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AN OPTIMUM REACTANCE ONE-PORT COMPENSATOR FOR HARMONIC MITIGATION

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Summary**: The installation of electronic devices, digital equipment, and nonlinear loads in both industrial and domestic applications have dramatically increased in recent years, which in turn increased the level of harmonics in the system. Harmonic distortion is widely recognized as a significant cause of damage to, and mal-operation of electrical equipment. A harmonic filter can eliminate the potentially dangerous effects of harmonic currents created by nonlinear loads. There are two types of harmonic filters: passive filters and active filters. Passive filters are inexpensive compared with most mitigating devices. In this paper, reactance one-port compensator is designed for current harmonic mitigation. The optimal parameters the filter are determined using the Branch and Reduce Optimization Navigator (BARON) Nonlinear Programming Solver in the General Algebraic Modeling System (GAMS).**

Key words:
Filter
Harmonics
Nonlinear loads
Optimization

1. INTRODUCTION

During the last decade a major twist in the electricity system has taken place as deregulation is being adopted by most electrical utilities. This move has brought lots of changes to the way utilities are conducting business including more emphasis on economical operation. As a result Power Quality (PQ) gains more importance in the deregulated market as a differentiating factor between successful utilities [1]. Moreover, the last decade has witnessed a huge proliferation of sensitive loads that are greatly affected by different PQ problems. Those loads mostly share a common characteristic which is being electronic-based and hence, non-linear. Harmonic distortion has been realized historically as one of the most disturbing PQ problems that can lead to lots of system and equipment problems. Increased losses, overheating of neutral conductors and transformers, malfunction of microprocessor-based equipment, deterioration of power factor capacitors and erratic operation of breakers and relays are just examples of such problems [2]. Although the amounts of harmonics injected by individual loads are not significant enough to cause any problem, it is anticipated that the cumulative effect of large number of non-linear loads will be capable of raising the amount of harmonic distortion to the extent that necessitate a mitigation action to be taken in order to meet the recommended standards [3–5].

On the distribution level, harmonic mitigation is normally achieved by installing corrective equipment that can be either active or passive. Active power filters have been realized to be a very effective solution for harmonic mitigation especially if lots of dynamics are involved. However, given the complexity, switching non-linearties, high filter cost and the economical burden imposed by the deregulation process, active filters become less attractive. Zigzag transformers can be used effectively to reduce triplen harmonics, but can not address other harmonics such as the fifth and seventh. Neutral blocking filters can be used, however, on the expense of increasing voltage distortion. On the other hand, passive filters can provide reasonable harmonic mitigation at a very cheap price, however, on the expense of not being adaptive. Notch filters as a passive solution suffer from major drawbacks, which are the possibility of being overloaded and the chance to resonate with the system [6]. In [7], reactance one-port compensator was implemented, as a passive solution, to achieve total compensation of system harmonics without the possibility of being overloaded or creating resonance problems. However, the compensator poles and zeros were generated by trial and error [7].

In this paper, the reactance one port compensator is designed to mitigate current and voltage harmonics. In order to avoid the trial and error process, the compensator design problem is formulated in the form of an optimization problem and the compensator parameters are optimally determined. The filter design problem is formulated as a Nonlinear Programming Problem (NLP), where the main objective is to minimize the difference between the compensator and load susceptance values in order to achieve minimum Total Harmonic Distortion (THD).

The paper is organized as follows. Section 3 presents an overview on the reactance one port compensator. Section 4 provides the compensator design problem formulation. Section 5 presents the load and system models studied in this paper. Section 6 presents the optimal compensator parameters and simulation results. Lastly section 7 draws the conclusions.

2. REACTANCE ONE-PORT COMPENSATOR

Reactance one-port compensator can be applied in linear as well as non-linear circuits to achieve total harmonic compensation. In linear circuits, the application is easier since the linear load susceptance can be calculated directly by dividing the harmonic current by the harmonic voltage. This compensator has been applied successfully to reduce harmonics in circuits having linear loads and supplied from a periodic nonsinusoidal source [7]. In non-linear circuits, susceptance calculation is more complex and a tuning process is required to estimate the non-linear load susceptance at different harmonics. The application of this compensator was extended to circuits with non-linear loads supplied from a practical sinusoidal source with inductive impedance [8]. However, the compensator poles and zeros were chosen randomly within a specified range governed by the compensator design. The following subsections will be dedicated to the filter design procedures.

