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MATHEMATICAL PROGRAMMING MODEL OF COST OPTIMIZATION FOR SUPPLY CHAIN FROM PERSPECTIVE OF LOGISTICS PROVIDER

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Abstract
The article presents the problem of optimizing the supply chain from the perspective of a logistics provider and includes a mathematical model of multilevel cost optimization for a supply chain in the form of MILP (Mixed Integer Linear Programming). The costs of production, transport and distribution were adopted as an optimization criterion. Timing, volume, capacity and mode of transport were also taken into account. The model was implemented in the environment of LINGO ver. 12 package. The implementation details, the basics of LINGO as well as the results of the numerical tests are presented and discussed. The numerical experiments were carried out using sample data to show the possibilities of practical decision support and optimization of the supply chain. In addition, the article presents the current state of logistics outsourcing.

Keywords
supply chain, MILP – Mixed Integer Linear Programming, optimization, 3PL-Third Party Logistic, multimodal transport.

Introduction

The issue of the supply chain is the area of science and practice that has been strongly developing since the '80s of the last century. Numerous definitions describe the term, and a supply chain reference model has also been designed [1, 2]. The supply chain is commonly seen as a collection of various types of companies (raw materials, production, trade, logistics, etc.) working together to improve the flow of products, information and finance. As the words in the term indicate, the supply chain is a combination of its individual links in the process of supplying products (material and services) to the market.

Huang et al. [3] studied the shared information of supply chain production. They considered and proposed four classification criteria: supply chain structure, decision level, modeling approach and shared information.

Supply chain structure: It defines the way various organizations within the supply chain are arranged and related to each other. The supply chain structure falls into four main types [4]: Convergent: each node in the chain has at least one successor and several predecessors. Divergent: each node has one predecessor, at least, and several successors. Conjoined, which is a combination of each convergent chain and one divergent chain. Network: this cannot be classified as convergent, divergent or conjoined, and is more complex than the three previous types.

Decision level: Three decision levels may be distinguished in terms of the decision to be made; strategic, tactical and operational; and its corresponding period, i.e., long-term, mid-term and short-term.

Supply chain analytical modeling approach: This approach consists in the type of representation, in this case, mathematical relationships, and the aspects to be considered in the supply chain. Most literature describes and discusses the linear programming-based modeling approach, mixed integer linear programming models in particular [5–9].

Shared information: This consists in the information shared between each network node determined by the model, which enables production, distribution and transport planning in accordance with the purpose drawn up. The shared information process is vital for effective supply chain production, distribution and transport planning. In terms of centralized planning, this information flows from each node of the network where the decisions are made. Shared information includes the following groups of parameters: resources, inventory, production, transport, demand, etc. Minimization of total costs is the main purpose of the models presented in the literature [9–13] while maximization of revenues or sales is considered to a smaller scale [7, 14].

This paper deals with a mathematical model for supply chain costs optimization in the form of MILP (Mixed Integer Linear Programming Problem) [15] from the perspective of logistic provider. In this model, shared process information includes such parameters as resources, inventory, production, transport, demand etc. In previous works, we have studied models and algorithms for combinatorial optimization of cost in supply chain. In this paper, we focus on the multimodal transport in the supply chain and its implementation aspects. It should be emphasized that the presented model can be the basis for the decision support in the supply chain management. Optimization results of this model relate to two types of decision. These are short-term decisions about how to supply at minimum cost (operational level). One can also support the long-term decision on the capacity of individual distributors or production capacity of individual producers (tactical and strategic level). The article also presents various models of outsourced logistics management. The rest of the paper is organized as follows: Section 2 describes the problem of SCM (Supply Chain Management) from the logistic provider perspective. Section 3 analyses the state of the art in this domain. Section 4 gives the problem statement and provides an optimization model for the considered supply chain with multimodal transport. The implementation aspects of the optimization model are explained briefly in Sec. 5. Computational examples and tests of the implemented model are presented in Sec. 6. The discussion on possible extensions of the proposed approach and conclusions is included in Sec. 7.

Supply Chain Management

The aim of supply chain management (SCM) is to increase sales, reduce costs and take full advantage of business assets by improving interaction and communication between all the actors forming the supply chain. The supply chain management is a decision process that not only integrates all of its participants but also helps to coordinate the basic flows: products/services, information and funds. Changes in the global economy and the increasing globalization lead to the widespread use of IT tools, which enables continuous, real-time communication between the supply chain links. One of the objectives is to optimize logistics and entrust it to specialized companies.

This direction contributed to the development of logistics outsourced operators known as 3PL, 4PL, or 5PL [16]. The term 3PL (Third Party Logistics) refers to the use of external companies and organizations to carry out logistic functions that can involve the entire logistics process or its selected features. The company offers and provides 3PL services using its own means of transport, warehouses, equipment and other necessary resources, and acts as a “third party” between a producer and a customer. The resulting model with the supply chain logistics services outsourced to specialized 3LP companies is shown in Fig. 1. This kind of cooperation is frequently referred to as the logistics alliance.

