Management and Production Engineering Review
Volume 3 • Number 3 • September 2012 • pp. 3–10
DOI: 10.2478/v10270-012-0018-9

JOINT ALLOCATION AND TRANSPORTATION PROBLEM IN SUPPLY NETWORK OPTIMISATION

Grzegorz Filcek

Wrocław University of Technology, Institute of Informatics, Poland

Corresponding author:
Grzegorz Filcek
Wrocław University of Technology
Institute of Informatics
Wyb. Wyspianskiego 27, 50-370 Wrocław, Poland
phone: +48 71 330-41-12
e-mail: grzegorz.filcek@pwr.wroc.pl

Received: 1 July 2012
Accepted: 30 August 2012

Abstract
The paper concerns joint allocation and transportation as an optimization problem in selected supply networks. The network consists of a set of suppliers of the raw material, a set of production units and a set of product receivers. The raw material is treated as fast perishing good like vegetables or fruits. The production units are described by time models. In the optimization process, the time of the production and cost of the transportation is taken into account. The objective function is in general non-convex function of raw material allocation and transportation plans of the raw material and the product. To solve the problem considered, exact and heuristic algorithms have been developed and presented. To solve convex problems, solver Lingo developed by Lindo systems is proposed. The idea of a computer decision supported system integrating all presented algorithms is presented as well as four numerical examples illustrating some properties of the assumed supply network model.

Keywords
supply networks, optimization, convex optimization, heuristic algorithms, decision support systems.

Introduction

In the paper we are considering supply networks that deal with fast perishing material. Such networks can be build for the needs of sugar beet or fruit and vegetables processing industries. In such industries time of the production is very important, and transportation costs must be also taken into consideration. The production time is determined by the raw material allocation to production units, and transportation cost by transportation plans. The raw material allocation is obtained by solving allocation problem, the transportation plans by solving the transportation problem. In the paper we consider those two problems jointly.

The problem of raw material allocation to production units is based on the problem of task allocation in a complex of operations and is well known and solved (see [1–3]). It finds its applications mainly in computer systems (e.g. task allocation among parallel processors), and in manufacturing systems, where the task consists in processing of a raw material in the given amount into a product. The aim of the task allocation is to find such an allocation that minimizes a time needed to finish all tasks (or to finish the processing of a raw material in the case considered). In manufacturing systems, finding a solution for allocation of a material is only one step in very complex decision making process. After finding the allocation of a raw material, another problem appears. Namely, how to deliver a raw material to production units, and distribute manufactured product to the receivers. This problem is well known in Operation Research as a transportation problem and solved (see e.g. [4]). Those two problems often exist in one system and are interdependent.

The solution of raw material allocation problem gives the data for the transportation problem, and the change in transportation plans may cause the change in the allocation and in processing times as a consequence. In such systems, we are dealing, in fact, with multi-criteria problem, when we want to minimize the processing time on the one hand, and to minimize the transportation costs on the other hand. It is justifiable to treat the processing time as a cost since a material spoils with time, and operational costs, like energy cost or employment costs, depend on time. To compare cost of the production and transportation, we have to determine the appropriate coefficient cost/time that enable us to measure the production cost in the same units as the transportation cost. This coefficient can be given by an expert or estimated using statistics. The results of the research presented in [5] show that, in many cases, slight deterioration of the optimal raw material allocation can result in the improvement of the total cost of production and transportation. So, much better results might be achieved by solving problems of allocation and transportation jointly. In the literature, joint consideration of different well known elementary sub-problems are often considered to improve the quality of a decision for the problem as a whole. Let us present just a few elementary sub-problems important for manufacturing and logistic systems which are the area of interest for the paper: facility location, vehicle routing, task scheduling, assignment, queuing, inventory management, task allocation, raw material allocation, resource allocation. The selected joint problems which are mainly the connection of two selected elementary sub-problems are as follows: location-routing [6–9], location-scheduling [10, 11], inventory-location [12, 13], inventory-routing [14, 15], routing-scheduling [16–18], production-inventory [19], production-transportation [20].

In this paper we are dealing with the minimization of the total cost of the production and transportation. The minimization of cost is one of the main aims in supply chain management, see the surveys of [21–23].

In the paper, the production cost is determined by the longest processing time on all production units. In a consequence, the total cost is strongly non-linear and non-differentiable function. So, the dedicated solution algorithms have to be developed.