2.1. Load Susceptance Calculation

For linear loads, the determination of the load susceptance at different frequencies could be determined easily by measuring both the voltage and current at different harmonic frequencies. On the contrary, for systems with nonlinear loads, calculating the load susceptance accurately is not straightforward. The load susceptance was calculated by, first, measuring the voltage and current angles at the different harmonic frequencies. The phase angle relationship between the voltage and current gives an indication on whether the nonlinear load is inductive or capacitive. For each harmonic frequency, depending on whether the phase angle is lagging or leading, a variable capacitance C_h or inductance L_h element is connected in parallel with the nonlinear load. The added element (C_h or L_h) is varied until the value of the current at harmonic *h* is negligible and this value corresponds to the load susceptance at that harmonic.

2.2. Compensator Design

Once the equivalent load susceptances at different harmonic frequencies are measured, the compensator can be designed. The filter susceptance should be equal in magnitude and opposite in sign to the equivalent load susceptance at each harmonic frequency such as:

$$B_{Cn} = -B_{Ln} \tag{1}$$

where:
B_{Cn} = compensator susceptance at harmonic "*n*".
B_{Ln} = load susceptance at harmonic "*n*".

The next step is to identify the reactance one-port compensator complexity. The compensator complexity "*M*" according to [7] is $N \le M \le 2N$. The reactance one-port compensator complex admittance, $Y_c(s)$, or complex impedance, $Z_c(s)$, where "*s*" is the complex variable, is determined by the number of parameters equal to its complexity, that is, $2N$. According to equation (1) we have only *N* equations. The remaining *N* parameters can be chosen arbitrary by assuming the values of the compensator poles. The synthesis procedure of a reactance one-port compensator [9] can be found in [8].

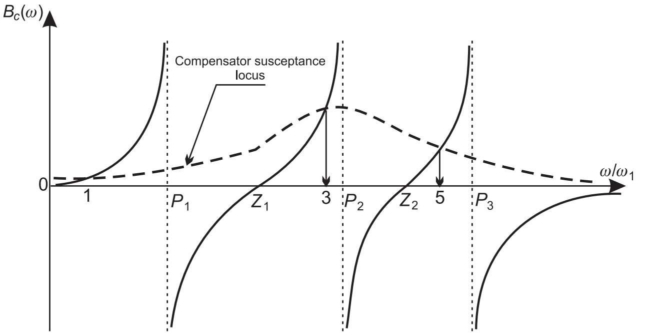

Fig. 1. Compensator Susceptance locus for 1st, 3rd, and 5th harmonic compensation

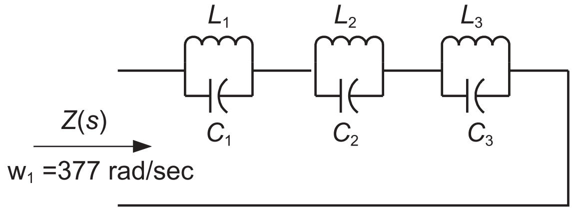

Fig. 2. Reactance One Port Compensator

3. OPTIMUM COMPENSATOR PARAMETERS

As stated earlier, for the compensator to efficiently mitigate harmonics, it should be designed such that both the compensator and load susceptance are equal. Figure 2 shows a reactance one-port compensator with a complexity of 3. This filter will be designed to mitigate both 3rd and 5th harmonic component. Besides, the filter should not draw any fundamental current component. The impedance of the filter can be written in the s domain as follows:

$$Z_c(s) = As\frac{(s^2+z_1^2)(s^2+z_2^2)}{(s^2+p_1^2)(s^2+p_2^2)(s^2+p_3^2)} \tag{2}$$

