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SINGLE-PHASE POWER ACTIVE FILTER USING INSTANTANEOUS REACTIVE POWER THEORY —THEORETICAL AND PRACTICAL APPROACH

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Summary: The paper deals with the new method of analysis, synthesis and experimentation of single-phase power active filters. By using a new particular transformation theory, the ordinary single-phase system can be transformed into an equivalent two-axes orthogonal one. The new original thought is based on the idea that ordinary single-phase quantity can be complemented by fictitious second phase so that both of them will create an orthogonal system, as is usual in three-phase systems, in spite of [1]. Application of the above theory makes it possible to use complex methods of analysis as the instantaneous reactive power method, which have not been usable for single-phase systems so far. Both, the active and reactive powers can be determined by this way. Practical application of the method is outlined for the case of reference current determination for single-phase power active filter. The paper shows some examples of the simulation verification results, which proved a high accuracy and extremely fast response of the single-phase active filter with control, based on the introduced method. The effectiveness of proposed control algorithm is also demonstrated by experimental results, which were carried out on the single – phase active parallel filter, power rate of 25 kVA and controlled by 32 – bit floating point digital signal processor TMS 320C31.

Key words:
Power active filter
P-q power theory
Orthogonal transformation
Single-phase system
Fictitous-virtual quantity

1. USING ORTHOGONAL TRANSFORMATION FOR SINGLE PHASE POWER ACTIVE FILTER

This method was first time described by authors in [3]. Now, we revise just the basic of that theory. So, let's have a single-phase system defined as:

$$u(t) = U\cos(\omega t); \qquad i(t) = I\cos(\omega t - \varphi) \tag{1}$$

After complementing by fictitious imaginary phase defined as:

$$u_i(\mathrm{t}) = U_i\sin(\omega t); \qquad i_i(t) = I_i\sin(\omega t - \varphi) \tag{2}$$

we obtain an orthogonal co-ordinate system with:

$$u_\alpha = u\,(t) \quad \text{and} \quad u_\beta = u_i(t) \tag{3}$$

Generally, the fictitious phase can be created by shifting the ordinary single-phase quantity to the right with phase shift equal to $-\pi/2$. It follows out from the 4-side symmetry of vector quantity trajectory in Gauss plain [2], see Figure 1.

The following equation must be valid for quantity with 4-side symmetry:

$$x^*(t) = x^*(t - \mathrm{T}/4)\,\exp(\mathrm{j}\pi/2) \tag{4}$$

and also:

$$x(t) = -x(t - \mathrm{T}/2) \quad \text{and} \quad x_i(t) = x(t - \mathrm{T}/4) \tag{4a}$$

Finally, the general transform equation can be introduced for single-phase system:

$$x^*(t) = \mathrm{K}\,[x(t) + \exp(\mathrm{j}\pi/2).x_i(t)] \tag{5}$$

where K is multiplicative constant (equal to 1 for single-phase system) and:

$$x_\alpha = x(t) \quad \text{and} \quad x_\beta = x_i(t) \tag{6a,b}$$

Note, that now, the Fourier analysis of investigated quantity is possible to do in 1/4 of time period as seen in Figure 1.

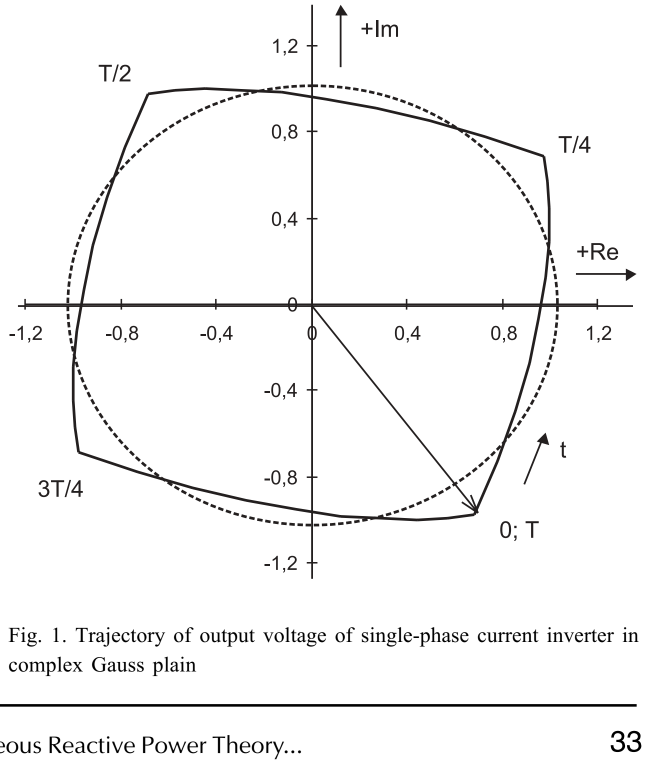

Fig. 1. Trajectory of output voltage of single-phase current inverter in complex Gauss plain

