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GEOID IN THE AREA OF POLAND IN THE AUTHOR'S INVESTIGATIONS

Adam Łyszkowicz

Chair of Land Surveying
University of Warmia and Mazury in Olsztyn

Key words: geopotential models, geoid, quasigeoid, deflections of the vertical.

Abstract

The present paper describes the results of the author's work related to the geoid determination in the area of Poland. Beginning from the geoid model *geoid92* worked out in 1992, various geoid models calculated from various data sets, using various methods, are presented. Additionally, the evaluation of the accuracy of the determined geoid models is given. Next, the necessity of the fitting of computed geoid models to the national vertical reference system and the evaluation of the accuracy of the functionals N, Δg, ξ, η of the gravity field calculated from the various geopotential models are presented.

It results from the presented investigations, that at present the accuracy of gravimetric geoid/quasigeoid models is ± 1.4 cm and accuracy of geoid computed from geopotential model EGM08 is ±2.4 cm. It is stated additionally that accuracy of gravity anomalies computed from the EGM08 model is ± 25 μm s^{-2} , while accuracy of the deflections of the vertical ξ and η is from ±0.6'' to ± 0.7''.

BADANIE PRZEBIEGU GEOIDY NA OBSZARZE POLSKI

Adam Łyszkowicz

Katedra Geodezji Szczegółowej
Uniwersytet Warmińsko-Mazurski w Olsztynie

Słowa kluczowe: modele geopotencjału, geoida/quasi-geoida, składowe odchyleń pionu.

Abstrakt

W pracy przedstawiono wyniki badań dotyczących przebiegu geoidy na obszarze Polski. Poczynając od modelu *geoid92,* opracowanego w 1992 roku, w artykule przestawiono kolejne modele geoidy liczone z różnych danych i różnymi metodami wraz z ich charakterystyką dokładnościową. Uzasadniono konieczność dopasowania wyliczonych modeli geoidy do krajowego układu wysokościowego oraz podano dokładności charakterystyk pola siły ciężkości N, Δg, ξ, η obliczanych z kolejnych modeli geopotencjału.

Z wyników badań wynika, że obecnie dokładność grawimetrycznych modeli geoidy/quasi-geoidy wynosi ±1,4 cm, a dokładność geoidy wyliczonej z modelu geopotencjału EGM08 ±2,4 cm. Dodatkowo stwierdzono, że dokładność anomalii grawimetrycznych wyliczonych z modelu EGM08 wynosi ±25 μm s^{-2}, a dokładność składowych odchyleń pionu ξ i η od ±0.6'' do ±0.7''.

Introduction

Geoid has the basic role in geodesy, oceanography and geophysics. It serves in geodesy and oceanography, as the reference surface describing the topography of continents and oceanic surfaces. One uses geoid in geophysics, as the representation of the gravity field which reflects the expansion of earth mass situated in the depth of the Earth.

The paper is based on report presented at the conference dedicated to the fifth anniversary of Institute of Geodesy and Geoinformatics in Wroclaw. The main aim of this paper is to show, how in author's investigations, methods and data in geoid/quasigeoid models computation in the area of Poland has changed and how the accuracy of successive models have increased. Beside geoid determination it presents computation of gravimetric deflections of the vertical e.g. (ŁYSZKOWICZ 1996a) and recapitulations of these investigations are also in this paper. More and better geopotential models create a new possibility in getting the gravity anomalies and the deflections of the vertical without necessity of laborious field measurements. In the present paper the newest author's investigations in this matter are presented.

The paper consists of nine sections. After the introduction, in the second section the accuracy evaluation of geoid undulations, deflections of the vertical and gravimetric anomalies are presented. In the next section, briefly an overview of the basis of geoid calculation from gravimetric data and then geoid models computed by the author in the last 20 years are presented. Section four contains information relating to the determination of geoid from the deflections of the vertical by the collocation method. Section five deals with the necessity of fitting computed geoid models to the vertical reference system, and then various ways of such fitting are considered. Then it is shown how the accuracy of geoid computed from various geopotential models increased in the last 20 years. Last section contains recapitulation and conclusions.

