SINGULAR VECTORS IN THE SYNTHESIS OF INDICES FOR ACOUSTIC ASSESSMENT OF OBJECTS

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Summary

The SVD (Singular Value Decomposition) technique makes it possible to transform a set of correlated data into uncorrelated ones without the loss of any information. The new system of mutually uncorrelated variables is comparable to the initial one. Apart from decorrelation of data, SVD makes full variable correlation possible, which ensures no copying of certain information during the addition of such new variables.

This article presents the application of the SVD technique for the construction of single-number indices for assessment of correlated object acoustic parameters using the methods of decorrelation and full correlation. Verification of the proposed single-number assessment indices was carried out using the example of sacral buildings. For one Roman Catholic church with flawed acoustics, the application of a single-number index for assessment of the variant of proposed acoustic adaptation of its interior was shown. The synthetic index applied for sacral objects, being an approximated general measure of assessment, can be a helpful tool for designers and during assessment of church interiors in terms of the acoustic functioning of the object.

Keywords: Singular Value Decomposition (SVD), decorrelation, index method of assessment.

WEKTORY SZCZEGÓLNE W SYNTEZIE WSKAŹNIKÓW OCENY AKUSTYCZNEJ OBIEKTÓW

Streszczenie

Technika rozkładu względem wartości szczególnych (SVD) umożliwia przekształcenie zbioru danych skorelowanych w dane nieskorelowane, bez utraty jakiejkolwiek informacji. Nowy układ zmiennych wzajemnie nieskorelowanych jest porównywalny z układem wyjściowym. Oprócz dekorelacji, SVD umożliwia korelację zupełną zmiennych, zapewniającą brak powielenia pewnych informacji przy sumowaniu doskonale skorelowanych nowych zmiennych.

W artykule pokazano wykorzystanie techniki SVD do konstrukcji jednoliczbowych wskaźników oceny skorelowanych parametrów akustycznych obiektów metodami dekorelacji i korelacji zupełnej. Weryfikację zaproponowanych jednoliczbowych ocen wskaźnikowych przeprowadzono na przykładzie kilku obiektów sakralnych. Dla jednego kościoła-rzymsko katolickiego o wadliwej akustyce pokazano zastosowanie jednoliczbowego wskaźnika do oceny wariantu zaproponowanej adaptacji akustycznej tego wnętrza. Syntetyczny wskaźnik zastosowany dla obiektów sakralnych, będący przybliżoną miarą ogólną oceny, może być pomocnym narzędziem dla projektantów oraz przy ocenie wnętrz kościelnych, związanej z prawidłowym pod względem akustycznym funkcjonowaniem obiektu.

Słowa kluczowe: Rozkład względem wartości szczególnych, dekorelacja, wskaźnikowa metoda oceny.

1. INTRODUCTION

Assessment using indices is very useful from the perspective of diagnostics as a description of the current state of quality or usefulness of a given object for fulfilling its utilitarian function. Indices make it possible to show and analyze complicated phenomena in simplified way and measure changes in these phenomena. Index methods also have applications in the study of vibroacoustic processes [3]. In singular cases, single-number indices are used to evaluate these processes. Global indices being an approximated general measure of the assessment are a function of several partial indices, giving more accurate information on the phenomenon. A quality of a good index is that it can be understood not only by specialists. Depending on the degree of aggregation, indices can be oriented towards the general public, politicians, scientists, and researchers [1]. Sometimes indices that are understood by the general public do not necessarily have to be sufficient for scientific research.

The construction of a synthetic global assessment can take place by means of different methods. One of them is the statistical method that is multi-variate analysis of comparative, related to the analysis of complex phenomena. According to the definition given by K. Kukula [12], a complex phenomenon is understood as an abstract construct representing the directly measurable state of quality of actual objects, described by a certain number, that is greater than one, of so-called diagnostic variables. Multi-dimensional methods of statistical analysis applied, among other fields, in econometrics, make it possible to present variables (object qualities) in such a form so that they can be compared directly. When the additive formula is used, the single-number index is based on adding the products of the values of standardized qualities and their corresponding weights. According to this method, diagnostic variables should be weakly correlated with each other and strongly correlated with the variable being interpreted.

