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A STRATEGY FOR AUTOMATIC ELIMINATION OF MECHANICAL INSTABILITY IN STRUCTURAL ANALYSIS OF SPATIAL TRUSS TOWER MODEL

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Abstract

A method for structural analysis of lattice transmission towers considered to be spatial trusses is proposed. It consists in automatic elimination of mechanical instability from the structure. The instability result from the existence of the so-called out-of-plane nodes in a structure model. The method is based on blocking the possible displacements of out-of-plane nodes and the information concerning the need for blocking the specific displacement is obtained from analysis of the system stiffness matrix. Through application of the spatial rod finite elements lattice transmission towers may be analysed with the use of spatial truss models. The short characteristic of the in-house computer program applied for identification of truss members not satisfying the load capacity criterion, comparison of the results obtained by use of the method and ones obtained by applying the plane truss model and the frame model with the reduced member flexural stiffnesses are also presented.

Introduction

In engineering practice there exist structures, which have computational models in form of spatial trusses. Due to numerical difficulties associated with these models, they are commonly replaced with frame models. It can be shown that the member forces in frame model differ from those that occur in the spatial truss model under the same loading conditions. So the question is: do computational difficulties have to dictate the need to seek a substitute for the spatial truss model?

In the case of applying additional loading to existing lattice transmission towers (e.g. with optical fibre cables) and assessment of their condition or determination of safe work time period for them, it is necessary to check the load capacity criteria for all members of the structure. In many countries the standard regulations recommend assuming the truss model of the tower. Then it becomes an important task to determine efficiently the values of member forces in the truss subjected to load combination specified in the regulations. During the times preceding the computer era the task was simplified, i.e. plane trusses corresponding to the individual faces of a tower were analysed. The plane truss member forces were being determined generally by the use of graphical or analytical methods. Currently computer programs, mainly based on the finite elements method (FEM), are used for that purpose (RAKOWSKI, KACPRZYK 2005). Breaking the spatial truss down into plane substructures is not only an arduous method of low effectiveness but also it may be a source of errors due to both the simplifications related to that method and the process of specifying loads applied to plane trusses. Lattice transmission towers in their nature are spatial structures and treating them as three-dimensional systems should be the natural consequence of that fact.

Recently a large number of papers have been published on many important aspects of the analysis of towers for overhead transmission lines such as: non-linear analysis (ALBERMANI, KITIPORNCHAI 2003), prediction of structure limit load (KEMPNER et al. 2002) or dynamic analysis (GANI, LÉGERON 2010). We should, nevertheless, differentiate between the research papers from specific requirements formulated in the standard regulations in force in a given country. The former ones allow better understanding of the structure behaviour and more precise assessment of the safety reserve as well as improving the applicable standards while the latter ones must be met absolutely in relation to both the designed structures and the existing ones but additionally loaded or undergoing a change caused by the influence of various factors (e.g. corrosion, ageing).

Truss is one of the simplest load bearing structure but structural analysis of a three-dimensional truss tower model by FEM encounters serious difficulties related to the mechanical instability of the system resulting from the occurrence of the so-called out-of-plane nodes. Attempts at overcoming that difficulty were undertaken in various ways, e.g. by using beam elements for analysis of the spatial truss or by inserting internal dummy rods with lower longitudinal stiffnesses than the truss members (DA SILVA et al. 2005). Manual addition of the dummy rods to the computational model is a time consuming and arduous work. Additionally, a priori prediction of the rods stiffnesses is not an easy task and it may yield erroneous results. A similar situation appears in the analysis of truss by the use of beam elements but reduction of members bending stiffnesses is required. Excessively small reduction results in a stiffer frame structure and the axial force values obtained may be different than for the truss. Excessively large reduction on the other hand may cause numerical difficulties resulting from poor conditioning of the system stiffness matrix. The use of beam elements requires additional input data for each element. It is related to the necessity of entering the points coordinates to define the orientation of the principal inertia axes of the beam cross section in space.

This paper presents the effective method for overcoming the inherent difficulties encountered in spatial truss analysis by FEM. It consists in automatic blocking of the additional degrees of freedom of the structure related to the out-of-plane nodes by means of the additional external dummy rod elements. Their locations are determined on the analysis of the stiffness matrix of the structure.