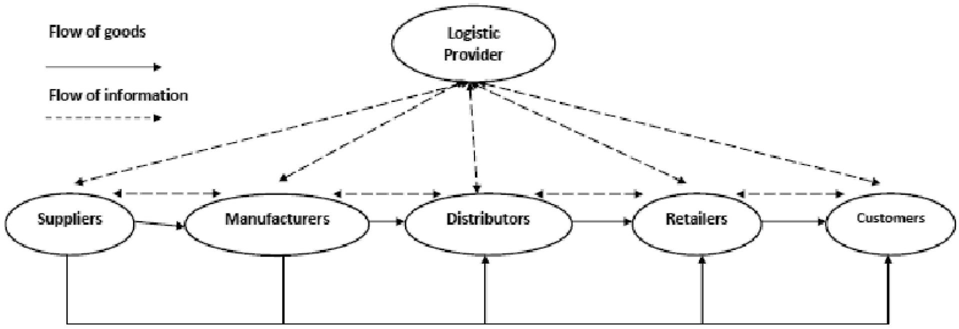

Fig. 1. The chart of the supply chain with logistics services outsourced to a logistic provider – based on [23].

4PL (Fourth-Party Logistics) is a certain evolution of the 3PL concept to provide greater flexibility and adaptation to the needs of the client. 4PL companies and organizations operate primarily by managing the information flow within the entire supply chain. Unlike the 3PL, responsible for only a selected segment, a 4PL coordinates logistics processes along the whole length of the chain (from raw materials to end-buyers). The 4PL model enables the 3PL operator to become a coordinator and integrator of the flows, not just an operator of physical displacement of goods. Very often, its subcontractors are 3PL or even 2PL (Second Party Logistics) operators, i.e., transport companies and warehouses. The company that uses the services of a 4PL provider is in contact with only one operator who manages and integrates all types of resources and oversees the entire functionality across the supply chain. 4PL providers, having a complete picture of the supply chain and large IT capabilities may offer optimization and decision support advisory services [17]. Further development of logistics outsourcing resulted in the creation of 5LP model (Fifth Party Logistics) – providers of integrated logistics services that can design and implement flexible and networked supply chains to cater to the needs of all participants (manufacturers, suppliers, carriers and end users).

State of art and motivation

Simultaneously considering the supply chain production, distribution processes in distribution centers and transport-planning problems greatly advances the efficiency of all processes. The literature in the field is vast, so an extensive review of existing research on the topic is extremely helpful in modeling and research. Comprehensive surveys on these problems and their generalizations were published, for example in [3].

In our approach, we are considering a case of the supply chain where:

- The shared information process in the supply chain consists of resources (capacity, versatility, costs), inventory (capacity, versatility, costs, time), production (capacity, versatility, costs), product (volume), transport (cost, mode, time), demand, etc.(Fig. 2, Fig. 3).
- The transport is multimodal. (Several modes of transport.A limited number of means of transport for each mode).
- Different products are combined in one batch of transport.
- The cost of supplies is presented in the form of a function (in this approach linear function of fixed and variable costs).
- Different decision levels are considered simultaneously.

Decision levels in supply chains are mainly classified by the extent or effect of the decision to be made in terms of time. For instance, at the strategic level, the decisions made in relation to selecting production, storage and distribution locations, etc should be identified. At the tactical level however, the aspects such as production and distribution planning, assigning production and transport capacities, inventories and managing safety inventories are identified. Finally, at the operational level, replenishment and delivery operations are classified [3]. Most of the reviewed works focus on the tactical decision level [6–8, 10–12, 18, 19]. Only few works deal with the problems taken together for the different decision levels [5, 13].

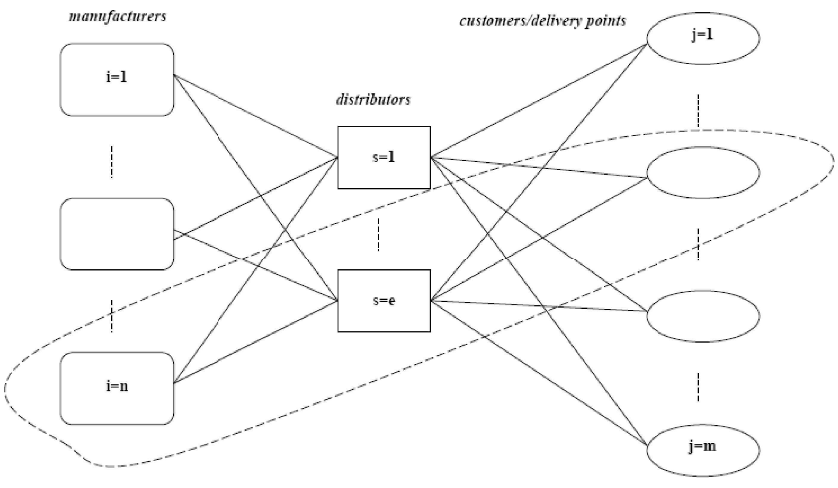

Fig. 2. The part of the supply chain network with marked indices of individual participants (elements). Dashed line marks one of the possible routes of delivery.