Assume now, for simplicity, harmonic waveforms of phase-voltage and rectagonal phase-current, Fig. 2a,b,c:

$$u_{\mathrm{RE}}(t) = U\cos(\omega t) \qquad u_{\mathrm{IM}}(t) = U\sin(\omega t) \tag{7a,b}$$

2. USING OF INSTANTANEOUS REACTIVE POWER METHOD FOR COMPENSATING AND FILTERING HIGH ORDER HARMONICS

Using time-sub-optimal analysis in transformed orthogonal co-ordinates for 4-side symmetry an average value of active power P_{AV} of an original (real) phase:

$$P_{AV} = P_{\alpha\beta AV}/2 = (2/T)\int_0^{T/4}\left[u_\alpha i_\alpha + u_\beta i_\beta\right]dt \tag{8}$$

Average values of active power of fictitious phase P_{iAV} and reactive powers of both original Q_{AV} and fictitious Q_{iAV} phases can be determined by similar way.

Utilisation of the instantaneous reactive power method used in [1] for three-phase systems, and above theory is allowed its use for single-phase systems as well, thus:

$$p_{\alpha\beta} = u_\alpha i_\alpha + u_\beta i_\beta$$

$$q_{\alpha\beta} = u_\alpha i_\alpha - u_\beta i_\beta \tag{9 a,b}$$

where $p_{\alpha\beta}$ and $q_{\alpha\beta}$ are the instantaneous active and reactive powers of both phases in orthogonal coordinates, respectively. For harmonic waveforms they are equal to constants, and the power factor is possible to determine by phase shifting:

$$\varphi = \mathrm{arctg}\,(q_{\alpha\beta}/p_{\alpha\beta}) \tag{10}$$

It's important that $p_{\alpha\beta}$, $q_{\alpha\beta}$ and φ are in this harmonic case determined instantaneously, what is the essential contribution of the introduced method.

As an example the instantaneous $p_{\alpha\beta}$, $q_{\alpha\beta}$ components of the power are depicted in Figure 3.

Time-waveforms of instantaneous p- and q-components of the power are shown in Fig. 4a, 4b, respectively.

3. CREATING OF VIRTUAL *p–q* MODEL OF SINGLE-PHASE PAF

The arrangement of real- and fictitious imaginary phases of single-phase system is depicted in Figure 5, where imaginary phase is shifted by 90 degrees el. Note that both phases are completely separated, however they are synchronised by signal SYNC, see Fig. Such arrangement implies that zero component of any quantity of x_0 (e.g. u_0, i_0) is *a priori* zero. It is also clear that active and reactive power component of real phase is a one half of total active and reactive power of the system, following Eq. (8).

Anyway, the fictitious imaginary phase cannot be built in fact, it's not necessary, because it can be created by software, using by memorizing of the sampled data of the real phase. Memorizing of data is necessary in any case for computing of average integrate values of the power.