Automatic elimination of the mechanical instability of the truss tower model

The process of standard structural analysis of a lattice tower consists of some stages: assuming the computational model of the structure as the truss, determining the loading, applying forces to nodes, computing truss member forces and verifying the load capacity criterion for all the members.

Figure 1 presents a fragment of the truss tower model with two adjacent faces. The nodes marked with black circles referred to as the spatial nodes are characterised by the rule that displacement of such a node in any direction requires elongation/shortening of the members connected to the node. The other nodes marked by circles (out-of-plane nodes) have constrained mobility in the plane of one face but in the direction perpendicular to that face they can displace. The occurrence of the out-of-plane nodes in the truss structure results in its mechanical instability, i.e. system stiffness matrix becomes singular.

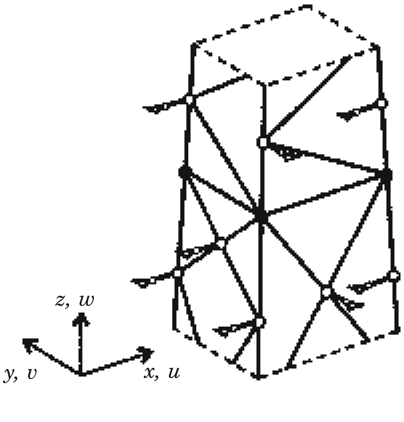

Fig. 1. Out-of-plane nodes and dummy rods

On the whole three-dimensional truss model cannot be analysed in a simple way by FEM using the truss elements, i.e. 2-noded truss elements with six degrees of freedom. To overcome the numerical difficulties it is possible to apply additional constraints on the degrees of freedom of out-of-plane nodes. For example, it is possible to impose the condition that the out-of-plane nodes are subject to displacements assuring that they remain within the plane of the face in which the members connected to the nodes are located. Commercial general-purpose FEM programs allow applying constraints of that type, although manual identification of out-of-plane nodes within the tower computational model and defining for each of them additional constraints may be a highly work-intensive task. That is why a method was proposed for automatic blocking of out-of-plane node displacements by attaching dummy rods to the nodes to eliminate their undesirable mobility (Fig. 1). Each such a rod blocks one degree of freedom of a node. The method, however, should be found for identification of the degrees of freedom that require blocking. It can be noticed that in the truss with out-of-plane nodes, considered within the global system of coordinates x, y, z, on the main diagonal of the structure stiffness matrix, in cells corresponding to degrees of freedom related to the possible displacements of those nodes, terms of low values, differing clearly from the other terms, occur. In case of the out-of-plane nodes located on the non-tapered tower segments the values of those terms can be zero. For the purpose of determining the condition of blocking the excess degrees of freedom the value of the so-called blocking threshold must be determined. It can be set quite precisely on the examination of the main diagonal terms of the stiffness matrix of structure with out-of-plane nodes not blocked.

In-house computer program for structural analysis of lattice towers

The presented method for dealing with out-of-plane nodes may be applied if access to the system stiffness matrix is possible. Most FEM programs give such a possibility. In the in-house computer program targeted to the effective structural analysis of lattice transmission towers the presented method for automatic elimination of the mechanical instability from the spatial truss model was applied. The program was developed considering the necessity for efficient analysis of a large number of truss towers. The computations were performed mainly because of the additional loading of existing structures with optical fibre cables and in the process of determination of safe work time period for old towers.

The analysis of a truss tower model is performed in two stages. During the first stage, after assembling the structure stiffness matrix and application of the boundary conditions, the program subroutine examines the terms on the main diagonal of the matrix and if the term value is lower than the blocking threshold the given degree of freedom is being blocked. This involves generating the additional massless rod and setting it perpendicularly to the appropriate tower face. One end of the rod is located in the node to be blocked and the other is pinned supported. When the automatic addition of all the necessary dummy rods is completed the second stage of the analysis follows which is typical for the finite element analysis of a structure. The in-house program is provided with the procedure analysing the values of forces in dummy rods. In case of blocking an inappropriate degree of freedom a noticeable force value occurs and the user is alerted. The practice shows that only in a few cases minor modifications (addition or removal) of dummy rods in the tower model were necessary.