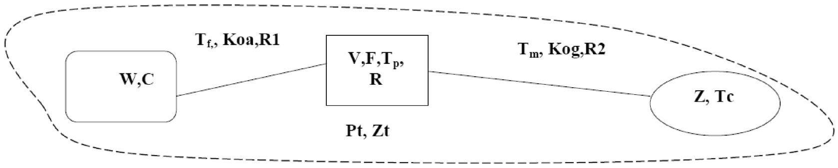

Fig. 3. The selected path of the supply chain along with the parameters that describe the individual elements and its dependencies (shared information).

Therefore, the motivation behind this work is to suggest an approach of multilevel supply chain cost optimization with multimodal transport from the perspective of a logistics provider, and to propose an optimization model in the form of integer programming problem [20] which facilitates its solution using specialized software available on the market (LINGO, CPLEX). Many aspects of implementing the proposed model are featured, including additional decision variables introduced at the level of implementation, optimization model in LINGO language, etc.

The aim of this paper is to design and implement the model that can become the basis for making optimal decisions at different levels of supply chain management. The proposed solution will also enable a comprehensive examination of the impact on cost and performance of various parameters of the shared information.

Problem statement

Background

A key step in many decision-making and design processes is the optimization phase, which itself contains several stages. The purpose of the optimization process in our approach is to help determine realistic and practical outcomes of management decision-making and design processes in the supply chain. There are two basic ways to optimize the problem, either the qualitative approach or the quantitative approach. Using only a qualitative approach, the problem optimization, when making a decision, relies on personal judgment or experience acquired in dealings with similar problems in the past. In a few cases this approach may be adequate; however, there are many situations where a quantitative approach to the problem provides a better-structured and logical path through the decision-making process.

We propose the quantitative approach for the cost optimization supply chain network model (Fig. 2) designed from a perspective of 3PL/4PL/-5PL providers.

Problem formulation

The mathematical optimization model was formulated as an integer linear programming problem [20] with the minimization of costs (1) under constraints (2) .. (23). Indices, parameters and decision variables in the model together with their descriptions are provided in Table 1. The proposed optimization model is a cost model that takes into account three other types of parameters, i.e., the spatial parameters (area/volume occupied by the product, distributor capacity and capacity of transport unit), time (duration of delivery and service by distributor, etc.) and transport mode. The position of each parameter against the subsequent links of the supply chain is shown in Fig. 3.

Optimization criteria

The objective function (1) defines the aggregate costs of the entire chain and consists of four elements. The first is the fixed costs associated with the operation of the distributor involved in the delivery (e.g. distribution center, warehouse, etc.). The second component determines the cost of supply from the manufacturer to the distributor. Another component is responsible for the costs of supply from the distributor to the end user (the store, the individual client, etc.). The last component of the objective function determines the cost of manufacturing the product by the given manufacturer.

$$\sum_{s=1}^{E} F_s * Tc_s + \sum_{i=1}^{N}\sum_{s=1}^{E}\sum_{d=1}^{L} Koa_{i,s,d} + \sum_{s=1}^{E}\sum_{j=1}^{M}\sum_{d=l}^{L} Kog_{s,j,d} + \sum_{i=1}^{N}\sum_{k=1}^{O}(C_{ik} * \sum_{s=1}^{E}\sum_{d=1}^{L} X_{i,s,k,d}). \tag{1}$$

Table 1

Summary indices, parameters and decision variables of the mathematical optimization model.