The main objective is to minimize the difference between the load and filter susceptance and this is represented by an error function as given below:

$$min \sum (error)^2 = min \sum_{n=1,3,5,..}^{m} (B_{L_n} - B_{C_n})^2 \tag{3}$$

where B_{Ln} and B_{Cn} are the load and compensator susceptance and m represents the maximum order of harmonic to be compensated. The objective is minimized under the following constraints; according to Figure 1:

$$1 \le p_1 \le z_1 \le 3 \tag{4}$$

$$3 \le p_2 \le z_2 \le 5 \tag{5}$$

$$p_3 \ge 5 \tag{6}$$

$$A \ge 0 \tag{7}$$

The problem is formulated as a Nonlinear Optimization Programming (NLP) problem and is solved using the BARON Solver in GAMS. In order to calculate the values of the *L* and *C* of each compensator tank, the compensator impedance can be written in terms of *L* and *C* as follows:

$$Z_c(s)=\frac{\frac{1}{C_1}s}{s^2+\frac{1}{L_1C_1}}+\frac{\frac{1}{C_2}s}{s^2+\frac{1}{L_2C_2}}+\frac{\frac{1}{C_3}s}{s^2+\frac{1}{L_3C_3}} \qquad (8)$$

By applying partial fractions to (2):

$$Z_c(s)=\frac{k_1s}{(s^2+p_1^2)}+\frac{k_2s}{(s^2+p_2^2)}+\frac{k_3s}{(s^2+p_3^2)} \qquad (9)$$

where:

$$k_1=\frac{A(-p_1^2+z_1^2)\cdot(-p_1^2+z_2^2)}{(-p_1^2+p_2^2)\cdot(-p_1^2+p_3^2)} \qquad (10)$$

$$k_2=\frac{A(-p_2^2+z_1^2)\cdot(-p_2^2+z_2^2)}{(-p_2^2+p_1^2)\cdot(-p_2^2+p_3^2)} \qquad (11)$$

$$k_3=\frac{A(-p_3^2+z_1^2)\cdot(-p_3^2+z_2^2)}{(-p_3^2+p_2^2)\cdot(-p_3^2+p_1^2)} \qquad (12)$$

By equating (8) and (9), for each tank, the values for L and C could be calculated.

4. LOAD MODELS

Different single phase loads which are normally located on practical distribution system have been developed in the time-domain in order to conduct this study. The time-domain method has the advantage of accounting for phase angle diversity and harmonic voltage-current interactions; two phenomena that can lead to partial harmonic cancellation as well as harmonic attenuation. The EMTDC/PSCAD has been used to develop the non-linear loads as well as the distribution system. It should be mentioned that since we are dealing with balanced system, then single-phase representation will be adequate. Nevertheless, the same procedure can be implemented for unbalanced three-phase systems. The non-linear loads utilized in this study have been divided into three distinct groups.

4.1. Load Group I

This load group contains magnetic ballast compact fluorescent lamps (CFL) and other discharge lamps with characteristics as shown in Figure 3. The lamp voltage rating is 120 volt (RMS) and power rating of 40 watts. The model for this load that consists of two switched non-linear resistors is shown in Figure 4.