The paper is organized as follows. After Introduction we make a model definition and problem formulation. After it, solution for the convex version of the problem is presented. The next section is devoted to briefly describe solution algorithms developed to deal with non-convex version of the problem. Then the idea of computer system integrating the developed algorithms with numerical examples are described. Final remarks complete the paper.

The model and problem formulation

Let us consider a supply network which consists of three sets: suppliers of a raw material, production units, and receivers of the product. Each set is a part of at least one from the consecutive stages: transportation of the raw material, production, transportation of the product. Paths between every production unit and supplier or receiver connect the stages The structure of the supply network under consideration is given in Fig. 1. The parameters and variables described in Table 1 can be gathered as appropriate column vectors and matrices, i.e.

$$\mathbf{w} = [w_1, w_2, ..., w_I]^T, \qquad \overline{\mathbf{v}} = [\overline{v}_1, \overline{v}_2, ..., \overline{v}_K]^T,$$
$$\mathbf{e} = [e_1, e_2, ..., e_R]^T, \qquad \mathbf{v} = [v_1, v_2, ..., v_R]^T,$$
$$\overline{\gamma}(\overline{\mathbf{w}}) = [\overline{\gamma}_1(\overline{w}_r), \overline{\gamma}_2(\overline{w}_r), .., \overline{\gamma}_R(\overline{w}_r)]^T,$$
$$\mathbf{c}' = [c'_{i,r}]_{\substack{i=1,2,...,I \\ r=1,2,...,R}},$$
$$\text{and} \quad \overline{\mathbf{c}} = [\overline{c}_{r,k}]_{\substack{r=1,2,...,R \\ k=1,2,...K}},$$

are vectors or matrices of: available raw material, demand for the product, production units productivities, allocation of the raw material, production units time models, unit transportation costs of a raw material, and unit transportation costs of a product, respectively. The decision variables are expressed by $\overline{\mathbf{w}} = [\overline{w}_1, \overline{w}_2, ..., \overline{w}_R]^T$, $\mathbf{x}' = [x'_{i,r}]_{\substack{i=1,2,...,I \\ r=1,2,...,R}}$, and $\overline{\mathbf{x}} = [\overline{x}_{r,k}]_{\substack{r=1,2,...,R \\ k=1,2,...K}}$ which denote production plan, transportation plan of a raw material, and transportation plan of a product, respectively.

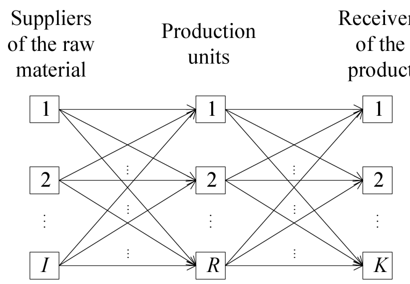

Fig. 1. Structure of the supply network under consideration.

It is important to note that production units can play the role of both the receivers for a raw material and the suppliers for a product. The condition $\exists_{r\in\overline{1,R}}\ (\overline{V} \leq e_r W)$ enables us to satisfy the demand $\overline{V}$.

Table 1
Notation.

Symbol	Description
I	number of warehouses of a raw material (raw material suppliers)
R	number of production units
K	number of warehouses of a product (product receivers)
$i \in \{1,\ 2,\ ..., I\} \triangleq \overline{1, I}$	index of the warehouse of a raw material
$r \in \{1,\ 2,\ ..., R\} \triangleq \overline{1, R}$	index of the production unit
$k \in \{1,\ 2,\ ..., K\} \triangleq \overline{1, K}$	index of the warehouse of a product
w_i	amount of a raw material available in the ith warehouse
$\overline{v}_k$	demand of a product in the kth warehouse
V	total amount of a raw material allocated
$\overline{V}$	total demand of the product receivers
W	total supply of a raw material
$e_r \in (0,1]$	productivity of the rth production unit
$v_r \in [0, V]$	amount of a raw material allocated to the rth production unit
$\overline{w}_r \triangleq e_r v_r$	amount of a product planned to be manufactured by the rth production unit
$x'_{i,r} \in [0, w_i]$	amount of a raw material transported from the ith supplier to the rth production unit
$\overline{x}_{r,k} \in [0, \overline{v}_k]$	amount of a product transported from the rth production unit to the kth receiver
$T_r = \gamma_r(v_r)$	processing time of the allocated raw material to the rth production unit
$\gamma_r(\cdot)$	continuous, strictly increasing function satisfying the condition $\gamma_r(0) = 0$
$\overline{\gamma}_r(\overline{w}_r) \triangleq \gamma_r\left(\frac{\overline{w}_r}{e_r}\right)$	time needed to manufacture the amount $\overline{w}_r$ of the product on the rth production unit
$c'_{i,r}$	unit transportation cost of a raw material from the ith supplier to the rth production unit
$\overline{c}_{r,k}$	unit transportation cost of a product from the rth production unit to the kth receiver
$\pi > 0$	production cost coefficient