Symbol	**Description**
	Indices
k	product type ($k = 1..0$)
j	delivery point/customer/city ($j = 1..M$)
i	manufacturer/factory ($i = 1..N$)
s	distributor /distribution center ($s = 1..E$)
d	mode of transport ($d = 1..L$)
N	number of manufacturers/factories
M	number of delivery points/customers
E	number of distributors
O	number of product types
L	number of mode of transport
	Input parameters
F_s	the fixed cost of distributor/distribution center s ($s = 1..E$)
P_k	the area/volume occupied by product k ($k = 1..O$)
V_s	distributor s maximum capacity/volume ($s = 1..E$)
W_{ik}	factory production capacity for product k ($i = 1..N$) ($k = 1..O$)
C_{ik}	the cost of product k at factory i ($i = 1..N$) ($k = 1..O$)
R_{sk}	if distributor s ($s = 1.E$) can deliver product k ($k = 1..O$) then $R_{sk} = 1$, otherwise $R_{sk} = 0$
Tp_{sk}	the time needed for distributor s ($s = 1..E$) to prepare the shipment of product k ($k = 1..O$)
Tc_{jk}	the cut-off time of delivery to the delivery point/customer j ($j = 1..M$) of product k ($k = 1..O$)
Z_{jk}	customer demand/order j ($j = 1..M$) for product k ($k = 1..O$)
Zt_d	the number of transport units using mode of transport d ($d = 1..L$)
Pt_d	the capacity of transport unit using mode of transport d ($d = 1..L$)
Tf_{isd}	the time of delivery from manufacturer i to distributor s using mode of transport d ($i = 1..N$) ($s = 1..E$) ($d = 1..L$)
$K1_{iskd}$	the variable cost of delivery of product k from manufacturer i to distributor s using mode of transport d d ($d = 1..L$) ($i = 1..N$) ($s = 1..E$) ($k = 1..O$)
$R1_{isd}$	if manufacturer i can deliver to distributor s using mode of transport d then $R1_{isd} = 1$, otherwise $R1_{isd} = 0$ ($d = 1..L$) ($s = 1..E$) ($i = 1..N$)
A_{isd}	the fixed cost of delivery from manufacturer i to distributor s using mode of transport d ($d = 1..L$) ($i = 1..N$) ($s = 1..E$)
Koa_{sjd}	the total cost of delivery from distributor s to customer j using mode of transport d ($d = 1..L$) ($s = 1..E$) ($j = 1..M$)
Tm_{sjd}	the time of delivery from distributor s to customer j using mode of transport d ($d = 1..L$) ($s = 1..E$) ($j = 1..M$)
$K2_{sjkd}$	the variable cost of delivery of product k from distributor s to customer j using mode of transport d d ($d = 1..L$) ($s = 1..E$) ($k = 1..O$) ($j = 1..M$)
$R2_{sjd}$	if distributor s can deliver to customer j using mode of transport d then $R2_{sjd} = 1$, otherwise $R2_{sjd} = 0$ ($d = 1..L$) ($s = 1..E$) ($j = 1..M$)
G_{sjd}	the fixed cost of delivery from distributor s to customer j using mode of transport d ($s = 1..E$) ($j = 1..M$) ($k = 1..O$)
Kog_{sjd}	the total cost of delivery from distributor s to customer j using mode of transport d ($d = 1..L$) ($s = 1..E$) ($j = 1..M$) ($k = 1..O$)
	Decision variables
X_{iskd}	delivery quantity of product k from manufacturer i to distributor s using mode of transport d
Xa_{isd}	if delivery is from manufacturer i to distributor s using mode of transport d then $Xa_{isd} = 1$, otherwise $Xa_{isd} = 0$
Xb_{isd}	the number of courses from manufacturer i to distributor s using mode of transport d
Y_{sjkd}	delivery quantity of product k from distributor s to customer j using mode of transport d
Ya_{sjd}	if delivery is from distributor s to customer j using mode of transport d then $Ya_{sjd} = 1$, otherwise $Ya_{sjd} = 0$
Yb_{sjd}	the number of courses from distributor s to customer j using mode of transport d
Tc_s	If distributor s participates in deliveries, then $Tc_s = 1$, otherwise $Tc_s = 0$
CW	Arbitrarily large constant

Constraints

The model was developed subject to constraints (2) .. (23).

Constraint (2) specifies that all deliveries of product k produced by the manufacturer i and delivered to all distributors s using mode of transport d do not exceed the manufacturer's production capacity.

$$\sum_{s=1}^{E}\sum_{d=1}^{L} X_{i,s,k,d} \le W_{i,k} \quad \text{for } i = 1..N,\ k = 1..O. \tag{2}$$

Constraint (3) covers all customer j demands for product k ($Z_{j,k}$) through the implementation of supply by distributors s (the values of decision variables $Y_{i,s,k,d}$). The constraint was designed to take into account the specificities of the distributors (i.e., whether the distributor s can deliver the product k or not).

$$\sum_{s=1}^{E}\sum_{d=1}^{L} (Y_{s,j,k,d} * R_{s,k}) \ge Z_{j,k} \quad \text{for } j = 1..M,\ k = 1..O. \tag{3}$$

The balance of each distributor s corresponds to constraint (4).

$$\sum_{i=1}^{N}\sum_{d=1}^{L} X_{i,s,k,d} = \sum_{j=1}^{M}\sum_{d=1}^{L} Y_{s,j,k,d} \quad \text{for } s = 1..E,\ k = 1..O. \tag{4}$$

The possibility of delivery in due to its technical capabilities – in the model, in terms of volume/capacity of the distributor's is defined by constraint (5).

$$\sum_{k=1}^{O}\left(P_k * \sum_{i=1}^{N}\sum_{d=1}^{L} X_{i,s,k,d}\right) \le Tc_s * V_s \quad \text{for } s = 1..E. \tag{5}$$

Constraint (6) ensures the fulfillment of the terms of delivery time.

$$Xa_{i,s,d}{*}Tf_{i,s,a} + Xa_{i,s,d}{*}Tp_{s,k} + Ya_{s,j,d}{*}Tm_{s,j,d} \le Tc_{j,k} \quad \text{for } i = 1..N,\ s = 1..E,\ j = 1..M,\ k = 1..O,\ d = 1..L. \tag{6}$$

Constraints (7a) (7b), (8) guarantee deliveries with available transport taken into account.

$$R1_{i,s,d} * Xb_{i,s,d} * Pt_d \ge X_{i,s,k,d}{*}P_k \quad \text{for } i = 1..N,\ s = 1..E,\ k = 1..O,\ d = 1..L, \tag{7a}$$

$$R2_{s,j,d} * Yb_{s,j,d} * Pt_d \ge Y_{s,j,k,d}{*}P_k \quad \text{for } s = 1..E,\ j = 1..M,\ k = 1..O,\ d = 1..L, \tag{7b}$$

$$\sum_{i=1}^{N}\sum_{s=1}^{E} Xb_{i,s,d} + \sum_{j=1}^{M}\sum_{s=1}^{E} Yb_{j,s,d} \le Zt_d \quad \text{for } d = 1..L. \tag{8}$$