A CAD type program (in this case IntelliCAD) equipped with a set of functions written in the AutoLISP language performs the pre-processor and post-processor tasks in the tower analysis. On completing the tower finite element model its database is exported in DXF format, which is then read and processed by external program (data generator) to write the model data text file readable for the in-house program. The program is based on the FEM using three-dimensional truss-type elements (axial tension or compression), however, the method of blocking out-of-plane nodes and analysis of load capacity criterion, according to the standard regulations in force, were implemented. The program determines safety factor for each tower member in case of the given load combination. The factor depicts the degree of satisfying the member load capacity criterion. Program also generates the drawings of all the tower faces and the cross-arms and linking elements in which the hazardous members are marked in the appropriate colour. The drawings allow rapid location of unsafe members in the structure analysed.

Results generated by 3d truss tower model vs. the results obtained using the plane model and frame model

For the verification of the presented method of handling with out-of plane nodes in the three-dimensional truss tower model the results of computations obtained using the method were compared with those obtained with the use of the plane truss model and the spatial frame model. The spatial truss model was computed using the in-house program while the plane and frame models were solved using MSC.Marc Mentat system.

The actual transmission tower of 38.83 m high loaded with wind acting in the direction perpendicular to the transmission line (direction of the x axis depicted in Fig. 2a) was chosen for the comparative study. Under conditions of normal rime that load case represents in general the most rigorous test of load

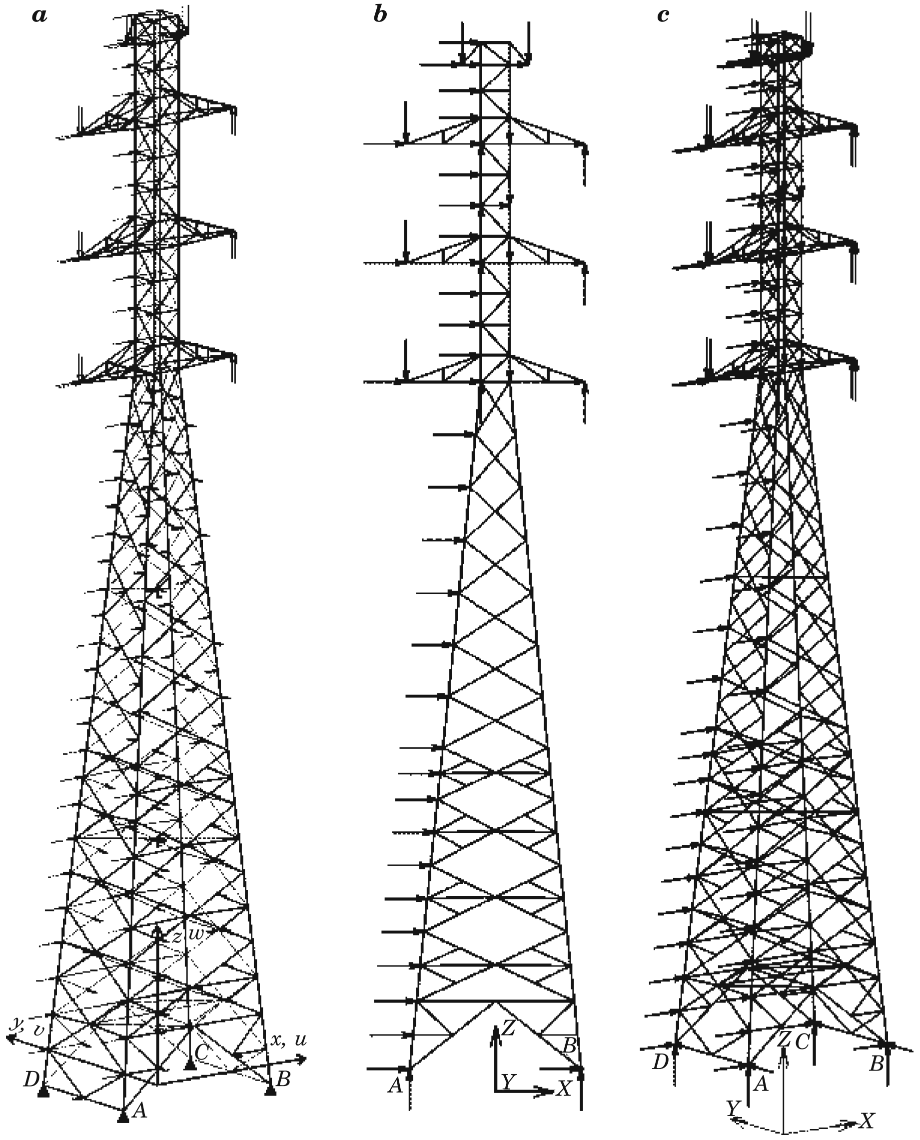

Fig. 2. Transmission tower models: *a* – 3D truss with out-of-plane nodes degrees of freedom blocked, *b* – plane truss, *c* – 3D frame

bearing capacity of tower. The plane model of the tower was defined by the *AB* tower face with out-of-plane nodes removed.