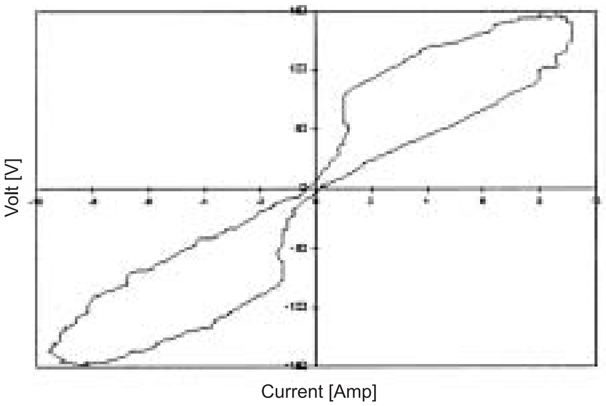

Fig. 3. CFL characteristics

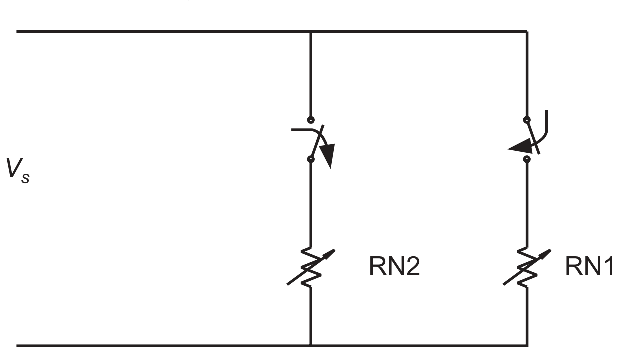

Fig. 4. CFL circuit model

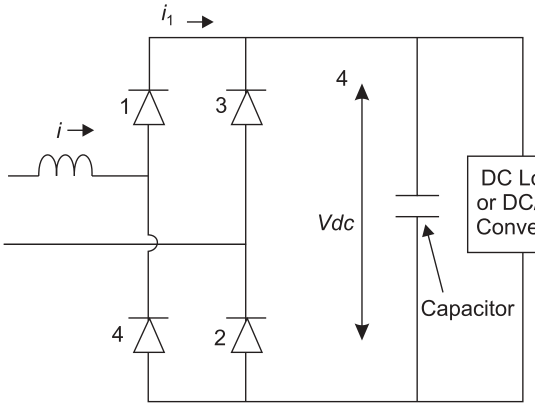

Fig. 5. DBR circuit model

4.2. Load Group II

The second load group contains those loads utilizing single-phase diode bridge rectifier DBR as their front-end power supply. Loads such as TV sets, computers, battery chargers and small adjustable speed drives are examples of this group. The circuit model for this group is given in Figure 5.

4.3. Load Group III

The third load group comprises those loads utilizing the Thyristor phase angle to control its input voltage. Phase angle AC voltage controller PAVC can be found in light dim-

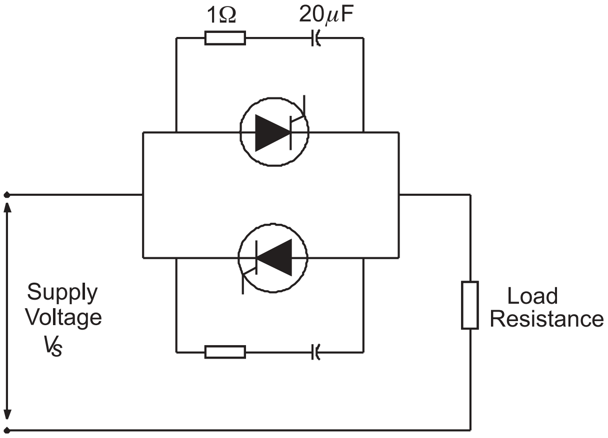

Fig. 6. PAVC circuit model

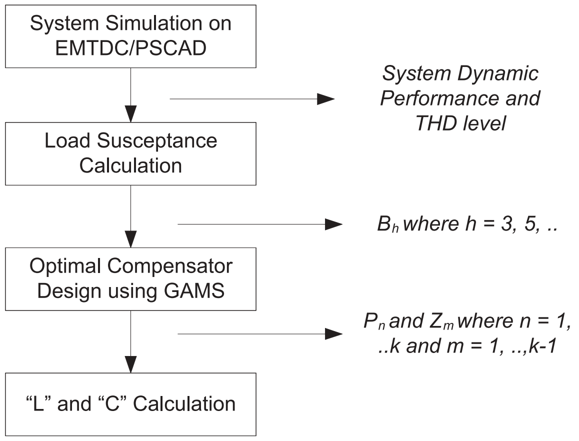

Fig. 7. Summary of the procedure used for compensator parameters

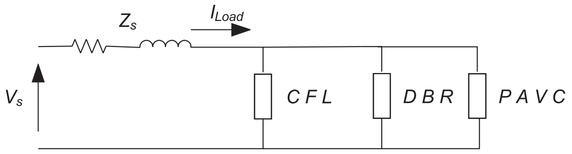

Fig. 8. System consisting of three loads connected in parallel

mers, heating loads and controls of single-phase induction motors. The circuit representation of such load group is shown in Figure 6.