Constraints (9)–(11) respectively set values of decision variables based on binary variables Tc_s, $Xa_{i,s,d}$, $Ya_{s,j,d}$.

$$\sum_{i=1}^{N}\sum_{d=1}^{L} Xb_{i,s,d} \le CW * Tc_s \quad \text{for } s = 1..E, \tag{9}$$

$$Xb_{i,s,d} \le CW * Xa_{i,s,d} \quad \text{for } i = 1..N,\ s = 1..E,\ d = 1..L, \tag{10}$$

$$Yb_{s,j,d} \le CW * Ya_{s,j,d} \quad \text{for } s = 1..E,\ j = 1..M,\ d = 1..L. \tag{11}$$

Dependencies (12) and (13) represent the relationship by which total costs are calculated. In general, these may be any linear functions.

$$Koa_{i,s,d} = A_{i,s,d} * Xb_{i,s,d} + \sum_{k=1}^{O} K1_{i,s,k,d} * X_{i,s,k,d} \quad \text{for } i = 1..N,\ s = 1..E,\ d = 1..L, \tag{12}$$

$$Koa_{s,j,d} = G_{s,j,d}{*}Yb_{j,s,d} + \sum_{k=1}^{O} K2_{s,j,k,d} * Y_{s,j,k,d} \quad \text{for } s = 1..E,\ j = 1..M,\ d = 1..L. \tag{13}$$

The remaining constraints (14)..(23) arise from the nature of the model.

$$X_{i,s,k,d} \ge 0 \quad \text{for } i = 1..N,\ s = 1..E,\ k = 1..0,\ d = 1..L, \tag{14}$$

$$Xb_{i,s,d} \ge 0 \quad \text{for } i = 1..N,\ s = 1..E,\ d = 1..L, \tag{15}$$

$$Yb_{s,j,d} \ge 0 \quad \text{for } s = 1..E,\ j = 2..M,\ d = 1..L, \tag{16}$$

$$X_{i,s,k,d} \in C \quad \text{for } i = 1..N,\ s = 1..E,\ k = 1..0,\ d = 1..L, \tag{17}$$

$$Xb_{i,s,d} \in C \quad \text{for } i = 1..N,\ s = 1..E,\ d = 1..L, \tag{18}$$

$$Y_{s,j,k,d} \in C \quad \text{for } s = 1..E,\ j = 1..M,\ k = 1..0,\ d = 1..L, \tag{19}$$

$$Yb_{s,j,d} \in C \quad \text{for } s = 1..E,\ j = 1..M,\ d = 1..L, \tag{20}$$

$$Xa_{i,s,d} \in \{0,1\} \quad \text{for } i=1..N,\ s=1..E,\ d=1..L, \tag{21}$$

$$Ya_{s,j,d} \in \{0,1\} \quad \text{for } s=1..E,\ j=2..M,\ d=1..L, \tag{22}$$

$$Tc_s \in \{0,1\} \text{ for } s=1..E. \tag{23}$$

Method developed

The model was implemented in "LINGO" by LINDO Systems [21]. "LINGO" Optimization Modeling Software is a powerful tool for building and solving mathematical optimization models. "LINGO" package provides the language to build optimization models and the editor program including all the necessary features and built-in "solvers" in a single integrated environment. "LINGO" is designed to model and solve linear, nonlinear, quadratic, integer and stochastic optimization problems. Model implementation is possible in two basic ways. The first way is to enter the model into the "LINGO" editor in the explicit form, that is, a full function of the objective with all the constraints, parameters, etc. Although this is an intuitive approach and consistent with the standard form of linear programming [20], it is not very useful in practice. This is due to the size of models implemented in practice. For the small example presented in chapter Computational examples, the number of decision variables and constraints was 592 and 1125, respectively. The other way is to use the "LINGO" language of mathematical modeling, an integral part of the "LINGO" package, whose basic syntax elements are shown in Table 1. For the real examples with sizes exceeding several decision variables, the construction and implementation of the model is only possible using the modeling language (Table 2, Fig. 7). The basic elements of mathematical modeling language syntax of "LINGO" are presented in Table 1.

Table 2
The basic syntax of "LINGO" mathematical modeling language

Mathematical nomenclature	LINGO syntax
Minimum	MIN =
$\sum Z_{jkt}$	@sum(ORDER (j,k,t))
$j=1..M$ for each customer (j) in the set of customers	@FOR(CUSTOMERS (j))
•	*
=	=
X ∈integer	@gin(X)
$X \in \{0,1\}$	@bin(X)
Load input parameters p from the file dane.ldt	p=@file(dane.ldt)

The model can be saved in a text file using any text editor and with a standard extension *. lng and *. ldt data file. The structure of the model is composed of sections. The main section is the MODEL section, which begins with the word MODEL: and ends with the word END. Other sections may be integrated in this section. The most important sections, highlighted by the relevant keywords are: section SETS (SET: ENDSETS) and DATE (DATE: enddate). In the SETS section one can define types of simple or complex objects, and their mutual relationships. In the implemented model, simple objects are exemplified by types such as products, factories, etc.; complex objects: production, distribution, etc. In this section, the parameters and variables of the model are assigned to particular types. DATA section allows initiating or assigning values to individual parameters of the model. There are two methods to do it in the "LINGO" package. Either place the numerical data directly in the section or make references to the place where those data files are included. This method of model construction ensures the separation of data from the relevant model, which is very important because the change in data values or even their size does not require any changes in the objective function or constraints. Only the model implemented in the implicit form has such a feature.