Three-dimensional truss tower model

To determine the value of the blocking threshold for the structure analysed in the global coordinate system (x, y, z) shown in Figure 2a, the values of k_{ii} terms located on the main diagonal of the structure stiffness matrix are being watched. After arranging those terms in ascending order the graph of their variability can be drawn. It can be seen from Figure 3 that some terms have low values (the initial part of the graph) and they correspond to the undesirable degrees of freedom of the model. For the tower under consideration the blocking threshold has the value $7.2 \cdot 10^6$ N/m and on its base the program has generated dummy rods that allow blocking the excess degrees of freedom of the truss model. Given small taper of the tower body (1:10 in xz plane and 1:14 in yz plane) the rods are rotated by small angles about the x or y axis to set them perpendicular to the appropriate tower face. The taper of the tower body is defined as the value of the tangent of the tower face slope to the vertical plane. In the case considered 148 dummy rods were generated: 54 in the direction close to the x axis and 92 to y axis and 2 in the direction of z axis.

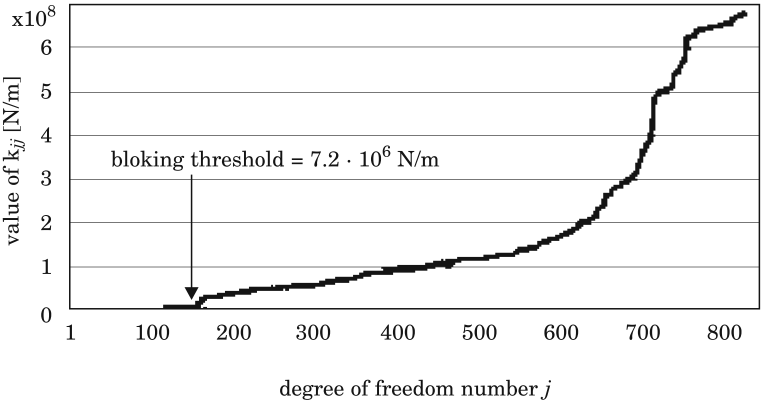

Fig. 3. Stiffness matrix diagonal terms values for the 3D truss tower model

External loads are applied to the nodes in the directions that do not correspond with the blocked degrees of freedom of the model. Nevertheless, as a result of self weight loading vertical forces are applied to all nodes, including the out-of-plane nodes. As the dummy rods are not positioned horizontally, negligible forces due to the structure self weight occur in them. Maximum values of the reaction forces components at the supports of dummy rods (0.099, 0.025, 0.343 kN) divided by the minimum values of reaction forces in nodes at the tower base (19.03, 8.98, 129.67 kN) are: $R_{fx}/R_{x\min} = 0.52\%$, $R_{fy}/R_{y\min} = 0.28\%$ and $R_{fz}/R_{z\min} = 0.26\%$ respectively. The possibility of shifting the gravity forces from the out-of-plane nodes to the adjacent "normal" nodes could be considered, however, as a consequence of minor values of the forces in dummy rods induced by the gravity forces that action is not necessary. Very low values of reaction forces by the dummy rods indicate also blocking the appropriate, i.e. excess, degrees of freedom of the structure.

Plane tower model

The plane model was obtained by projecting the AB tower face, shown in Fig. 2a, on the vertical plane containing points A and B. The arrangement of members in this model is presented in Fig. 2b.

Three-dimensional frame tower model

The frame model shown in Fig. 2c can well predict the values of member axial forces in the lattice tower treated as the truss if the flexural stiffnesses of the members is very low. It cannot, however, be excessively small because then the mechanical instability may occur as in the case of the truss model with out-of-plane nodes. The issue of reducing the tower beams flexural stiffnesses must be solved then.

The members of the analysed tower have angle sections but for the purpose of reduction of data input their cross sections were substituted with circles. The cross section area of a circle was equal to the actual one whereas moment of inertia was assumed to be equal to the minor principal moment of inertia for the member angle section.