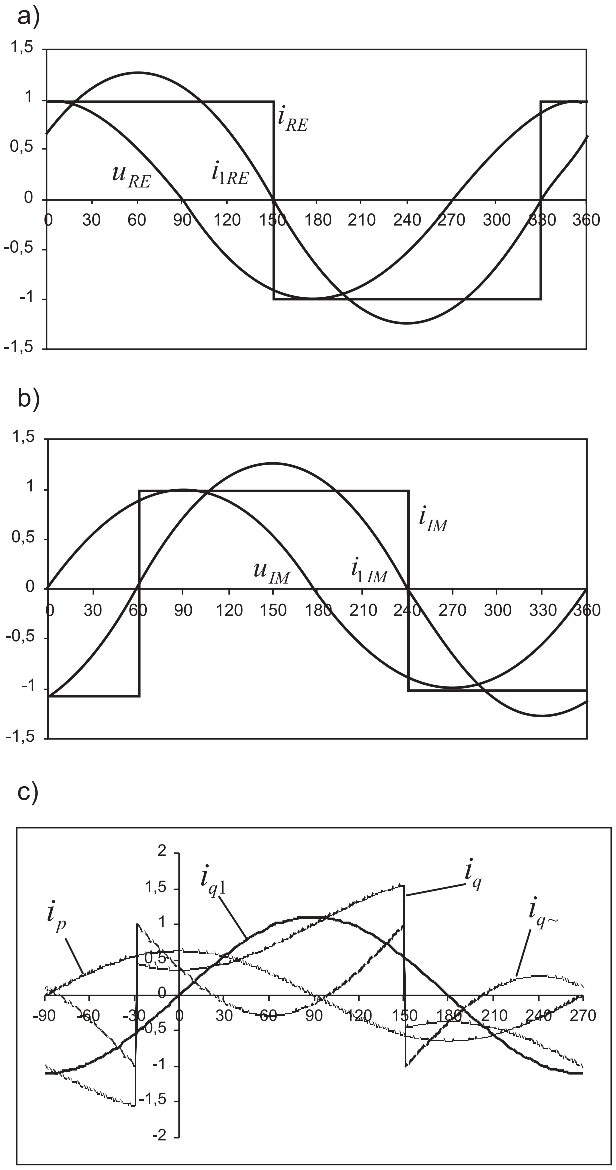

Fig. 2 a, b) Time—waveforms of real- and imaginary phase voltages and currents; c) active- and reactive components of phase current

Using *p-q-r* power theory introduced by Kim and Akagi *et al.* [8], [13] allows to present the power situation in rotation frame, Figure 6.

The denotes in this figure mean:

$$u_{\alpha\beta} = \sqrt{u_\alpha^2 + u_\beta^2} \tag{10}$$

and

$$\theta = \tan^{-1}(u_\beta/u_\alpha) \tag{10a}$$

Due to zero value of u_0, i_0 the r-axis will be identical one to 0-axis and $u_{\alpha\beta 0}$ voltage will be the same as $u_{\alpha\beta}$. Then the following transform relations will be valid:

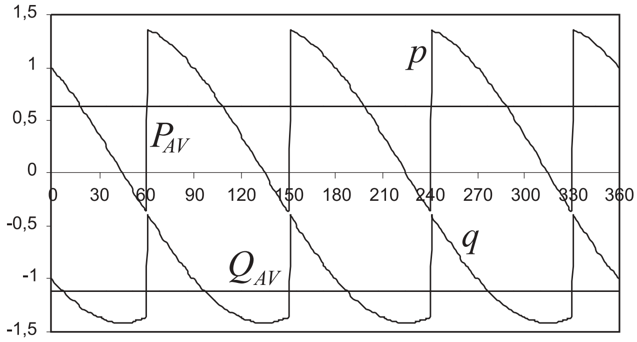

Fig. 3. Time-dependence of instantaneous $p_{\alpha\beta}$, $q_{\alpha\beta}$ components of the power for non-harmonic load

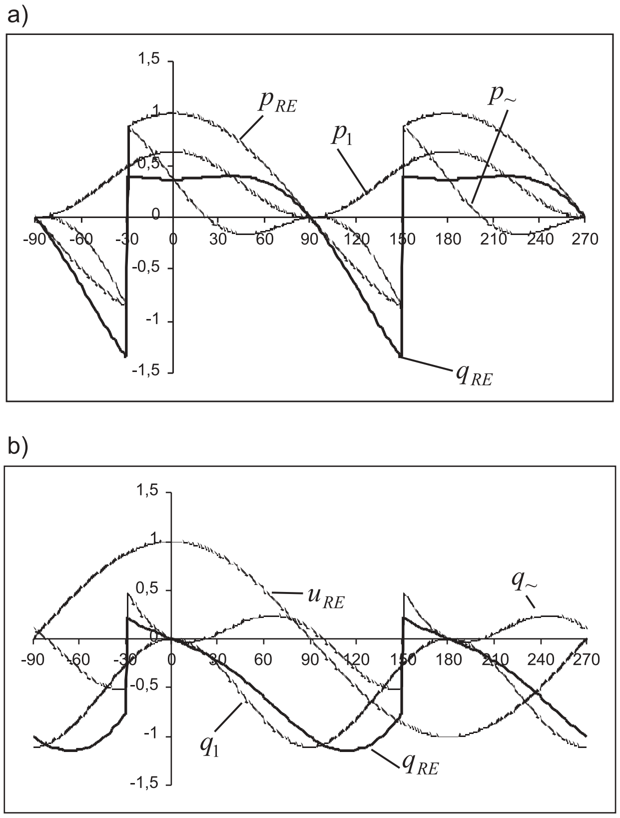

Fig. 4 a, b) Time-waveforms of instantaneous *p*- and *q*-components of the power