Computational examples

The cost optimization model (1)..(23) was implemented in the "LINGO" environment. Figure 7 shows the implicit model. Optimization was performed for three examples: P1, P2 and P3.

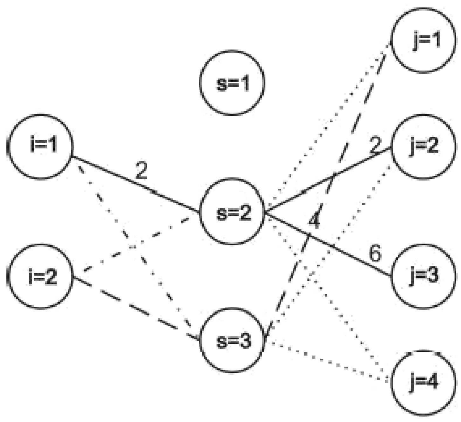

Fig. 4. Transport network of multi-modal optimal solution ($Fc^{opt} = 74810$) for P1. The number of hauls is 16.

All cases relate to the supply chain with two manufacturers ($i = 1..2$), three distributors ($s = 1.3$), four recipients ($j = 1..4$), four mode of transport ($d = 1..4$) and five types of products ($k = 1..5$). The examples differ in capacity available to the distributors (V_s) and number of transport units using mode of transport $d(Zt_d)$. The numeric data for all the model parameters from Table 1 are presented in Appendix A (Table 3)

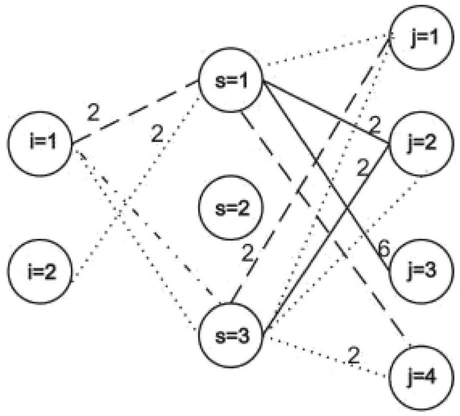

Fig. 5. Transport network of multi-modal optimal solution ($Fc^{opt} = 69000$) for P2. The number of hauls is 18.

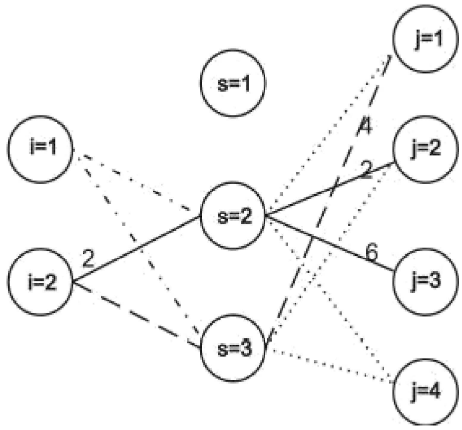

Fig. 6. Transport network of multi-modal optimal solution ($Fc^{opt} = 75420$) for P3. The number of hauls is 16.

