G)\right] \tag{58}$$

and differential-angular coefficient of power asymmetry by the formula:

$$\underline{k}_p=k_pe^{j\alpha_p}=\frac{k_{U^2}e^{j\alpha_{U^2}}+k_Ge^{j\alpha_G}+\frac{1}{2}k_{U^2}k_Ge^{-j(\alpha_{U^2}+\alpha_G)}}{1+\frac{1}{2}k_{U^2}k_G\cos(\alpha_{U^2}-\alpha_G)} \tag{59}$$

6. CONCLUSION

Authors believe that proposed method of the assessment of line voltage unbalance in power networks treating as a supplement of the method based on symmetrical component theory will be useful in practice and find the application into sinusoidal and non-sinusoidal waveform analysis in unbalanced working states of three-phase circuits, and electrothermal devices in particular.

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