For the purpose of selecting the appropriate degree of reduction for members flexural stiffnesses the frame model was computed several times changing the values of the moment of inertia. The values of actual minor moments of inertia were assumed to be the reference values. Changed values of moments of inertia were obtained by multiplying the reference values with a factor (tab. 1).

The consecutive rows of the table present the values of the: extreme axial force, extreme bending moment, square root of the sum of squares of displacements for all the nodes of the structure and displacements of nodes that revealed large variations. Analysis of Table 1 indicates that the frame with the

Table 1

Influence of the frame members flexural stiffness on section forces and displacements of selected nodes

	$I/I_{\min}$	0.01	0.02	0.1	0.2	0.5	1	2	5	10
$N_{\max}$	kN	-164.7	-164.7	-164.7	-164.7	-164.7	-164.6	-164.5	-164.2	-163.7
$M_{\max}$	kNcm	7.2	7.2	7.2	7.7	13.9	25.7	49.1	111.5	198.6
Δ	cm	148.49	143.95	142.43	142.36	142.28	142.17	141.97	141.42	140.65
u_2	cm	8.17	4.09	0.82	0.41	0.16	0.08	0.04	0.02	0.01
v_{169}	cm	0.58	0.29	0.05	0.02	0.00	0.00	0.00	-0.01	-0.01
w_2	cm	-0.78	-0.38	-0.07	-0.03	-0.01	0.00	0.00	0.01	0.01

lowest member flexural stiffnesses but still assuring the stable solution should have the elements with the values of the moments of inertia reduced 5-fold in relation to the values of the minor moments of inertia of the angle cross sections. In particular the monotonically decreasing function Δ has the inflection point for $I/I_{\min} = 0.2$ as shown in Figure 4. Additionally, the bending moment starts changing its value significantly at that point. The results obtained for that frame model were further compared with those for the truss models of the tower.

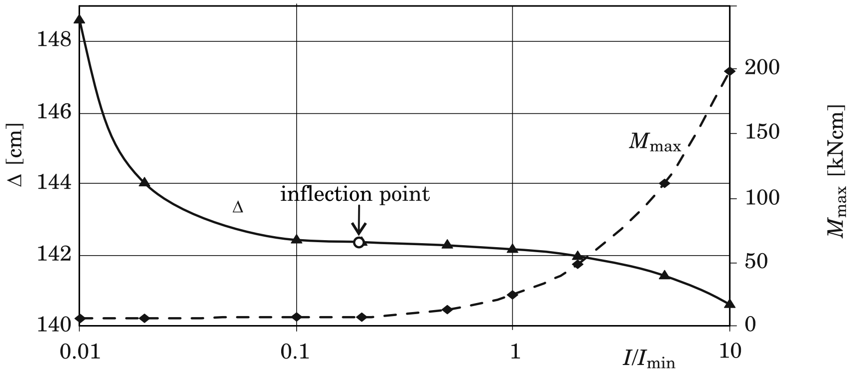

Fig. 4. Variations in the root from the sum of squares of displacements for all nodes and the extreme bending moment in the function of the members flexural stiffness parameter

Computational results

Table 2 presents the parameters of the tower models and the extreme values of the tower crown displacement and member axial forces obtained. Deformation of the tower model with the dummy rods attached to it and a real tower deformation do not coincide; nevertheless, the value of crown displacement is almost the same as for the plane model and for the frame model.

Table 2

Specification of the differences in the tower models analysed

Tower Model	3D truss	Plane truss	3D frame
Number of elements	829	217	681
Number of nodes	424	110	276
Number of degrees of freedom	1272	220	1656
Crown displacement [cm]	21.26	20.53 (-3.4%)	21.26 (0.00%)
Max axial force [kN]	127.00	125.57 (-1.13%)	126.90 (-0.08%)
Min axial force [kN]	-164.58	-164.15 (-0.26%)	-164.69 (0.07%)

The values of extreme axial forces determined using the analysed models are very close. The percentage differences do not exceed 1.2%. That observation, however, do not apply to all the members. In Figure 5 and 6 the percent differences in force values are presented. The differences are small in significantly loaded members. Larger differences occur in slightly loaded members, which, however, are not critical elements in the assessment of the entire structure load capacity. It should be noticed, nevertheless, that in the plane model of the structure the distribution of axial forces is different than in the spatial model as a consequence of neglecting the slope of the tower face and disregarding the influence of the internal members. It was established that the points farthest away from the horizontal axis in the Figure 5 correspond to the members possessing common nodes with the members connecting opposite faces of the tower that are not considered in the plane model.