Evaluation of geoid accuracy

The estimation of geoid/quasigeoid accuracy was for many years unusually difficult. At the moment when appeared the possibility of the establishment of geodetic networks by the satellite GPS method appeared, the estimation of geoid accuracy simplified considerably and it looks as follows.

The relationship between geoid undulation N, ellipsoidal height h and orthometric height H is.

$$h = H + N \tag{1}$$

From this formula it follows, that if the points of a satellite network at which ellipsoidal heights h were determined from satellite observation are tied to the national levelling network, then the geoid/quasigeoid heights at these points can be computed from relationship.

$$N_i^{\text{gps/levelling}} = h_i - H_i \tag{2}$$

From the available (e.g. gravimetric) geoid/quasigeoid models, the distance between geoid and ellipsoid N_i^{grav} can be computed at the points of a satellite network and then the differences can be created

$$\delta N_i = N_i^{\text{gps/levelling}} - N_i^{\text{grav}} \tag{3}$$

from which mean value $\hat{x}$ and the empirical standard deviation $\hat{\sigma}_N$ can be computed. So computed empirical standard deviation $\hat{\sigma}_N$ is an estimation of absolute accuracy of geoid models. The method of estimation of the relative accuracy of geoid/quasigeoid models is more complicated and is given e.g. in the paper (ŁYSZKOWICZ 2009a).

In a case of estimation of gravity anomalies Δg_{GM} computed from geopotential models, these anomalies are compared with gravity anomalies Δg obtained from terrestrial measurements and then the differences are created as

$$\delta \Delta g_i = \Delta_i^{\text{GM}} - \Delta g_i \tag{4}$$

Empirical standard deviation $\hat{\sigma}_{\Delta g}$ of differences is an estimate of the accuracy of gravimetric anomalies computed from geopotential models.

In similar way is estimated the accuracy the deflections of the vertical computed from geopotential model or gravity data is estimated

$$\begin{aligned} \delta\xi_i &= \xi_i^{\text{GM}} - \xi_i \\ \delta\eta_i &= \eta_i^{\text{GM}} - \eta_i \end{aligned} \tag{5}$$

where ζ_i^{GM}, η_i^{GM} are computed from a geopotential model or gravity data and ζ_i, η_i are astrogeodetic deflections of the vertical. Empirical standard deviations $\hat{\sigma}_\xi$, $\hat{\sigma}_\eta$ of differences are estimate of the accuracy of deflections.

In the present work to estimate the accuracy of geoid/quasigeoid models the POLREF network and control traverse were used. POLREF network consists of 360 points measured in campaigns from July 1994 till May 1995 (ZIELIŃSKI et al. 1997). Traverse is about 868 km length, running through the whole area of Poland, consisting from 190 points, measured in five campaigns in the years 2003–2004 (KRYŃSKI et al. 2005). The points of the POLREF network and traverse were tied up to the precise levelling network whose heights are expressed in Kronsztadt86 system.

The gravimetric data used for the evaluation of accuracy of anomalies computed from successive geopotential models and also to calculate the successive versions of gravimetric geoid/quasigeoid models contains data from the following area: Poland, Czech Republic, Slovakia, Hungary, Romania, Ukraine, Germany and Denmark and they are describes in detail in (ŁYSZKOWICZ 1994).

To estimate the accuracy of deflections of the vertical determined from geopotential models and to compute astrogeodetic geoid by the collocation method, 171 astrogeodetic deflections of the vertical measured during years 1967–1981 were used (*Katalog* 1981). These deflections were then improved (ROGOWSKI et al. 2003) because of the new star catalogue, movements of the pole, time system and horizontal reference system and these improved deflections were used in our computations.