Constraints imposed on all decision variables, which allow determining feasible solutions, are as follows:

$$W = \sum_{i=1}^{I} w_i \geq \sum_{r=1}^{R} v_r = V, \tag{1}$$

$$\sum_{r=1}^{R} x'_{i,r} \leq w_i, \quad i = 1,\ 2,\ ..., I, \tag{2}$$

$$\sum_{i=1}^{I} x'_{i,r} = v_r, \quad r = 1,\ 2,\ ..., R, \tag{3}$$

$$\sum_{k=1}^{K} \overline{x}_{r,k} = \overline{w}_r, \quad r = 1,\ 2,\ ...,\ R, \tag{4}$$

$$\sum_{r=1}^{R} \overline{x}_{r,k} = \overline{v}_k, \quad k = 1,\ 2,\ ...,\ R, \tag{5}$$

$$\overline{V} = \sum_{r=1}^{R} \overline{w}_r = \sum_{r=1}^{R} e_r v_r = \sum_{k=1}^{K} \overline{v}_k. \tag{6}$$

Constraints (1)–(3) guarantee the allocation of a raw material $\mathbf{v}$ for all production units is not exceeding the supply. Constraints (4)–(6) assure that the manufactured amounts of a product fulfil the demand of the product receivers.

The process of transportation generates costs that can be expressed as

$$\overline{J}_1(\overline{\mathbf{w}}, \mathbf{x}') \triangleq \sum_{i=1}^{I} \sum_{r=1}^{R} c'_{i,r} x'_{i,r} \triangleq J_1(\mathbf{v}, \mathbf{x}'), \tag{7}$$

$$\overline{J}_3(\overline{\mathbf{w}}, \overline{\mathbf{x}}) \triangleq \sum_{r=1}^{R} \sum_{k=1}^{K} \overline{c}_{r,k} \overline{x}_{r,k} \triangleq J_3(\mathbf{v}, \overline{\mathbf{x}}) \tag{8}$$

where $\overline{J}_1(\overline{\mathbf{w}}, \mathbf{x}')$ (or $J_1(\mathbf{v}, \mathbf{x}')$), and $J_3(\overline{\mathbf{w}}, \overline{\mathbf{x}})$ (or$J_3(\mathbf{v}, \overline{\mathbf{x}})$) are transportation cost of a raw material, and of a product, respectively, and they are dependent on $\overline{\mathbf{w}}$ (or $\mathbf{v}$) via constraints. It is assumed that production cost is proportional to the time, which is necessary to manufacture the desired amount of a product, and is denoted as

$$\overline{J}_2(\overline{\mathbf{w}}) \triangleq \pi \max_{r \in \overline{1,R}} T_r = \pi \max_{r \in \overline{1,R}} \overline{\gamma}_r(\overline{w}_r) \triangleq J_2(\mathbf{v}) \tag{9}$$

where π is a non-negative time-cost coefficient. Strictly increasing function $\overline{\gamma}_r(\overline{w}_r = v_r e_r)$ expresses the relationship between the processing time of the rth production unit and the amount of a raw material allocated to it. Its form depends on the kind of process modeled. For example, when considering simple cutting process, the function is approximately linear i.e. $\overline{\gamma}_r(\overline{w}_r) = \alpha \cdot \overline{w}_r$. For more complex processes the function takes often the polynomial forms $\overline{\gamma}_r(\overline{w}_r) = \alpha \cdot \overline{w}_r^\beta$ where $\beta \geq 0$.

The total cost is expressed by the sum

$$\overline{J}(\overline{\mathbf{w}}, \mathbf{x}', \overline{\mathbf{x}}) \triangleq \overline{J}_1(\overline{\mathbf{w}}, \mathbf{x}') + \overline{J}_2(\overline{\mathbf{w}}) + \overline{J}_3(\overline{\mathbf{w}}, \overline{\mathbf{x}}) \\ = J_1(\mathbf{v}, \mathbf{x}') + J_2(\mathbf{v}) + J_3(\mathbf{v}, \overline{\mathbf{x}}) \triangleq J(\mathbf{v}, \mathbf{x}', \overline{\mathbf{x}}). \tag{10}$$

The optimization problem is formulated as follows.