$$\begin{bmatrix} u_p \\ u_q \\ u_r \end{bmatrix} = \frac{1}{u_{\alpha\beta}} \begin{bmatrix} u_\alpha & u_\beta & 0 \\ 0 & 0 & 0 \\ 0 & 0 & u_{\alpha\beta} \end{bmatrix} \begin{bmatrix} u_\alpha \\ u_\beta \\ 0 \end{bmatrix} = \begin{bmatrix} u_{\alpha\beta} \\ 0 \\ 0 \end{bmatrix} \tag{11}$$

and similarly:

$$\begin{bmatrix} i_p \\ i_q \\ i_r \end{bmatrix} = \frac{1}{u_{\alpha\beta}} \begin{bmatrix} u_\alpha & u_\beta & 0 \\ 0 & 0 & 0 \\ 0 & 0 & u_{\alpha\beta} \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \\ 0 \end{bmatrix} = \begin{bmatrix} i_{\alpha\beta} \\ 0 \\ 0 \end{bmatrix} \tag{12}$$

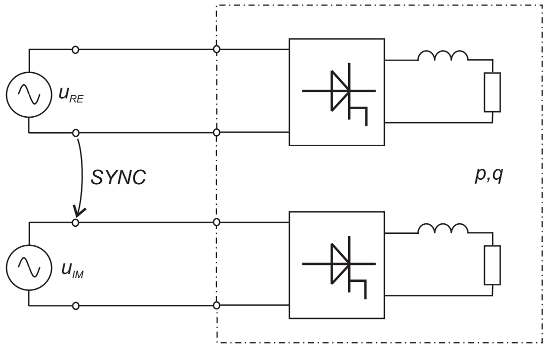

Fig. 5. Arrangement of real- and fictitious imaginary phases of single-phase system

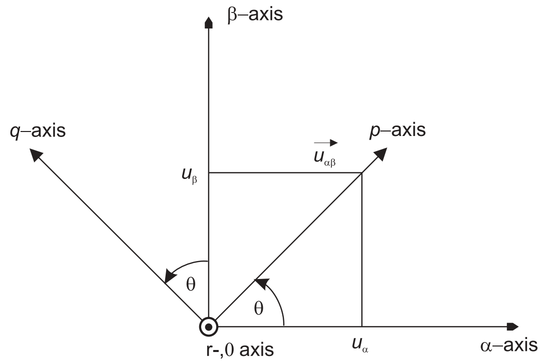

Fig. 6. Power co-ordinates of single-phase active filter in rotary frame

where:

$$i_{\alpha\beta} = p_{\alpha\beta} / u_{\alpha\beta} \tag{a}$$

whereas:

$$p_{\alpha\beta} = u_\alpha i_\alpha + u_\beta i_\beta = p \tag{b}$$

Finally we can state:

$$u_p = u_{\alpha\beta} \tag{13a}$$

$$i_p = i_{\alpha\beta} \tag{b}$$

$$u_q = u_r \equiv i_q = i_r = 0 \tag{c}$$

$$p = u_p \, i_p \tag{d}$$

Reference current determination

From (9a,b) can be obtain

$$\begin{matrix} i_\alpha \\ i_\beta \end{matrix} = \frac{1}{u_{\alpha\beta}^2} \begin{bmatrix} u_\alpha & -u_\beta \\ u_\beta & u_\alpha \end{bmatrix} \begin{bmatrix} P_{AV} + p_\sim \\ Q_{AV} + q_\sim \end{bmatrix} \tag{14}$$