5. SIMULATION RESULTS

To verify and test the performance of the designed filter, two systems were simulated. While the first system represents a simple parallel combination of non-linear loads, the second system represents a more practical secondary distribution system. Both system are simulated on EMTDC/ PSCAD to determine the Total Harmonic Distortion (THD) and the system' dynamic performance. Then, the load susceptance values are calculated using the method previously described in Section II. The compensator poles and zeros are then optimally determined using the BARON solver in GAMS [12]. The values of L and C of each tank could then be calculated from the optimal poles and zeros. The procedure for determining the compensator parameters is illustrated in Figure 7.

5.1. Case I

The first system is a simple system consisting of just three loads (CFL, DBR, and PAVC) connected in parallel and interconnected to a source with an impedance $Zs = 0.8 + j0.9424$ ohms as shown in Figure 8. The load susceptances at different harmonic frequencies are calculated using the previously explained method and are given below:

$$B_1 = 0.00315$$

$$B_3 = 17.8$$

$$B_5 = -27.5$$

Once the susceptance values are determined, the values are inputted to the compensator design problem formulation to determine the optimal values for the compensator's poles and zeros. The BARON solver in Gams is used to solve the compensator NLP problem [12]. Baron is a computational system for solving non convex optimization problems to global optimality. Purely continuous nonlinear programs, purely integer, and mixed-integer nonlinear programs (MINLPs) can be solved with the software [13]. The optimal poles and zeros values for the compensator are given in Table 1.

Using the equations presented in (8) through (12), the compensator *L* and *C* parameters could be calculated as given in Table 2.

To test the effectiveness of the designed compensator, the system is simulated again but with the compensator connected at $t = 2$ sec. The total current drawn by the three loads, voltage at the load terminals and the current THD are presented in Figure 9.

It can be seen that once the compensator is introduced in the system, both the current and voltage become much more sinusoidal. This in turn reflects on the value of the THD. The THD value has decreases from more than 16 % to 2.1 %.

Table 1. Optimal Compensator Design Parameters

P1	498.850
P2	1201.888
P3	1954.676
Z1	1130.624
Z2	1885.377
A	11104.832

Table 2. Optimal L and C Values

$L1$	35.6 mH
$L2$	0.94903 mH
$L3$	0.223168 mH
$C1$	113.0106 μF
$C2$	729.4441 μF
$C3$	1129.7 μF

5.2. Case II

A practical distribution system is shown in Figure 10 with combined loads from the previous three load groups. The system supplies balance three-phase and single-phase commercial and residential customers. The primary and secondary distribution transformers data [10] are given in Figure 10. Table 3 lists the distribution system feeder sections data. The system is further loaded with balanced three-phase star-connected linear load with power factor of 0.9 lagging to represent resistive heating loads.

The load susceptances at different harmonic frequencies are given below:

$$B_1 = 0.225$$

$$B_3 = -45$$

$$B_5 = 35$$

In a similar manner, the optimal poles and zeros are determined and consequently, the L and C parameters for the reactance one-port compensator are calculated as shown in Table 4 and Table 5, respectively.

Similarly, the system is simulated again but with the compensator connected at $t = 2$ sec. The total current drawn by the three loads, voltage at the load terminals and the current THD are presented in Figure 11. In this case, the current THD improves from more than 34 % to 3.8 %.