For given: I, R, K, $\mathbf{w}$, $\overline{\mathbf{v}}$, $\mathbf{e}$, $\overline{\gamma}(\overline{\mathbf{w}})$, $\mathbf{c}'$, $\overline{\mathbf{c}}$, π for which (1) is true, determine vector $\overline{\mathbf{w}}^*$(or $\mathbf{v}^*$) as well as matrices $\mathbf{x}'^*$ and $\overline{\mathbf{x}}^*$ feasible with respect to constraints (2)-(6) to minimize the total cost i.e.

$$\overline{J}^* \triangleq \overline{J}(\overline{\mathbf{w}}^*, \mathbf{x}'^*, \overline{\mathbf{x}}^*) = \min_{\overline{\mathbf{w}}, \mathbf{x}', \overline{\mathbf{x}}} \overline{J}(\overline{\mathbf{w}}, \mathbf{x}', \overline{\mathbf{x}}) \\ = \min_{\overline{\mathbf{w}}, \mathbf{x}', \overline{\mathbf{x}}} (\overline{J}_1(\overline{\mathbf{w}}, \mathbf{x}') + \overline{J}_2(\overline{\mathbf{w}}) + \overline{J}_3(\overline{\mathbf{w}}, \overline{\mathbf{x}})) \\ = J^* \triangleq J(\mathbf{v}^*, \mathbf{x}'^*, \overline{\mathbf{x}}^*) = \min_{\mathbf{v}, \mathbf{x}', \overline{\mathbf{x}}} J(\mathbf{v}, \mathbf{x}', \overline{\mathbf{x}}) \\ = \min_{\mathbf{v}, \mathbf{x}', \overline{\mathbf{x}}} (J_1(\mathbf{v}, \mathbf{x}') + J_2(\mathbf{v}) + J_3(\mathbf{v}, \overline{\mathbf{x}})). \tag{11}$$

In general, problem has strongly non-linear, non-differentiable, and non-convex cost function. The difficulty of this problem occurs also in its high dimensionality. It is easy to see, that the problem is linear, convex or non-convex exactly when models of the production units are all linear, convex, and non-convex, respectively. For the first case we can use linear programming to solve the problem. For the second convex optimization can be performed, and known algorithms like interior point ([24]) can be applied. For the case, where model of at least one of the production units is non-convex, new algorithms must be developed. In the next section we describe how to solve the problem when it is convex. The following section describes briefly algorithms developed for the non-convex cases.

Solution for the convex version of the problem

For most of the systems production units are described by a time model functions in the polynomial form $\overline{\gamma}_r(\overline{w}_r) = \alpha \cdot \overline{w}_r^\beta$ where $\beta \geq 1$. It implies that, problem considered is convex and when $\beta = 1$ for all units, also linear. Those both cases can be solved using known optimization methods like interior point for convex optimization or even simplex for linear cases. There are many solvers available on the market, that solves this problems. We have chosen solver included in LINGO ver. 11 environment designed by LINDO Systems [25]. To solve our problem in this system, we had to implement the model in Lingo programming language.

LINGO Optimization Modeling Software is a tool for building and solving mathematical optimization models. It is designed for linear, nonlinear, quadratic, integer and stochastic optimization problems. The programming language provided by Lindo Systems lets to implement the model for many ways. One of it is to define the model in one file, and the data can be provided in other separate files. This enables us to change in easy way the data and the size of the problem. The size of the data is theoretically unbounded. The presentation of the solution can be very simple performed as well. In Table 2 we present the basic elements of LINGO mathematical modeling language syntax.

Table 2
The basic syntax of LINGO mathematical modelling language.

Mathematical nomenclature	**LINGO syntax**
Minimum	MIN =
$\sum x'_{i,r}$	@sum(X(i,r))
For each i in the set of suppliers	@FOR(Suppliers (i))
•	*
=	=
Exponent	ˆ
Load input parameters **W** from the file data.ldt	W=@file(data.ldt)
Write **W** to the output file out.ldt in append mode	@text('out.txt', 'a')=@write('Supply:',W);
Write to a file product transportation plans **X** as a table	@text('out.txt', 'a')=@table(Volume_Prod);

The structure of the model is composed of sections. The main section is the MODEL section. The most important sub-sections, highlighted by the relevant keywords are: SETS, DATA. In the SETS section types of simple or complex objects, and their mutual relationships are defined. DATA section allows initiating or assigning values to individual parameters of the model.