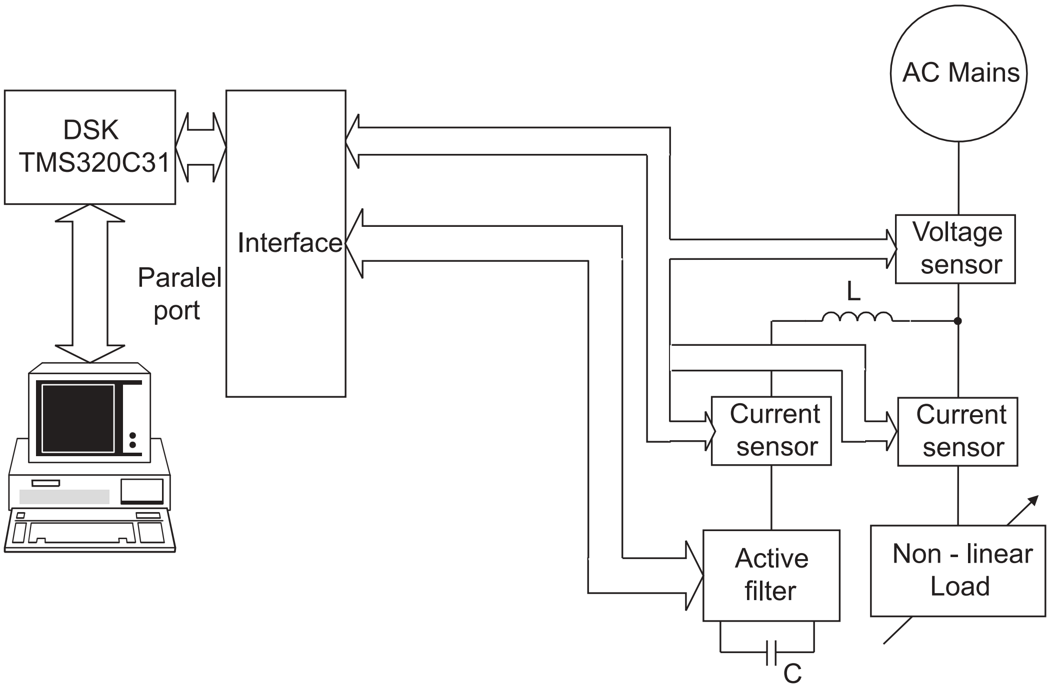

Fig. 7. Configuration of experimental system

Now, it depends on what we would like to compensate or to filter, respectively:

a) Case of compensation of whole reactive- and distortion powers. Then it is necessary to compensate Q_{AV} – component as well as to filter $q_\sim$- *and* $p_\sim$- components

$$i_{REF} = 1/u_{\alpha\beta}^2 \left[u_\alpha \left(p - P_{AV}\right) - u_\beta \left(Q_{AV} + q_\sim\right)\right] \quad (15)$$

b) Case of compensation of fundamental harmonic reac tive power only, that means to compensate Q_{AV} – componets

$$i_{REF} = 1/u_{\alpha\beta}^2 \left(-u_\beta Q_{AV}\right) = -\left(u_\beta / u_{\alpha\beta}^2\right) Q_{AV} \quad (16)$$

c) Case of filtration of distortion power only, i.e. $q_\sim$ – and $p_\sim$ – components of reactive power

$$i_{REF} = 1/u_{\alpha\beta}^2 \left(u_\alpha p_\sim - u_\beta q_\sim\right) \quad (17)$$

4. EXPERIMENTATION AND CARRIED-OUT RESULTS

The proposed control strategy was implemented into 32 – bit floating point digital signal processor TMS 320C31. The configuration of the used experimental system is shown in Figure 7.

The output current of the active power filter is controlled by a hysteresis comparator and with the maximum switching frequency of 15 kHz.

The time waveforms of load current, compensating current of power active filter and source current are depicted in the Figure 7.

Diode bridge rectifier with RL load on the DC side as a non – linear load was used. Power rate of the used inverter is 25 kVA, inductance L of output filter is 1.2 mH, and the capacity of condenser on the DC side is 10000 μF.

Those have been carried out on experimental test – rig at laboratory in the University of Zilina.

These results agree with theoretical assumptions.

5. USING HALF-BRIDGE CONNECTION OF POWER ACTIVE FILTER

The current of the load, which needs (considering its nonlinear nature) to be supplied by reactive and distortion powers will be composed from the fundamental harmonic component with unity power factor against the network supply voltage and harmonic components, to compensate the po-

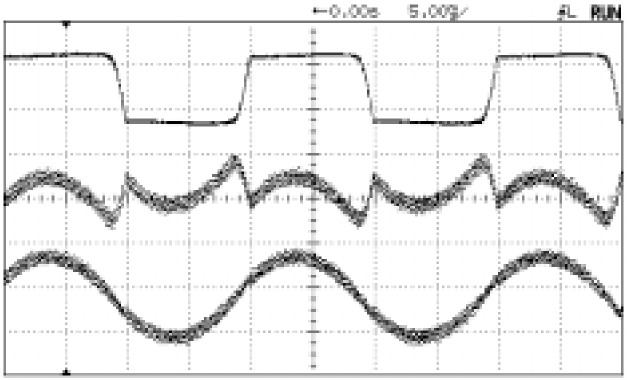

Fig. 8. Experimental results of single – phase power active filter with hardware hysteresis comparator