Geoid computation from Stokes integral and by the collocation method

The general adopted strategy of the geoid/quasigeoid calculation consists of combination of three effects: global, regional and local which are represented suitably through geopotential model GM, mean residual Faye anomalies and the topography

$$N = N_{\mathrm{GM}} + N_{\Delta g_{\mathrm{res}}} + N_H \tag{6}$$

Term N_{GM} represents the contribution of spherical harmonic coefficients, while the term $N_{\Delta g_{\mathrm{res}}}$ represents the contribution of residual Faye anomalies after removing the effects due to the geopotential model

$$\Delta g_{\mathrm{res}} = \Delta g_{\mathrm{FA}} - \Delta g^{\mathrm{GM}} \tag{7}$$

The term N_H in equation (6) is defined, as the indirect effect on geoid and represents the change of the equipotential surface after applying terrain reduction to Δg.

Component $N_{\Delta g_{\mathrm{res}}}$ can be computed from Stokes integral which is evaluated by the fast Fourier transforms (FFT) or by the collocation method. Computed geoid undulations can be when transform into quasigeoid according to the equation

$$\zeta = N - \frac{\Delta g_B}{\gamma} H \tag{8}$$

where Δg_B is Bouguer anomaly, γ is mean normal gravity and H is topographic height.

Gravimetric geoid models

In the present chapter are presented the successive versions of gravimetric geoid/quasigeoid models, which were computed by the author for the area of Poland in the last 20 years. In the description of the models, we pay attention to gravity data, methods of computation and obtained accuracy.

Model geoid92

The first gravimetric geoid model for Central Europe, including Poland, was computed in 1949 (TANNI 1949) and has very low accuracy. The first gravimetric geoid model for the area of Poland was computed at the Department of Planetary Geodesy, Polish Academy of Sciences in 1993. This model was computed by the combination of collocation and the integral method and it is based on the OSU81 geopotential model to degree and order 80 and on 6000 Faye gravity anomalies from the area of Poland. The estimation shows that the accuracy of this model is on the level of ± 26 cm (ŁYSZKOWICZ 1993)[1].

Model geoid94

The second gravimetric geoid model for the area of Poland *geoid94* was computed using Stokes integral which was evaluated by the FFT method and is based on a much bigger sets of gravimetric data (about 8000 additional mean anomalies), topographic data and geopotential model OSU91 to degree and order 360 (ŁYSZKOWICZ, DENKER 1994). The accuracy of this model is estimated[2] as ±14 cm.

Model quasi95

In 1994 within realization of the research project of Committee of Scientific Research (CSR) (ŁYSZKOWICZ 1995) a new set of mean Faye anomalies was used

[1] The evaluation of this model was done in 1993 on 10 points of the EUREF network, what means that estimation of accuracy is not realistic.

[2] New evaluation done in 2010 on the basis of all points of POLREF network.

in the geoid computation for the area of Poland in a grid 1' × 1'. These data together with gravimetric data collected from neighbouring countries and digital terrain model in a grid 1.5' × 3' interpolated from the terrain model in a grid 30' × 30' for Poland, and 5' × 7.5' for the neighbouring areas were used to compute the first gravimetric quasigeoid model named *quasi95* (ŁYSZKOWICZ, FORSBERG 1995). The calculation of this model was conducted with the utilization of fast Fourier transforms (FFT). In these calculations the global geopotential model OSU91 was used to degree and order 360. The accuracy of this model is estimated[3] as ± 8.7 cm.

This model was fitted to the vertical reference system Kronsztadt86 (section 7) and was made accessible to practical use by Head Office of Geodesy and Cartography in 1996 (ŁYSZKOWICZ 1997).

Model quasi97b

Next quasigeoid model named *quasi97b* was computed in 1997 from at least 140 000 point and mean gravity anomalies and a new geopotential model EGM96 to degree and order 360 (LEMOINE at al. 1998) by fast Fourier transforms (FFT) method. This model was realized within the project ordered by Committee of Scientific Research (ŁYSZKOWICZ 1998) and together with suitable software it was delivered to Head Office of Geodesy and Cartography to be used in geodetic practice. The accuracy of this model is estimated as ± 3.5 cm.

In order to use in practice *quasi97b* model, it was fitted to vertical reference system Kronsztadt86 by the collocation method. Accuracy of fitted *quasi97b* model is estimated as ± 1.0 cm (ŁYSZKOWICZ 2000)[4].