During diagnosis of the acoustic state of the object, there is often a need to evaluate many of its qualities (or indices describing those qualities), sometimes strictly related to the functions and scope of acoustic production of the object. These indices are not always independent from one another. During synthesis of correlated indices, the problem of copying certain information arises. An attempt to solve this problem, which was the goal of the many years of study of the author, has been shown in this article. The approach to the construction of the single-number index is based on the use of SVD (Singular Value Decomposition). SVD is a computing technique that is widely propagated in numerical linear algebra [6, 13] and has applications in many fields of science, such as diagnostics [2] and vibroacoustics [3, 4, 5].

Until now, attempts to apply SVD in single-number assessment of acoustic quality pertained to a certain group of objects, being sacral objects [7, 8, 9, 11]. Further studies related to the assessment of acoustic quality will apply to other public and industrial objects. In the article, a method of constructing the single-number global index of acoustic assessment of widely understood objects was shown, however verification of the proposed index was conducted using the example of sacral objects. The proposed single-number assessment of the acoustic quality of objects can be carried out using two methods, leading to the same results – the values of the single-number index. Factor analysis and Principle Component Analysis in particular (PCA), which is easier to use using SVD, makes it possible to transform a set of correlated data into uncorrelated ones, without the loss of any information. The new mutually uncorrelated system of variables (so-called common factors or principal components) is comparable to the starting system. Apart from index decorrelation, a second method is their full correlation, taking place with the application of SVD, ensuring no copying of certain information during the addition of perfectly correlated indices.

2. SINGLE-NUMBER ASSESSMENT OF THE ACOUSTIC QUALITY OF OBJECTS

2.1. Single-number index with the application of full correlation

Full correlation of partial assessment indices, dependent on one another, is carried out using the SVD technique [9]. In order to obtain singular vectors and values – the components of index W_{kz}, it is necessary to construct the so-called index and strucutre observation matrix **A** from partial indices, which will be decomposed to singular values. Singular value analysis makes it possible to identify the percentage share of the information interpreted by successive components. The first component is usually the most informative, and when it is large in comparison to the remaining components, the computational model can be simplified by using first-order approximation

$$\mathbf{A}^{*} = \sigma_1 \mathbf{u_1} \mathbf{v_1} \tag{1}$$

where: $\mathbf{A}^{*}$ - matrix obtained as a result of first-order approximation (k=1), $\mathbf{u_1}$ – first column of matrix **U**, $\mathbf{v_1}$- the first row of matrix $\mathbf{V^T}$, σ_1 the first singular value of matrix **Σ**.

Matrix $\mathbf{A}^{*}$, approximating matrix **A** (with a certain Frobenius error), has components in the form of partial indices that are perfectly correlated with each other (the linear correlation coefficient between indices is equal to 1). Because new indices are perfectly correlated with one another, their sums can be calculated without obtaining redundant information that could appear in the case of adding indices that are not fully correlated. It is proposed to standardize the vector of obtained W_{kz} indices using the zeroed unitarization method, making it possible to obtain values in the interval from 0 to 1, according to the formula:

$$z_{ij} = \frac{x_{ij} - \min_i x_{ij}}{\max_i x_{ij} - \min_i x_{ij}} \quad \max_i x_{ij} \neq \min_i x_{ij} \tag{2}$$

Therefore, the single–number index obtained from full correlation

$$W_{kz_i} = norm(\sum_{j=1}^{n} a_{ij}^{'}) \tag{3}$$

where: $a_{ij}^{'}$ – components of matrix $\mathbf{A}^{*}$ of the i-th object and j-th partial index.