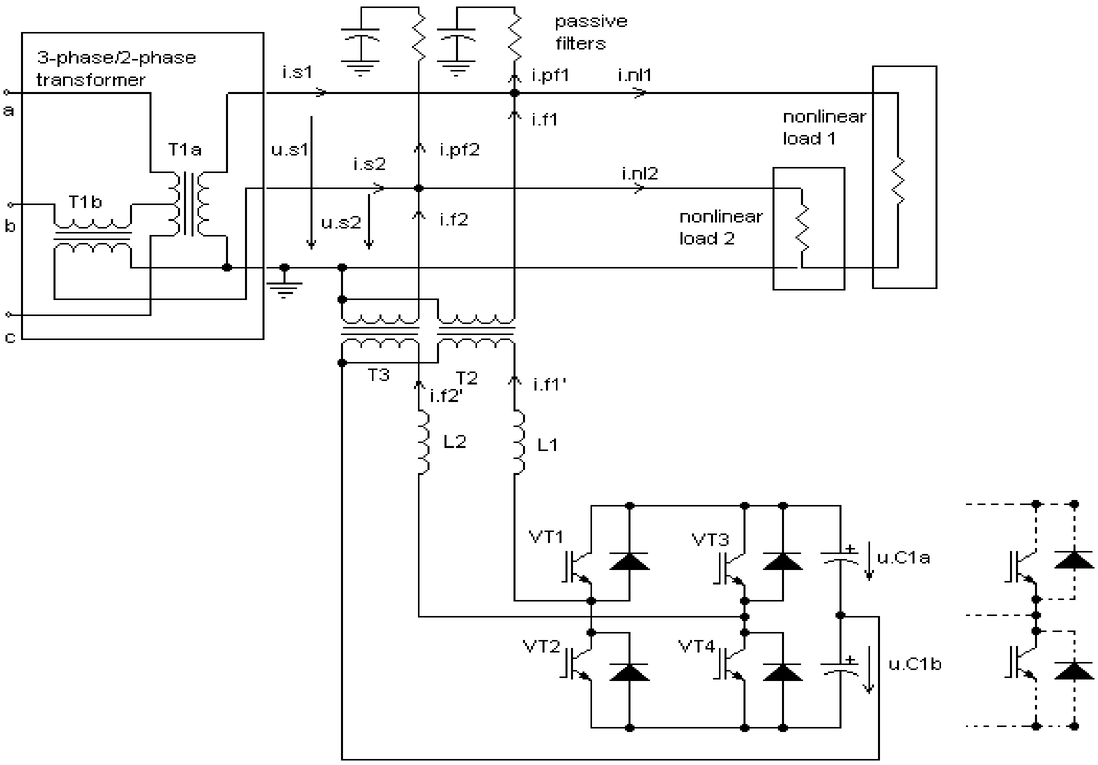

Fig. 9. Basic schemes of power active filtering for AC traction power supply

wer factor and to filter the harmonic components and it is the best to do it just in the point of their origin. Active filter can do this event in case of abrupt changing of the load. In a AC traction power supply of 25 kV/50 Hz the power active filter can both compensate fundamental harmonics and filter high order harmonic components [5].

Note, the power active filter works in half-bridge connection, which has been mostly used.

The simulation experiments of power active filter in half – bridge connection will be given in poster session, due to short connection (see Fig. 6), instead of a full bridge space of summary.

6. CONCLUSION

In this paper, the authors have proposed a new method to treat the single-phase system as the three-phase system. The virtual equivalent three-phase system is constituted of an ordinary single phase itself and a fictitious phase, which is created by shifting the ordinary single phase with a phase of p/2. Thereby, the way to deal with harmonics compensating and filtering for the single phase system can be carried out fast and accurately by applying the methods investigated extensively and used successfully for three-phase system. Especially, the *pq*-theory proposed by Akagi *et al.*[1], which has proved itself very robust even in case of the system voltage at the point of common coupling (PCC) is distorted and/or unbalanced, now can be applied for the single phase system. By simulation and experimental results the validity of the proposed theory is verified and shown a very good performance.

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