Implementation of the optimization model

The implementation of the optimization model to solve problem under consideration is given in Fig. 2. The model is constructed for time functions describing production units in form $\gamma_r(v_r) = Av_r^B$. The example of the data file is presented in Fig. 3. The file presents simple network consisting of two suppliers, two production units, and two product receivers. It is easy to see, that implementation of the model including the load and presentation of the data is very concise. For better orientation in model definition file as well as in data file, some comments are included.

```
Model:
Sets:
!Raw material suppliers
Suppliers /1..@file(data.ldt)/:Supply;
!Product receivers
Receivers /1..@file(data.ldt)/:Demand;
Units /1..@file(data.ldt)/:
                       A,B,Productivity;
!RMT -Raw material transportation;
RMT(Suppliers,Units):
           Cost_Raw_Mat, Volume_Raw_Mat;
!PT -Product transportation,;
PT(Units,Receivers):
                  Cost_Prod,Volume_Prod;
EndSets
Data: !Input data
Supply         = @file(data.ldt);
Demand         = @file(data.ldt);
A              = @file(data.ldt);
B              = @file(data.ldt);
Productivity   = @file(data.ldt);
Cost_Raw_Mat   = @file(data.ldt);
Cost_Prod      = @file(data.ldt);
Pi             = @file(data.ldt);
EndData
!The objective;
[Objective] Min=z
    +@sum(RMT(i,r):Cost_Raw_Mat(i,r)
          *Volume_Raw_Mat(i,r))
    +@sum(PT(r,k):Cost_Prod(r,k)
          *Volume_Prod(r,k));
!The constraints;
@for(Units(r):
z>=Pi*A(r)*(@sum(RMT(i,r):
            Volume_Raw_Mat(i,r)))^B(r));
@for(Units(r): (@sum(RMT(i,r):
   Volume_Raw_Mat(i,r)))*Productivity(r)
     =(@sum(PT(r,k):Volume_Prod(r,k))));
@for(Suppliers(i): @sum(RMT(i,r):
       Volume_Raw_Mat(i,r))<=Supply(i));
@for(Receivers(k): @sum(PT(r,k):
           Volume_Prod(r,k))=Demand(k));
Data: !Output data;
@text('out.txt')=@write(@TIME(),@newline
(1),Objective, ' $', @NewLine(1));
@text('out.txt', 'a')
                =@table(Volume_Raw_Mat);
@text('out.txt', 'a')
                   =@table(Volume_Prod);
enddata
end
```

Fig. 2. Optimization model described in Lingo programming language.

```
2~                       !I;
2~                       !R;
2~                       !K;
100 100~                 !Supply;
40 60~                   !Demand;
2 3~                     !a;
2 2~                     !b;
0.5 0.6~                 !e;
30 25 24 29~             !c';
31 19 24 19~             !c_;
0.0001~                  !Pi;
```

Fig. 3. Example of the data structure in the input data file to Lingo.

The solver is mainly used to solve convex optimization problem. For non-convex models, solver returns one of the local minima. The solver's output is determined by the initial conditions.

Algorithms solving the non-convex version of the problem

For non-convex optimization models the following algorithms have been developed:

- Evolution algorithm,
- Heuristic algorithm ,
- Scatter search algorithm.

The first algorithm is an implementation of a genetic algorithm with individual coded by real numbers and representing the allocation of the raw material to production units. To obtain the value of the fit function, two transportation problems have to be solved. The algorithm has been described in details in [26] and [27].

The second algorithm is based on the approximate algorithm developed for convex models and described in [26] and [28]. It has been extended to cope with non-convex models. The modified algorithm has been presented in [29]. It main idea is to change allocation of the raw material to production units in a way that brings the improvement of the quality expressed by the total cost function. Likewise it was done with evolutionary algorithm, to obtain the value of the total cost, two transportation problems have to be solved each time, new allocation values are examined. The size of change in the allocation values is not constant which gives the possibility to change search areas of the local minima. The comparison of those two algorithms presented in [29] show, that they give very similar results, but the heuristic algorithm is faster.

The third algorithm developed is the implementation of the scatter search algorithm. The scatter search provides the new allocation of the raw material being the basis for the initial conditions for the Lingo solver (start points). In fact, the algorithm works on the vector describing the amounts of the product manufactured by the production units, while the sum of its elements is constant. The linear combination of the values in this vector produce new initial conditions. This algorithm combines the power of Lingo solver, and the power of heuristic algorithm which is scatter search.