Index W_{kz} can also be obtained by using only the first left singular vector $\mathbf{u}_1$, which is usually best correlated with the initial partial indices (components of the observation matrix). Values of the single-number index correspond to elements of vector $\mathbf{u_1}$, which are to be standardized using formula (2).

$$W_{kz} = norm(\mathbf{u_1}) \tag{4}$$

where: W_{kz} – the vector containing single-number indices of objects.

2.2. Single-number index using decorrelation

The problem of index decorrelation was described in [8], where it was proposed to get rid of singular values and add independent products of vectors **u** and **v**, obtained from SVD. Currently, it is proposed to use all products obtained from SVD for single-number assessment. Observation matrix **A** can be written as follows

$$\mathbf{A} = \sigma_1 \mathbf{u_1 v_1} + \sigma_2 \mathbf{u_2 v_2} + \ldots + \sigma_n \mathbf{u_n v_n} \tag{5}$$

Successive products are related to independent object qualities, the variability of which is defined by singular values. The single-number index using decorrelation of W_d requires standardization. It is proposed to use the zeroed unitarization method for this purpose, as defined by formula (2). Index W_d is defined by the formula

$$W_d = norm\left(\sum_{j=1}^{n} \omega_j u_j v_j\right) \tag{6}$$

where: ω_j - weights of the j-th product corresponding to the j-th singular value, $\mathbf{u_j}$ - the j-th left singular vector (in matrix **U**, obtained from SVD of correlated indices of matrix **A**), $\mathbf{v_j}$ - the i-th right singular vector (in matrix $\mathbf{V^T}$, obtained from SVD of correlated indices of matrix **A**).

Weights ω_j are calculate from the formula:

$$\omega_j = \frac{\sigma_j}{\sum_{j=1}^{n} \sigma_j} \tag{7}$$

where: σ_j- the j-th singular value (in matrix Σ, obtained from SVD of correlated indices of matrix **A**).

Weights ω_j are calculated on the basis of a similar principle to that of the statistical method [12], in which quality variability coefficients (variance) are used for calculation of measures of relative informational value, being the weights. In the SVD method, measures of informational content are the singular components σ_j, used in formula (7).

3. VERIFICATION OF SINGLE-NUMBER ASSESSMENT OF CORRELATED INDICES USING THE EXAMPLE OF SACRAL OBJECTS

3.1. Studies of real objects

Values of calculated partial assessment indices have been determined on the basis of acoustic parameter measurements carried out in six Roman Catholic churches [9]. The strongly correlated three partial indices are: reverberation index W_P, music sound index W_M, and speech intelligibility index W_Z. The linear correlation coefficients between indices have been shown on table 1.

Table 1. Linear correlation coefficients between partial assessment indices

W_P <-> W_M	W_Z <-> W_P	W_M <-> W_Z
0. 9488	0. 9158	0. 9827

Using the Matlab software, SVD of observation matrix of three indices was carried out, as shown in Fig. 1. The first component is the most informational and contains 84% of information interpretation (Fig. 1). The methods of full correlation and decorrelation were used to determine single-number assessment indices W_{kz} and W_d (Fig. 2). In order to verify the full correlation of assessment indices, SVD was conducted again; this time, on matrix $\mathbf{A}^*$, containing perfectly correlated indices. A 100% information content of the first singular component confirms that the new variables in matrix $\mathbf{A}^*$ are perfectly correlated (Fig. 2a). Vector $\mathbf{u_1}$ can be successfully used to represent the data object of matrix **A** (formula (4)). The large values of linear correlation coefficients of vector $\mathbf{u_1}$ with initial indices (W_P, W_M, W_Z) as shown below are indicative of this: -0.9901, -0.9833 and -0.9598. Verification of index decorrelation (by carrying out SVD of matrix **B** – containing new uncorrelated variables) gave content of percent information interpretation by successive singular components equally divided between the new indices (Fig. 2b).