Model quasi09a

Computed in 2009 the next version of gravimetric quasigeoid model named *quasi09* was developed on the basis of gravimetric data, which consists of the set of mean anomalies from the area of Poland (KRYŃSKI 2007) and of data from neighbouring countries. Additionally during the calculations topographical data were used, that is DTED (*Digital Terrain Elevation Data*) model and SRTM (*Shuttle Radar Topography Mission*) model. The gravimetric model *quasi09a* was computed with the use of gravity data described above and geopotential model EGM08 to degree and order 2190 (PAVLIS et al. 2008) by the fast Fourier method (ŁYSZKOWICZ 2009b).

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[3] New evaluation done in 2010 on the basis of all points of POLREF network.

[4] This evaluation is probably too optimistic.

Accuracy of *quasi09a* model estimated at the points of POLREF network gives a mean error of ± 3.5 cm, while analogous estimation at the points of the traverse gives a mean error ± 2.7 cm.

Model quasi09c

In 2009, for the first time in Poland, a gravimetric quasigeoid model (*quasi09c*) was computed by the collocation method and, for the first time the mean square error of the term $N_{\Delta g_{res}}$ present in formula (6) was estimated. The error of this component over the area of Poland is from 0.3 cm to 0.4 cm (ŁYSZKOWICZ 2010a).

Estimation of the accuracy of *quasi09c* model on the points of the network POLREF gives the mean error ±3.2 cm, while analogous judgment conducted on the points of the traverse gives the mean error ±1.8 cm (ŁYSZKOWICZ 2010a). Additional data in the form of 171 astrogeodetic deflections of the vertical did not improve noticeably the accuracy of this model.

Geoid "GUGiK 2001"

The next published version of the quasigeoid model approved in 2001 by the Head Office of Geodesy and Cartography to be used in geodetic practice was the model named "*GUGiK 2001*" (PAŻUS 2001). This model came into being as a result of the fitting to the vertical reference system Kronsztad86 the gravimetric quasigeoid model *quasi97b* using spline functions of degree 3 (PAŻUS et al. 2002).

In the present work estimation of the accuracy of this model was done at 140 points of the traverse and we get the mean square error of this model ±1.8 cm. Estimation of accuracy of the deflection of the vertical computed from this model was done on 171 points of the astrogeodetic network and we got a mean error of the components ξ and η as 0.60'' and 0.68'' second of arc respectively.

Graphical illustration of the accuracy of the various gravimetric geoid/quasigeoid models are given in figure 1.

Geoid from deflections of the vertical by Helmert and collocation methods

The aim of this investigation was to use the first time in Poland the collocation method to determine a geoid model from astrogeodetic deflections of the vertical and show that this method is significantly better than the classical Helmert method.

Deflections of the vertical are traditionally used to the determination the geoid in a local or regional scale. The first astrogeodetic geoid model for the area of Poland was determined in 1961 (BOKUN 1961) and the next astrogeodetic geoid model was determined in 2005 (ROGOWSKI et al. 2005). This last model was an improved one due to the removal of systematic errors, in astrogeodetic and astrogravimetric deflections of the vertical.

The latest improved astrogeodetic geoid model is presented in the paper (ŁYSZKOWICZ 2010b). It was computed for the area of Poland, by the use improved data and a better computational algorithm, the collocation method. The theoretical fundamentals of the astronomical levelling and collocation method are given briefly e.g. in paper (ŁYSZKOWICZ 2010b, sec. I). In the same paper an evaluation of the accuracy of the deflections of the vertical and weights estimation is given.

Section II of the paper (ŁYSZKOWICZ 2010c) describes four computed geoid models, namely: two models from the improved astrogeodetic and astrogravimetric data by the Helmert method and models from the same two data sets but computed by the least squares collocation method.

These models were compared with quasigeoid undulation $N^{gps/levelling}$ at the points of the satellite POLREF network. The results of the comparison show, that geoid model determined by the collocation method is characterize by an accuracy of ±12 cm – ±7 cm and is 5–7 times better than the accuracy of the models computed by the classical method.