Of course, presented algorithms dealing with nonconvex models, do not guarantee finding the global optimum. However, most of them find good solutions in rather short time.

To integrate solvers developed by Lindo systems with C++ programs, we used Lindo API product that has the same solvers as Lingo.

For a special case of the supply network consisting of only one, or two production units, exact solution algorithms have been developed and described in details in [30].

The idea of the computer decision support system

The algorithms described can be integrated in one computer system treated as a decision support tool for managers or logistic providers. The system choose for the user, in a transparent way, the best solution algorithm to be executed. It takes into account the preferences of the user of which heuristic algorithm he prefer –Evolutionary, Heuristic, or Scatter Search. The decision process made by the system is described in Fig. 4.

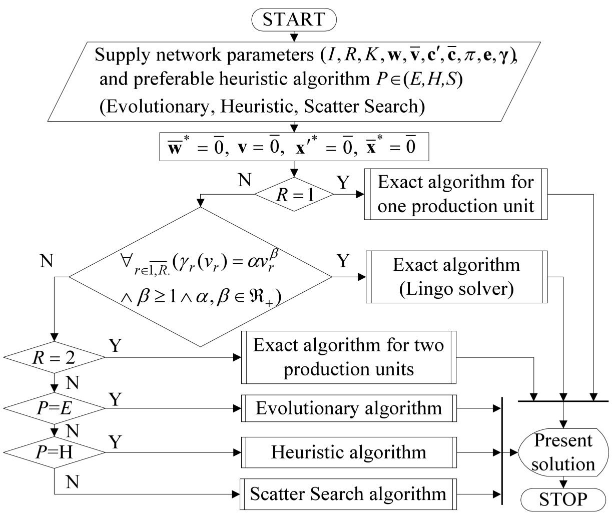

Fig. 4. Structure of the decision support system.

Numerical Examples

To show the main possibilities given by the developed algorithms and decision support system, we consider a network consisting of two suppliers, four production units and three product receivers. In the following numerical examples, the models of the production units and production cost coefficient π are to be changed. At first, all models are convex functions in the form $\gamma_r(v_r) = \alpha v_r^\beta$ where $\beta \geq 1$ and $\alpha, \beta \in \Re_+$, then we change model of a few production units in a way that $0 < \beta < 1$. This modification change the objective function from convex to non-convex. To handle the non-convex case of the problem, the Scatter Search was chosen as the preferable heuristic algorithm. For each example we were changing the value of π to see the sensitivity of the solution to this parameter, which describes how much production cost depends on overall production time. The fixed values of the supply network parameters are as follows: I=2, R=4, K=3, w_1=100, w_2=100, $\bar{v}_1$=20, $\bar{v}_2$=20, $\bar{v}_3$=60, $c'_{1,1}$=31, $c'_{1,2}$=31, $c'_{1,3}$=29, $c'_{1,4}$=30, $c'_{2,1}$=27, $c'_{2,2}$=29, $c'_{2,3}$=28, $c'_{2,4}$=27, $\bar{c}_{1,1}$=28, $\bar{c}_{1,2}$=26, $\bar{c}_{1,3}$=26, $\bar{c}_{2,1}$=33, $\bar{c}_{2,2}$=28, $\bar{c}_{2,3}$=31, $\bar{c}_{3,1}$=36, $\bar{c}_{3,2}$=32, $\bar{c}_{3,3}$=26, $\bar{c}_{4,1}$=35, $\bar{c}_{4,2}$=26, $\bar{c}_{4,3}$=29, e_1=0.5, e_2=0.6,

Table 3
Network parameters changed in the experiments.

Parameter	Values of the parameters in four experiments			
	Exp. 1	*Exp. 2*	*Exp.3*	*Exp. 4*
β_1	3		0.5	
β_2	3		0.6	
β_3	2		2	
β_4	2		2	
π	0.004	0.006	8.2	8.3

Table 4
Results of the experiments.