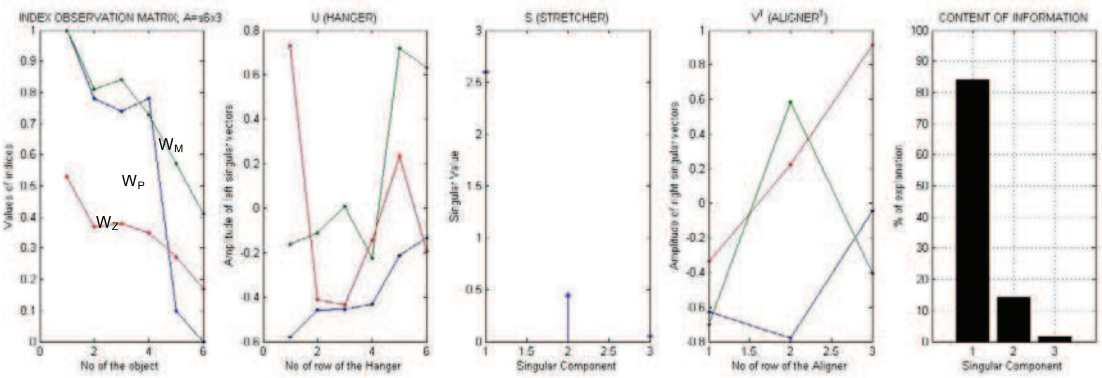

Fig. 1. SVD of a three-index observation matrix of sacral objects

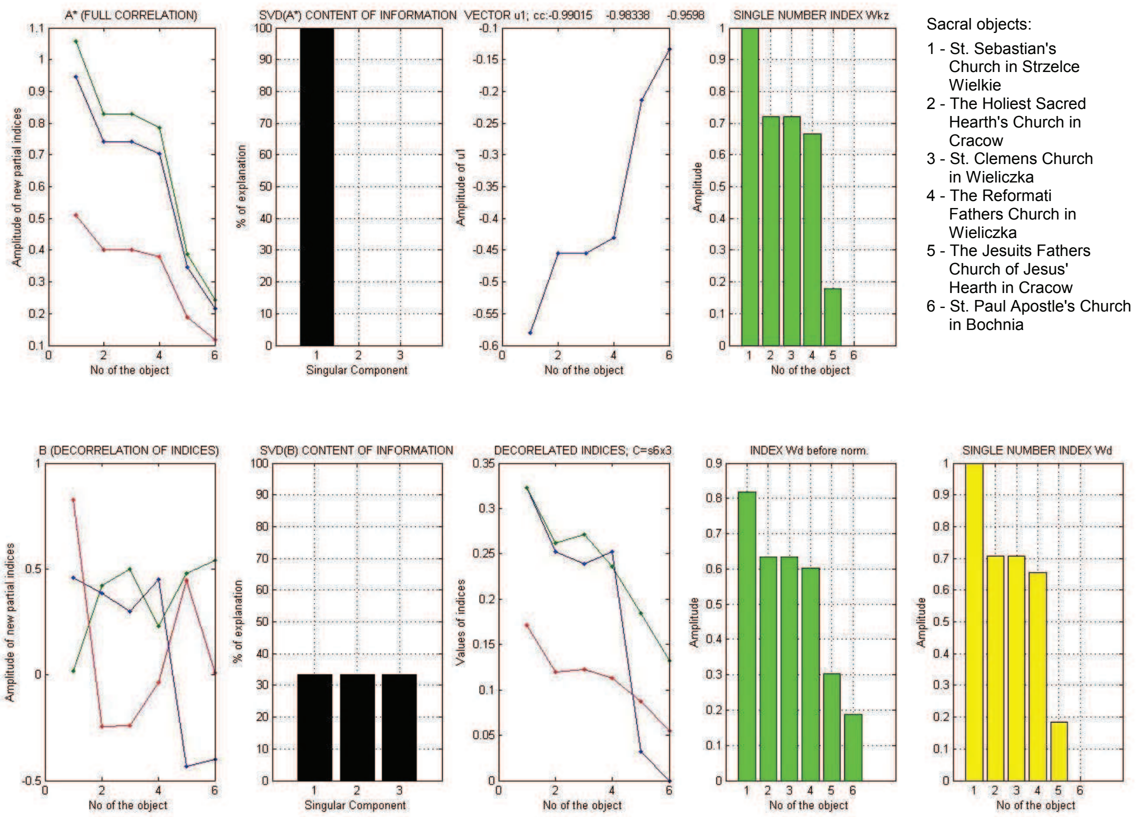

Fig. 2. Comparison of the full correlation method (Fig. a) and the decorrelation method (Fig. b) of correlated indices in the synthetic single-number assessment

The results obtained using the methods of full correlation and decorrelation in the form of single-number indices W_{kz} and W_d are identical (fig. 2). The best acoustic qualities were exhibited by the wooden church of St. Sebastian in Strzelce Wielkie (W_{kz}=1) and the worst qualities were exhibited by the churches of: the Jesuit Fathers in Cracow (W_{kz}=0.18) and the modern St. Paul the Apostle's Church in Bochnia, based on an elliptical lay-out (W_{kz}=0).

3.2. Simulation studies of the sacral object

The single-number index was applied for assessment of variants of proposed acoustic adaptation of the Church of St. Paul the Apostle in Bochnia [10], a object with flawed acoustics (W_{kz}=0). The observation matrix of 17 variants of adaptation and four correlated partial indices were subjected to SVD, as shown in Fig. 3. An additional partial index, besides the indices listed in section 3.2, was the sound strength index W_{St}, as defined in [10].

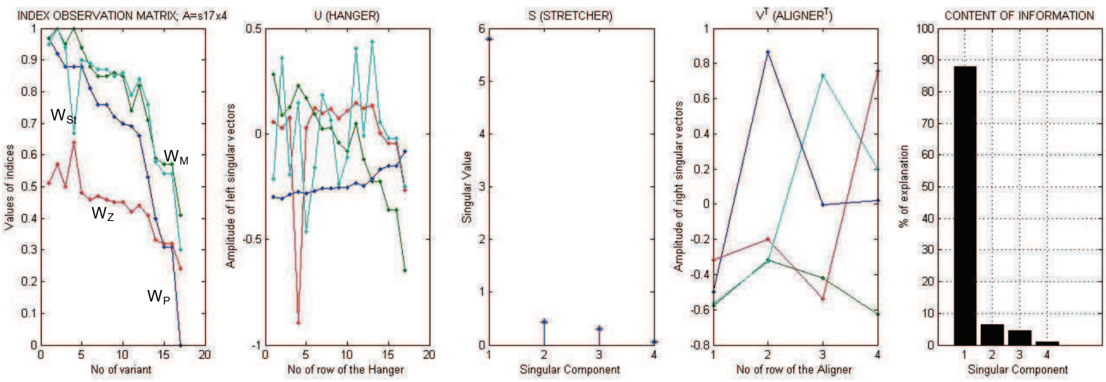

Fig. 3. SVD of the four-index observation matrix of acoustic adaptation variants for the church of St. Paul the Apostle in Bochnia

Simulation studies of adaptation variants were carried out using the CATT Acoustics v.8 software. Partial indices are strongly correlated with one another. Linear correlation coefficients between indices are shown on table 2. As shown in fig. 3, the first singular component contains 88% of information interpretation, and the three remaining components are below the limit of information noise, equal to 10%.

Table 2. Linear correlation coefficients between partial assessment indices used in the simulations tests

W_P <-> W_M	W_Z <-> W_P	W_M <-> W_Z	W_{St} <-> W_P	W_M <-> W_{St}	W_{St} <-> W_Z
0.9823	0.9188	0.9551	0.9207	0.8855	0.7475

The values of single-number indices for individual simulation variants using the methods of full correlation and decorrelation have been presented in Fig. 4. On the basis of the comparison of the values of indices W_{kz} and W_d (Fig. 4a and 4b), it can be stated that the obtained values for individual simulation variants are the same. The best simulation variant (variant 2) has a single-number index equal to 1, and the worst (wariant 17) – an index of 0. Selection of the variant of proposed acoustic adaptation of the church conducted using computer simulations along with a supporting tool – a single-number index, can facilitate the making of a decision by persons concerned with problems of adaptation and design of interiors of sacral objects.