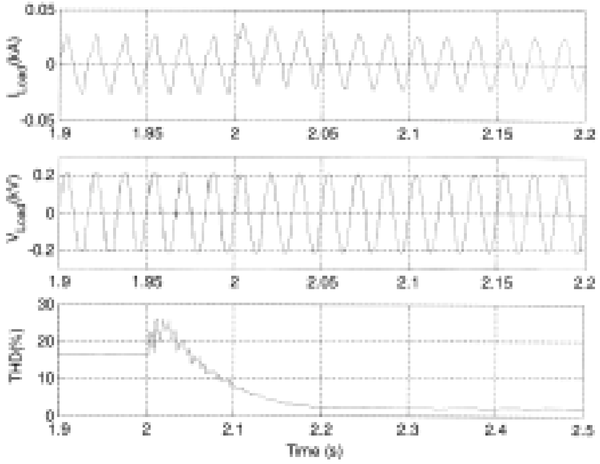

Fig. 9. Load Current, Voltage and current THD

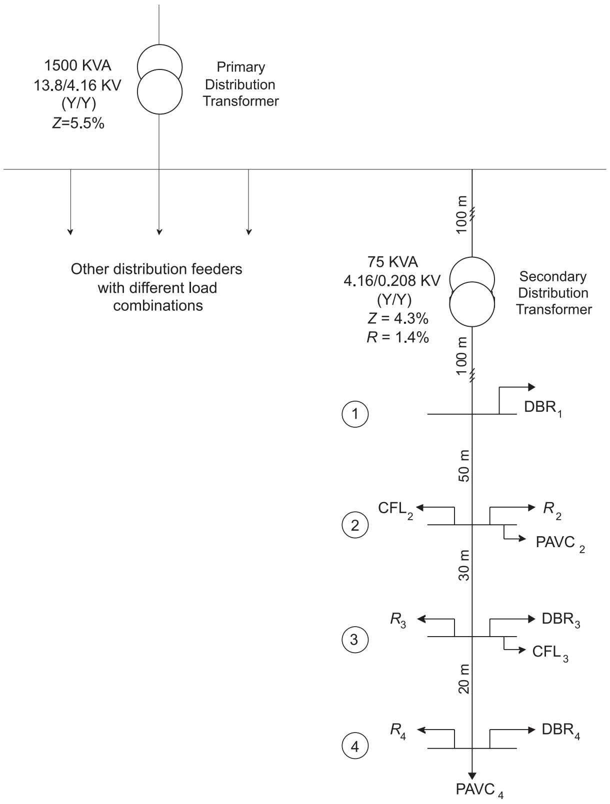

Fig. 10. Distribution system model

Table 3. Distribution system feeder data [11]

Section	Type	Length (m)	Cross sec. (mm^2)	R (? /km)	X (? /km)
1	3-phase	100.0	16.0	1.420	0.106
2	3-phase	100.0	95.0	0.239	0.081
3	1 core, PVC	50.0	50.0	0.464	0.112
4	2-cores, PVC	30.0	16.0	1.380	0.080
5	2-cores, PVC	20.0	16.0	1.380	0.080

Table 4. Optimal Compensator Design Parameters

P1	404.970
P2	1777.000
P3	21334.715
Z1	1131.377
Z2	1873.363
A	92338.476

Table 5. Optimal *L* and *C* Values

*L*1	1.5 mH
*L*2	0.014273 mH
*L*3	0.20221 mH
*C*1	3951.1 μF
*C*2	221.87 μF
*C*3	10.8648 μF

6. CONCLUSIONS

The paper presents a new problem formulation for designing a reactance one-port compensator. The system was modeled in time-domain in order to account for both the diversity and the attenuation phenomena while designing the compensator. The problem is formulated as an NLP problem where the main objective is to match the load and compensator susceptances at different harmonics by minimizing the error between them. The simulation results show that the compensator is capable of minimizing the THD value significantly.

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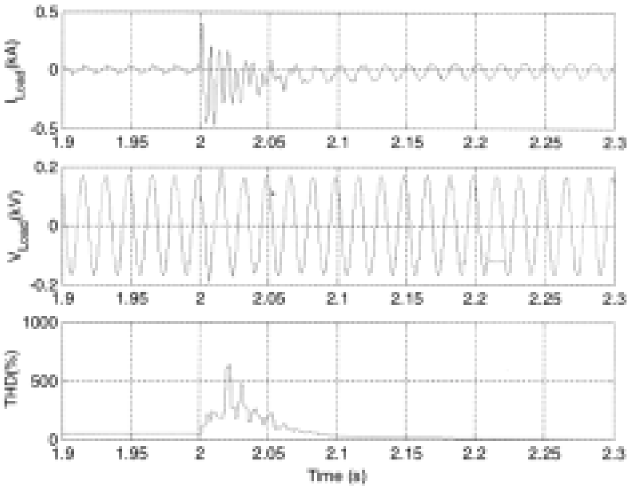

Fig. 11. Load Current, Voltage and current THD

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