Geoid fitting to the vertical reference system

In practice the relationship (1) is not fulfiled and each component in equation (1) is affected by systematic errors that can be written in the form

$$h + \delta h = (H + \delta H) + (N + \delta N) \tag{9}$$

Equation (9) can be written in the form

$$h = H + N + (\delta H + \delta N - \delta h) \text{ or } h = H + N + c \tag{10}$$

where c is correction surface that enables more precise transformation of heights H from spirit levelling into ellipsoidal heights h from satellite GPS observations and vice versa. Correction surface c can be determined from certain parametric models given in table 1 and additionally by the collocation method e.g. (ŁYSZKOWICZ 2000).

Model#1 in table 1 contains one parameter which can be interpreted as a constant shift between the considered surfaces. Model#1 was used to assess the accuracy of geoid models described in the previous sections of this paper.

Model#2 contains two additional parameters which represent mean north-south and east-west inclination e.g. between the gravimetric geoid and geoid from GPS and levelling.

Model#3 is commonly known as four parameter model which geometrically represents three shifts along the x, y z axis and scale change of reference systems i.e. geoid heights with respect to GPS heights or the opposite.

Finally, models #4, #5 and #6 represent height-dependent linear corrector surfaces that constrain the relation between ellipsoidal, normal and quasigeoidal heights in terms of the generalized equation

$$h - (1 + \delta_{S_H})H - (1\ \delta_{S_H}) = a_0 \tag{11}$$

The above equation takes into consideration the fact that the spatial scale of GPS heights does not necessarily match with the spatial scale induced by quasigeoid undulations of the geopotential model quasigeoid undulations and/or the inherent scale of normal heights obtained from spirit levelling. Moreover, the geopotential model of quasigeoid heights and/or local normal heights are often affected by errors that are correlated, to a certain degree, with the Earth's topography, what is a fact that can additionally justify the use of model #4 or #6 for optimal fitting between $\zeta^{\text{gps/levelling}}$ and ζ.

Table 1

List of various parametric models, KOTSAKIS (2008)

Number of the model	Type
1	$a_0 + (h_i - H_i - N_i) = v_i$
2	$a_0 + a_1 (\varphi_i - \varphi_0) + a_2 (\lambda_i - \lambda_0) \cos\varphi_i + (h_i - H_i - N_i) = v_i$
3	$a_0 + a_1 \cos\varphi_i \cos\lambda_i + a_2 \cos\varphi_i \sin\lambda_i + a_3 \sin\varphi_i + (h_i - H_i - N_i) = v_i$
4	$a_0 + \delta_{S_H} H_i + (h_i - H_i - N_i) = v_i$
5	$a_0 + \delta_{S_H} H_i + (h_i - H_i - N_i) = v_i$
6	$a_0 + \delta_{S_H} H_i + \delta_{S_N} + (h_i - H_i - N_i) = v_i$

In the paper (ŁYSZKOWICZ 2009a) the accuracy of fitting of the quasigeoid undulation computed from the EGM08 model to the vertical reference system Kronsztadt86 with the use of various parametric models (Table 1) was tested. From these investigations it can be concluded that the model #2 and #3 which are commonly used in practice e.g. (FORSBERG 1998) give somewhat better results than the remaining models. The best results are given by the parametric model #3 which makes possible to fit the quasigeoid from EGM08 to the vertical reference system in Poland with the accuracy ±2.4 cm (ŁYSZKOWICZ 2009a) and therefore it will be used in our further calculations.

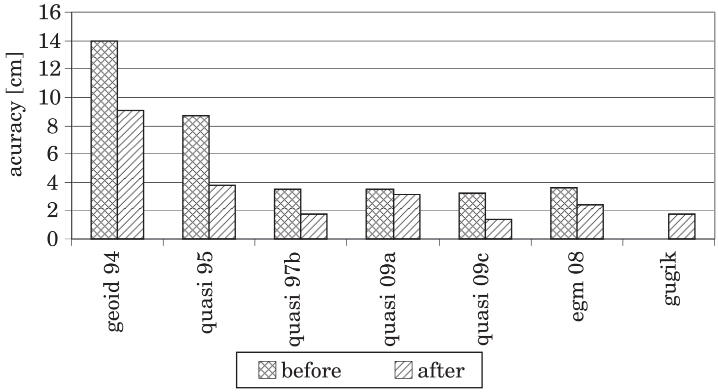

Fig. 1. Accuracy comparison of various geoid/quasigeoid models before and after fitting to the vertical reference system Kronsztad86 in Poland