Optimization started after the implementation of the model in the LINGO mathematical modeling language (Fig. 7). Optimization results are shown in Appendix B (Table 4) and Fig. 8 (only for P1) with the parameters of the process of searching for the optimal solution: the number of iterations, the optimization algorithm used (Branch-and-Bound) [20], the number of decision variables in the integer constraints, etc. The optimization process involves finding the global solution for the specific data Appendix A (Table 3), which in this case means the lowest cost of satisfying customer needs through the supply chain and amounts to $Fc^{opt} = 74810$ for P1, $Fc^{opt} = 69000$ for P2 and $Fc^{opt} = 75420$ for P3. Transportation networks diagrams showing the number of hauls (no number means one) corresponding to the optimal solutions for P1, P2, P3 are shown sequentially in Figs. 4–6.

```
Model:
 Sets:
products/1..@file(size.ldt)/:p;
factories/1..@file(size.ldt)/;
customers/1..@file(size.ldt)/;
distributors/1..@file(size.ldt)/:f,v,vx,T;
 mode /1..@file(size.ldt)/:pt,zt;
orders (customers,products):z,tc;
production (factories,products):c,w,wx;
locations (distributors,products):r,tp;
route_1(factories,distributors,mode):a,r1,
tf;
route_2(distributors,customers,mode):g,r2,tm
;
delivery_1(factories, distributors,
products,mode):X,Xb;
delivery_2(distributors,
customers,products,mode):Y,Yb;
delivery_3(factories, distributors,
products);
delivery_4(distributors,
customers,products);
 EndSets
Data:
 p  =@file(dane.ldt);
 f  =@file(dane.ldt);
 v  =@file(dane.ldt);
 pt =@file(dane.ldt);
 zt =@file(dane.ldt);
 z  =@file(dane.ldt);
 tc =@file(dane.ldt);
 c  =@file(dane.ldt);
 w  =@file(dane.ldt);
 r  =@file(dane.ldt);
 tp =@file(dane.ldt);
 a  =@file(dane.ldt);
 r1 =@file(dane.ldt);
 ...............
EndData

! Objective function;
Min= @sum(distributors(s):f(s)*T(s))+
@sum(delivery_1(i,s,k,d):a(i,s,d)*Xb(i,s,k,d
))+@sum(delivery_2(s,j,k,d):g(s,j,d)*Yb(s,j,
k,d))+@sum(production(i,k):c(i,k)*(@sum(dist
ributors(s):@sum(mode(d):X(i,s,k,d) ))));

! Constraint(1);

@for(production(i,k):@sum(distributors(s):
@sum(mode(d): X(i,s,k,d))) <=w(i,k) ;

! calculation of the auxiliary variable Wx;
 @sum(distributors (s):@sum(mode(d):
X(i,s,k,d))) =wx(i,k) ;
   );

! Constraint (2);
@for(orders(j,k):@sum(distributors
(s):@sum(mode(d): r(s,k)*Y(s,j,k,d)))
>=z(j,k));
................
! binary Ts;
 @for(distributors (s):bin(T(s)));
 End
```

Fig. 7. Part of the file scm.lng (the supply chain cost optimization model in LINGO).

At the same time, the specific values of decision variables that minimize the cost are determined (Table 4). These values represent, among other things, the volume of supplies from the manufacturer to the distributor of selected products using mode of transport ($X_{i,s,k,d}$) and the supply of products from specific distributors to selected customers/recipients ($Y_{s,j,k,d}$). Based on these variables, one can make a decision at the current operating level.

The values of decision variables $Y_{b,s,j,d}$, $X_{b,i,s,d}$ determine the number of courses using transport mode. Based on these variables one can make a decision from the tactical level, which includes the mode of transport and the need for different means of transport.

Another way to use the implemented model is to determine the effect of the change in the model parameters on the cost. One can analyze in detail the sensitivity of solutions depending on the parameters *Ko, A, G, C, T, V, Zt* etc. The article focused on the effect of parameter V and Zt.

Numerous analyses of that kind can be conducted. For further studies and especially long-term decision support, the optimization model was extended at the implementation stage. Auxiliary variables were introduced at implementation stage Vx_s (the value corresponds to the distributor's uptake capacity) and $Wx_{i,k}$ (production capacity utilization rates for manufacturer i of product k). The analysis of the decision variables values Vx_s and Wx_{ik} Appendix B (Table 5) has an impact on strategic decision making level of production capacity or dealer location and capacity.

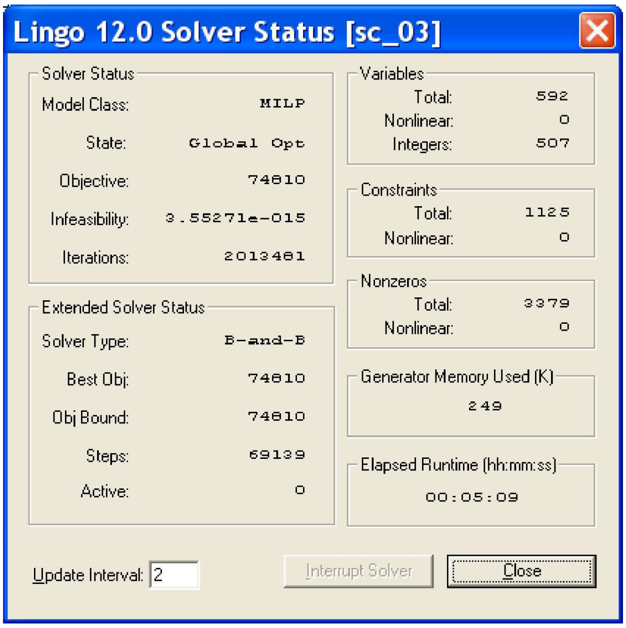

Fig. 8. LINGO results window, for P1.

Conclusions

The paper presents a model of optimizing supply chain costs. Creating the model in the form of a MILP problem undoubtedly facilitates its solution using mathematical programming tools available in "LINGO" package [21] or "CPLEX" [22] and others. Of course, the model should be implemented in one, selected environment package. Implementation of the model in the "LINGO" package and the computational experiments were presented. The approach from the perspective of an optimizing logistics provider that has access to all data and all participants in the downstream chain is very interesting.