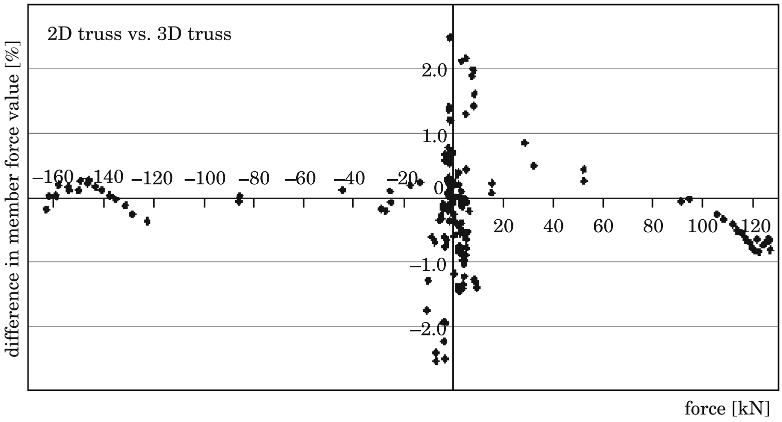

Fig. 5. Differences in the values of forces in the tower members obtained using the plane and the spatial truss models

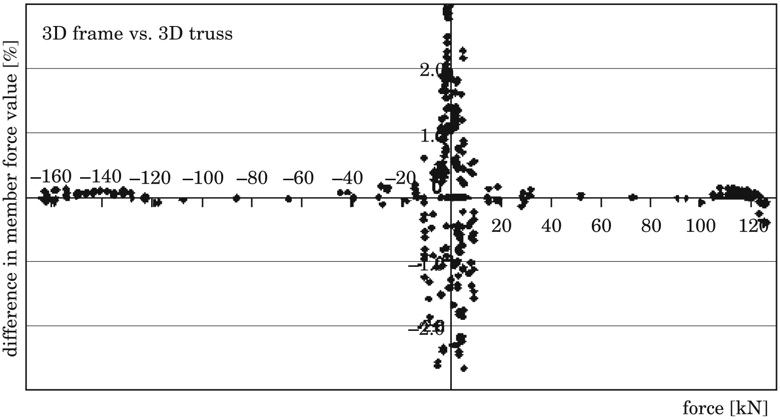

Fig. 6. Differences in the values of forces in the tower members obtained using the frame and the spatial truss models

Despite reduction of the members flexural stiffnesses, in the frame model the distribution of forces is different than in the truss model due to the influence of bending effects. The frame model gives low values of axial forces in the members that in the truss model are zero force members. In that type of members the percentage differences in force values are generally large. Opposite signs of forces in some insignificantly loaded members were also observed. For example, the member compressed by small force in the truss model is elongated by a small force in the frame model (e.g. member No. 21: N^{TRUSS} = -0.04 kN, N^{FRAME} = 0.03 kN). Then the corresponding points of the graph are outward the range presented and have not been shown in the figures.

Reasonably good correspondence between the values of the extreme displacements and forces (see Table 2) obtained using the analysed models confirms effectiveness of the presented method for automatic blocking of the mechanical instability in the three-dimensional truss tower model. It should be emphasised that the 3D truss tower model represents better the work of a spatial truss than a plane or frame model. This results from the fact that dummy rods are positioned perpendicularly to the members connected to the out-of-plane nodes and do not take any components of axial forces carried by the members.

Conclusions

Due to existence of the out-of-plane nodes in truss transmission tower model its numerical analysis, as three-dimensional structure, requires special approach. The proposed method of automatic removal of the mechanical instability from the finite element model with rod elements (tension/compression) has proved effective. The results obtained confirm the possibility of applying the described method of blocking out-of-plane nodes in models of typical lattice transmission towers.

Analysis of spatial lattice structures, especially with bolted or riveted joints, by the use of spatial truss model seems to be more natural approach then the use of substitute model i.e. frame model. Preparation of the truss tower model requires less time and is less arduous than determining of frame model because every frame element requires the coordinates of the additional node (point) defining the orientation of the neutral axes of the beam cross section in space and problems with reduction of beams flexural stiffnesses also appear. Models composed of truss elements allow shortening the work time necessary for preparing them.

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