Figure 1 represents the accuracy evaluation of various geoid/quasigeoid models before and after fitting to the vertical reference system Kronsztadt86. From figure 1 it can be seen that in the case of *geoid94* model if we apply the parametric model #3 then *geoid94* can be fitted to the vertical reference system Kronsztadt86 with an accuracy of ±9.1 cm.

If to the *quasi95* (= *quasi96*) model we apply the parametric model #3, then the fit to the vertical reference system is with accuracy ±3.8 cm. On the other hand *quasi97b* model was fitted to the vertical reference system by the collocation method and surprisingly good accuracy of the fitting ±1.0 cm was obtained (ŁYSZKOWICZ 2000).

Evaluation of accuracy of the fitted *quasi09a* model (parametric model#3) at the points of the POLREF network gives a mean error ±3.1 cm (ŁYSZKOWICZ 2009c). Accuracy of the *quasi09c* model computed from gravimetric data and geopotential model EGM08 by the collocation method after fitting to the vertical reference height system by the least squares collocation method, performed in this paper gives a mean error on the level of ±1.4 cm.

Accuracy of the fitting of EGM08 model to the vertical reference system Kronsztadt86 with the utilization of parametric model #3 is the same as by the collocation method and is equal to ±2.4 cm.

All accuracy evaluations given here were determined on the basis of the POLREF network. Investigations of the control traverse show, somewhat better accuracy. For example EGM08 model fits to the POLREF network with an accuracy ±2.4 cm while the fit to the points of the control traverse is ±1.9 cm.

Model "*GUGiK 2001*" approved in 2001 by the Head Office of Geodesy and Cartography is already a model which fits to the vertical reference system and therefore its new fitting was not done in the present paper leaving in the suitable place of figure 1 the value zero.

Geoid, gravity anomalies and deflections of the vertical from geopotential models

Information about the gravity field can be obtained from direct gravimetric measurements or computed from geopotential models. The first geopotential model up to degree and order 8 was elaborated by Żongołowicz in 1956 and characterized by low accuracy e.g. geoid heights could be computed with an accuracy of ±8 meters. The most recent geopotential model EGM08 enables to compute the geoid with the accuracy of few centimetres.

The aim of the preset section is to show how the accuracy of models: GEM-10B to the degree and order 36 (LERCH et al. 1978), OSU81 to the degree and order 180 (RAPP 1981), OSU91 to the degree and order 360 (RAPP et al. 1991), EGM08 to the degree and order 2100 (PAVLIS et al. 2008) during the last 50 years increased significantly.

The quasigeoid heights can be computed from the general formula

$$\zeta(r, \phi, \lambda) = \zeta_0 + \frac{\mathrm{GM}}{r\gamma} \sum_{n=2}^{n_{\max}} \left(\frac{a}{r}\right)^n \sum_{m=0}^{n} (C_{nm} \cos m\lambda + S_{nm} \sin m\lambda)\, P_{nm} (\sin \phi) \quad (12)$$

where C_{nm}, S_{nm} are fully normalized spherical harmonic coefficients of degree n and order m, $n_{\max}$ is the maximum degree of geopotential model, GM is the product of the Newtonian gravitational constant and mass of the geopotential model, r, ϕ, λ are spherical polar coordinates, a is the equatorial radius of the geopotential model and P_{nm} are the fully normalized associated Legendre functions. The term ζ_0 is the zero degree term due to the difference in the mass of the Earth used in IERS Convention and GRS80 ellipsoid. Detailed description of the term ζ_0 is given in the paper e.g. (ŁYSZKOWICZ 2009a).

In the present work in an analogous way, gravity anomalies Δg^{GM} and deflection of the vertical ζ^{GM}, η^{GM} were computed from various geopotential models. All computations were done using the program *geocol* (TSCHERNING et al. 1992).