Variable	Values of the variables in four experiments.			
	Exp. 1	*Exp. 2*	*Exp.3*	*Exp. 4*
Time [s]	< 1	< 1	3	3
Obj. value	8745.90	8911.25	8462.82	8478.06
v_1	34.65	33.61	0.00	158.42
v_2	30.27	29.36	183.33	34.65
v_3	0.00	9.87	0.00	0.00
v_4	129.02	123.25	0.00	0.00
$x'_{1,1}$	0.00	0.00	0.00	58.42
$x'_{1,2}$	30.27	29.36	83.33	34.65
$x'_{1,3}$	0.00	9.87	0.00	0.00
$x'_{1,4}$	63.67	56.87	0.00	0.00
$x'_{2,1}$	34.65	33.61	0.00	100.00
$x'_{2,2}$	0.00	0.00	100.00	0.00
$x'_{2,3}$	0.00	0.00	0.00	0.00
$x'_{2,4}$	65.35	66.39	0.00	0.00
$\bar{x}_{1,1}$	17.33	2.38	20.00	20.00
$\bar{x}_{1,2}$	0.00	0.00	0.00	0.00
$\bar{x}_{1,3}$	0.00	14.43	30.00	59.21
$\bar{x}_{2,1}$	2.67	17.62	0.00	0.00
$\bar{x}_{2,2}$	15.49	0.00	20.00	20.00
$\bar{x}_{2,3}$	0.00	0.00	30.00	0.79
$\bar{x}_{3,1}$	0.00	0.00	0.00	0.00
$\bar{x}_{3,2}$	0.00	0.00	0.00	0.00
$\bar{x}_{3,3}$	0.00	3.95	0.00	0.00
$\bar{x}_{4,1}$	0.00	0.00	0.00	0.00
$\bar{x}_{4,2}$	4.51	20.00	0.00	0.00
$\bar{x}_{4,3}$	60.00	41.63	0.00	0.00

e_3=0.4, e_4=0.5, α_1=2, α_2=3, α_3=4, α_4=5. The parameters values that change are gathered in Table 3. In the paper we present the results of four numerical experiments. All of them are gathered in Table 4.

As an output data, we selected the computation time used to obtain the solution, the value of the objective function in the minimum point, transportation plans of the raw material as well as the product, and the allocation of the raw material to production units.

We omit the units in the example because they do not have significant importance, and in fact depend from the real model and the currency used the country, or by logistic provider. In this particular case, we can assume, that production time is expressed in seconds cost in dollars, and amounts of the raw material as well as product in tons.

Corollaries

The sensitiveness of the solution depends on coefficient π. In some cases the change in the value of π may cause the need of opening or closing one or more production units from use. The manager have to take the final decision of the data execution, or of changing the input data. It is obvious, that it is not advisable to engage production unit into production process, if it is not planned to process on that unit enough amount of the raw material to cover all the costs and effort connected with running the unit. The benefit from including additional production unit into production process can be calculated as a difference between solution that takes this unit into account, and solution, that does not.

As it can be observed, little change in the values of the coefficient π does not change a lot the total cost, but can be the reason to take by the decision maker some strategic steps, like e.g. closing a factory. The change in the production unit model has a great impact for the solution, especially when coefficient π has greater value.

In the paper we present only a part of the research that can be done using developed algorithms. The most information gives the solution obtained by Lingo solver for convex, especially linear cases of the problem considered. For non-convex cases, we can achieve similar information, but concerning only the local minimum.

Conclusions

The paper concerns the optimization problem of joint allocation and transportation in a special case supply networks consisting of suppliers of the raw material, production units and receivers of the product. The supply networks under consideration are connected with the processing of fast perishing goods, like vegetables and fruits. The problem of joint allocation and transportation has been presented and solution algorithms described. The integration of the algorithms developed has been proposed. The special attention was put on the use of the commercial solver Lingo, developed by Lindo Systems inc. It was mainly used to model and solve convex cases of the problem. For non-convex cases, three heuristic algorithms have been presented. Some properties of the solution algorithms have been shown by numerical examples.

The further work will include the research dealing with parametric uncertainty in supply network under consideration. The results obtained from the numerical results, indicate, that the sensitiveness for expert's knowledge might be significant.

References

[1] Józefczyk J., *Decision Making Problems in Complex of Operations Systems* (in Polish), Wroclaw: Wroclaw University of Technology Press, 2001.

[2] Węglarz J., *Application of the convex sets theory in a certain problem of time-optimal control of a complex of operations*, Systems Science, 1, 1, 67–74, 1975.

[3] Węglarz J., *Time-optimal control of resource allocation in a complex of operations framework*. IEEE Trans. Systems, Man and Cybernetics, 6, 11, 783–788, 1976.

[4] Taha H.A., *Operations Research An Introduction*, Prentice Hall, Singapore, 1995.

[5] Bubnicki Z., Wołkowiński K., *On the allocation problem in a production system with transport of materials*, Systems Science, 24, 89–99, 1998.