4. CONCLUSIONS

The article shows that it is possible to evaluate object acoustic parameters using one number – a single-number index of selected correlated acoustic parameters. An example of public buildings in which correlated acoustic parameters are significant for the functioning of the building in terms of acoustics are sacral objects. Verification of the proposed computational procedures for the single-number assessment indices was carried out on Roman Catholic churches using two methods applying SVD: full correlation and decorrelation of indices. The article shows that the results obtained using both methods, in the form of single-number indices, are identical. Single-number indices were also used to evaluate simulations of adaptation variants of an acoustically flawed modern church. A synthetic index is a general measure that can support decision-making during selection of acoustic adaptation. Partial assessment indices can give more accurate information on acoustic parameters of a object. The single-number index of correlated acoustic parameters will be a component of the global acoustic assessment, which is a function of non-correlated indices and can be determined using the statistical method that is multi-variate analysis of comparative. Further studies will concern the verification of the proposed single-number assessment for other types of objects e.g. industrial halls, sports stadiums.

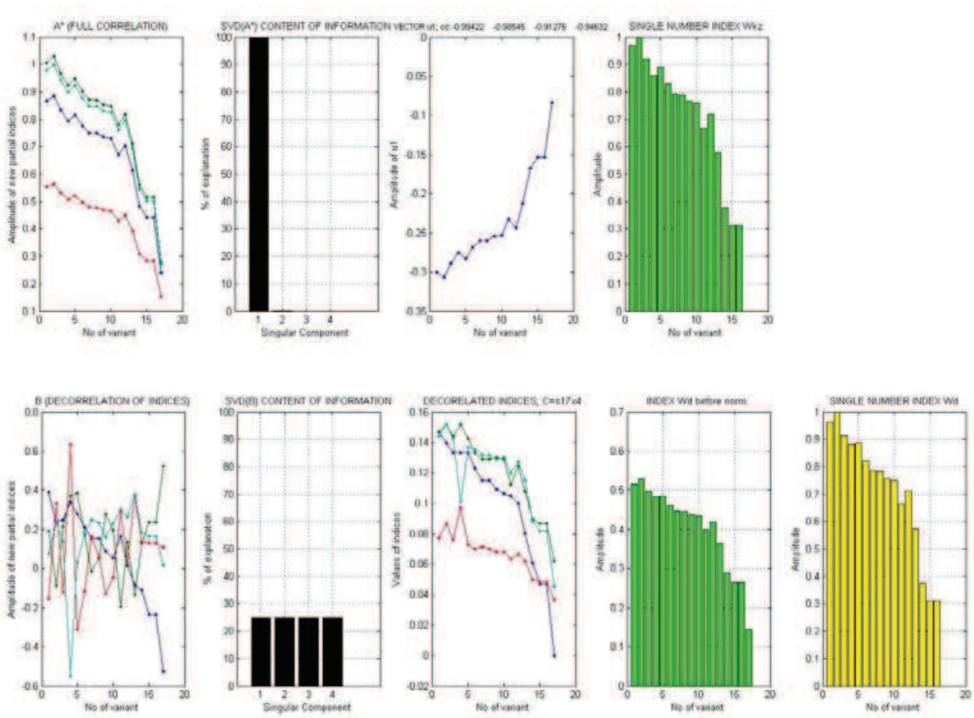

Fig. 4. Comparison of the full correlation method (a) and the decorrelation method (b) of correlated indices in the synthetic single-number assessment of acoustic adaptation of the studied church

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