Quasigeoid heights were computed at 360 points of the POLREF network for the various geopotential models and were compared with "true" heights according to the formula (3) and then empirical standard deviations were computed for each model (fig. 2). From figure 2 it results that the accuracy of ζ computed from geopotential models increased extremely (almost 40 times) and presently without difficulties ζ in the area of Poland can be computed with accuracy of ±3.6 cm.

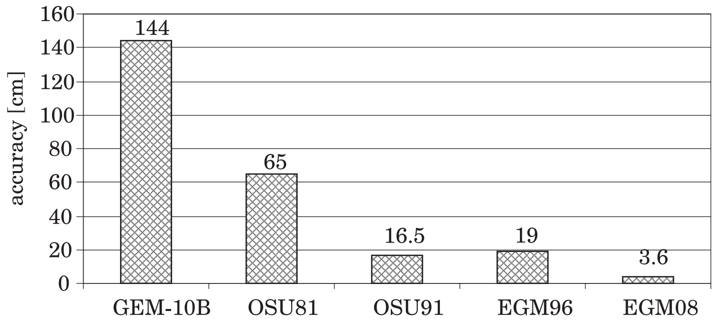

Fig. 2 Accuracy estimation of ζ from the various geopotential models

Terrestrial gravity data from the territory of Poland, used to evaluate the accuracy of gravity anomalies computed from geopotential models, consists of 147 530 mean gravity anomalies computed in a 1' × 1' grid. Gravity anomalies computed from the geopotential model were then compared with anomalies from terrestrial measurement, equation (4), and on the basis of such computed differences their accuracy was estimated (fig. 3). From that figure it can be concluded that at present, gravity anomalies in the area of Poland, can be computed from the EGM08 model with an accuracy of ±25 μm s^{-2}.

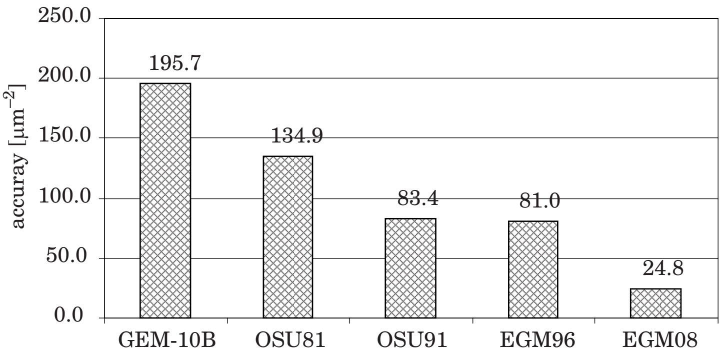

Fig. 3 Accuracy estimation of Δg from the successive geopotential models

171 astrogeodetic deflection of the vertical were used to estimate the accuracy of the deflections of the vertical ξ and η computed from the geopotential models. According to formula (5) appropriate differences were created and on their basis the accuracy was evaluated. From fig. 4 it can be seen that the accuracy of deflections of the vertical computed from geopotential models increase significantly and in of case the last EGM08 model they are ±0.6'' and ±0.7'' for the component ξ and η respectively.

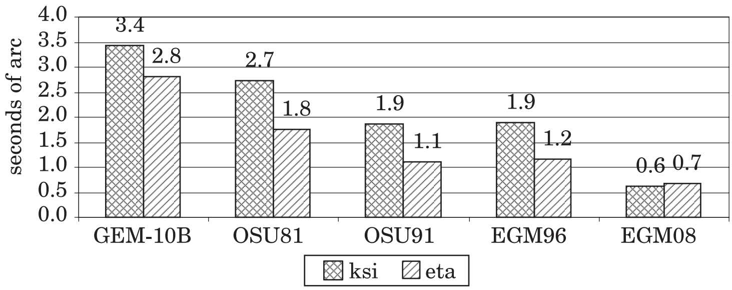

Fig. 4. Accuracy estimation of ξ η from the various geopotential models

Gravimetric deflections of the vertical

Traditionally deflections of the vertical are determined at the points of triangulation network from astronomical observations. The access to gravity data, which appeared in the last decade of the XX century, makes possible the computation of deflection of the vertical from Vening-Meinesz formulas.