[6] Karaoglana I., Altiparmak F., Kara I., Dengiz B., *A branch and cut algorithm for the location-routing problem with simultaneous pickup and delivery*, European Journal of Operational Research, 211, 318–332, 2011.

[7] Nagy G., Salhi S., *Location-routing: Issues, models and methods*, European Journal of Operational Research, 177, 649–672, 2007.

[8] Nguyen V., Prins C., Prodhon C., *Solving the two-echelon location routing problem by a GRASP reinforced by a learning process and path relinking*, European Journal of Operational Research, 216, 113–126, 2012.

[9] Prodhon C., *A hybrid evolutionary algorithm for the periodic location-routing problem*, European Journal of Operational Research, 210, 204–212, 2011.

[10] Hamacher H. W., Hennes H., *Integrated scheduling and location models: single machine makespan problems*, Studies in Locational Analysis, 16, 77–90, 2007.

[11] Kalsch M. T., Drezner Z., *Solving scheduling and location problems in the plane simultaneously*, Computerts & Operations Research, 37, 256–264, 2010.

[12] Bermana O., Krassa D., Tajbakhshb M.M., *A coordinated location-inventory model*, European Journal of Operational Research, 217, 500–508, 2012.

[13] Drezne Z., Scott C., Song J.-S., *The central warehouse location problem revisited*, IMA Journal of Management Mathematics, 14, 321–336, 2003.

[14] Li J., Chen H., Chu F., *Performance evaluation of distribution strategies for the inventory routing problem*, European Journal of Operational Research, 202, 412–419, 2010.

[15] Moin N. H., Salhi S., *Inventory routing problems: A logistical overview*, Journal of the Operational Research Society, 58, 1185–1194, 2007.

[16] Averbakh I., Berman O., *A simple heuristic for m-machine flow-shop and its applications in routing-scheduling problems*, Operations Research, 47, 165–170, 1999.

[17] Józefczyk J., *Scheduling tasks on moving executors to minimize the maximum lateness*. European Journal of Operational Research, 131, 171–187, 2001.

[18] Metters R. D., *Interdependent transportation and production activity at the United States postal service*, Journal of the Operational Research Society, 47, 27–37, 1996.

[19] Ben-Daya M., As'ad R., Seliaman M., *An integrated production inventory model with raw material replenishment considerations in a three layer supply chain*, International Journal of Production Economics, doi: 10.1016/j.ijpe.2010.10.024, 2010.

[20] Filcek G., *Algorithm of joint allocation and transportation in time varying supply network*, In: Z. Wilimowska, L. Borzemski, A. Grzech, & J. Świątek, eds., Information systems architecture and technology: Models of the Organisation's Risk Management, Wroclaw: Wroclaw University of Technology Press, pp. 129–139, 2008.

[21] Dolgui A., Proth J-M., *Supply chain engineering. Useful methods and techniques*, London Dordrecht Heidelberg New York: Springer, 2010.

[22] Meixell M. J., Gargeya V. B., *Global supply chain design: A literature review and critique*, Transportation Research Part E, 41, 531–550, 2005.

[23] Mula J., Peidro D., Diaz-Madronero M., Vicens E., *Mathematical programming models for supply chain production and transport planning*, European Journal of Operational Research, 204, 377–390, 2010.

[24] Boyd S., Vandenberghe L., *Convex optimization*, Cambridge, UK: Cambridge University Press, 2004.

[25] Lindo systems inc. website, 2012, 10.07.2012 www.lindo.com.

[26] Filcek G., *Algorithms of joint allocation of the raw material and transportation in production systems* (in Polish), Ph.D. Thesis, Wroclaw University of Technology, 2011.

[27] Filcek G., Szczepanik P., *Application of the Evolutionary Algorithm in Joint Problem of Raw Material Allocation and Transportation* (in Polish), Inzynieria Wiedzy i Systemy Ekspertowe IWiSE'09, Akademicka Oficyna Wydawnicza EXIT, Warsaw, pp. 441–452, 2009.

[28] Filcek G., Józefczyk J., *Integrated production and transportation in a class of supply networks : deterministic and uncertain cases*, Procedia: Social and Behavioral Sciences, 20, 197–206, 2011.

[29] Filcek G., Józefczyk J., *Heuristic algorithm for integrated allocation and transportation in three-stage supply network*, Procedia: Social and Behavioral Sciences, 2012, in press.

[30] Filcek G., *Algorithms for joint allocation and transportation problem in selected supply network*, 2012, unpublished.