First computation of gravimetric deflections of the vertical for the territory of Poland was done for the requests of the Head Office of Geodesy and Cartography in 1966 (ŁYSZKOWICZ 1996a). Deflections of the vertical ξ and η were computed from available at that time gravity anomalies[5] and geopotential model OSU91 by the fast Fourier method in a 1.5' × 3.0' grid. In order to assess their accuracy from the gridded data 171 gravimetric deflections of the vertical were interpolated at the points of astrogeodetic network and then the differences were computed according to formula (5) and their accuracy was estimated. This comparison gives mean errors ±0.59'' and ±0.47'' for components ξ and η respectively.

A second computation of gravimetric deflections of the vertical was realized in 2003 (ŁYSZKOWICZ 2003). In this computation the same gravity data set was used as in previous computation but the new and better geopotential model EGM96 available at that time. These calculations were made using Vening-Meinesz formulas, which were evaluated by the fast Fourier transform. The accuracy of compute deflections of the vertical is ±0.55'' and ±0.47'' for the component ξ and η respectively.

[5] At least 130 000 mean and point Faye anomalies.

Summary and conclusions

In the last years many geoid/quasigeoid models were computed for the territory of Poland. These models were computed using different methods and from various types of data. The computed geoid models i.e. astrogeodetic models and gravimetric models were computed using fast Fourier transform or the collocation method while geopotential models were computed from spherical harmonics coefficients.

Accuracy evaluation of these models was done on the basis of satellite networks such as POLREF network and control traverse, the points of which were connected to the national levelling network assuming that heights $N^{gps/levelling}$ are error-free.

As a result it was found that absolute accuracy of the astrogeodetic geoid model is ±7 cm, accuracy of gravimetric geoid models *quasi97b* and *quasi09a* is ±3.5 cm, while the accuracy of the quasigeoid model computed by collocation is ±3.2 cm. Quasigeoid model computed from the new geopotential EGM08 model has an accuracy ±3.6 cm. The accuracy of the mentioned quasigeoid models tested on points of the control traverse is much better and is at the level of ±2.5 cm.

By suitable fitting of the quasigeoid model to the national vertical system one can get the surface, which is obviously not an equipotential surface and in the literature is called the correction surface. Accuracy of such correction surface in the case of *quasi09c* model is ±1.4 cm, while in the case of quasigeoid from geopotential model EGM08 is ±2.4 cm. Accuracy of the model "*GUGiK 2001*", which is of course fitted to the vertical system Kronsztadt86 is ±1.8 cm. It means that the above mentioned models have almost the same accuracy, but the model *quasi09 c* is the most accurate model.

If we reject the hypothesis that the term $N^{gps/levelling}$ is error-free, then the accuracy of these geometrical distances depends on the accuracy of height h from satellite measurements and on the accuracy of connection to the vertical reference system. Accepting, that the accuracy of computation of h in the POLREF network is ±1 ... ±1.5 cm (ZIELIŃSKI et al. 1997) and, that the POLREF network was connected to the vertical reference system with the accuracy of ±1.5 cm (WYRZYKOWSKI 1998), (GELO 1994) it means that the quantity $N^{gps/levelling}$ is determined with an accuracy of ±2.1 cm, what further investigations show is a very optimistic opinion. From these evaluations it appear that the POLREF network is not very suitable to estimate the accuracy of recent quasigeoid models and new testing networks are indispensable.

This new network could be the ASG-EUPOS network with height component determined on the level of a few millimetres and connected to the vertical reference system. It should ensure credible evaluation of various geoid/quasigeoid models and their proper fit to the vertical system (various possibilities).

Additionally in the present work it was proved that the accuracy of gravity anomalies computed from EGM08 model is of the order of ±25 μm s^{-2} and the accuracy of the deflections of the vertical is 0.6'' – 0.7'', what can replace difficult astronomic field measurements in many cases.

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