After the implementation of the language from the mathematical modeling package "LINGO", a number of computational experiments were conducted. Three of them in the form of examples P1,P2 and P3 were described in the article. Based on the experimental results, analysis and previous experience, the authors can state that the proposed model and its implementation ensure a very large range of applications. First, they allow finding the distribution flows (decision variables) for the modeled supply chain, which minimize the global cost satisfying the needs of customers. Second, they offer numerous possibilities for decision support in supply chain management through the solutions sensitivity analysis, determination of the range and quality of the impact of various parameters on the cost and even on the structure of the supply chain. The analysis presented in the article, only in terms of the capacity available to distributors and the number of transport units fully confirms this statement.

Appendix A (data for computational examples P1, P2, P3)

Table 3
The set of parts of data tables for examples P1,P2 and P3.

s	F_s	V_s-*P1,P2*	V_s -*P3*
1	10 000	2 000	2 000
2	15 000	2 500	2 500
3	12 000	1 500	1 200

d	Pt_d	Zt_d -*P1,P3*	Zt_d -*P2*
1	50	10	10
2	200	4	8
3	800	2	1
4	60	5	5

j	k	Z_{jk}	Tc_{jk}	j	k	Z_{jk}	Tc_{jk}
1	1	20	10	2	1	10	10
1	2	10	10	2	2	0	10
1	3	15	10	2	3	10	10
1	4	20	10	2	4	10	10
1	5	15	20	2	5	15	20
3	1	10	10	4	1	20	10
3	2	20	10	4	2	0	10
3	3	0	10	4	3	10	10
3	4	20	10	4	4	0	10
3	5	0	20	4	5	20	20

i	k	C_{ik}	W_{ik}	i	k	C_{ik}	W_{ik}
1	1	100	100	2	1	150	100
1	2	200	100	2	2	210	100
1	3	200	100	2	3	150	100
1	4	300	100	2	4	250	100
1	5	300	100	2	5	350	100

s	k	R_{sk}	Tp_{sk}	s	k	R_{sk}	Tp_{sk}
1	1	1	2	2	1	1	1
1	2	1	2	2	2	1	1
1	3	1	2	2	3	0	1
1	4	1	2	2	4	1	1
1	5	0	2	2	5	1	1
3	1	1	3	$Tm_{sjd=}1,\ Tf_{isd}=1$			
3	2	1	3				
3	3	1	3				
3	4	0	3				
3	5	1	3				

i	s	d	A_{isd}	$R1_{isd}$
1	1	1	2	1
1	1	2	1	1
1	1	3	1	1
1	1	4	2	1
1	2	1	2	1
1	2	2	1	1
1	2	3	1	1
1	2	4	2	0
1	3	1	2	1
1	3	2	1	1
1	3	3	1	1
1	3	4	2	0
2	1	1	2	1
2	1	2	1	1
2	1	3	1	1
2	1	4	2	0
2	2	1	2	1
2	2	2	1	1
2	2	3	1	1
2	2	4	2	0
2	1	1	2	1
2	1	2	1	0
2	1	3	1	0
2	1	4	2	1

k	P_k
1	10
2	15
3	15
4	10
5	20

s	j	d	G_{isd}	$R2_{isd}$
1	1	1	1	1
1	1	2	1	1
1	1	3	1	0
1	1	4	1	1
1	2	1	1	1
1	2	2	1	1
1	2	3	1	1
1	2	4	1	0
1	3	1	1	1
1	3	2	1	1
1	3	3	1	1
1	3	4	1	0
1	4	1	1	1
1	4	2	1	1
1	4	3	1	1
1	4	4	1	1
2	1	1	1	1
2	1	2	1	1
2	1	3	1	0
2	1	4	1	1
2	2	1	1	1
2	2	2	1	1
2	2	3	1	1
2	2	4	1	0
2	3	1	1	1
2	3	2	1	1
2	3	3	1	1
2	3	4	1	0
2	4	1	1	1
2	4	2	1	1
2	4	3	1	1
2	4	4	1	1
3	1	1	1	1
3	1	2	1	1
3	1	3	1	0
3	1	4	1	1
3	2	1	1	1
3	2	2	1	1
3	2	3	1	1
3	2	4	1	0
3	3	1	1	1
3	3	2	1	1
3	3	3	1	1
3	3	4	1	0
3	4	1	1	1
3	4	2	1	1
3	4	3	1	1
3	4	4	1	1

Appendix B
(results of optimization for computational examples P1, P2, P3)

Table 4
The set of parts of tables with results for examples P1 , P2, P3

Example P1 $Fc^{opt} = 74810$

i	s	k	d	X_{iskd}
1	2	1	1	10.00
1	2	2	1	6.00
1	2	5	1	5.00
1	3	1	3	47.00
1	3	3	3	31.00
1	3	5	3	25.00
2	2	1	3	3.00
2	2	2	3	24.00
2	2	4	3	50.00
2	2	5	3	20.00
2	3	3	4	4.00

i	s	d	X_{bisd}
1	2	1	2
1	3	3	1
2	2	3	1
2	3	4	1

s	j	k	d	Y_{iskd}
2	1	2	2	10.00
2	1	4	2	20.00
2	1	5	2	10.00
2	2	4	1	10.00
2	2	5	1	5.00
2	3	1	1	10.00
2	3	2	1	20.00
2	3	4	1	20.00
2	4	1	2	3.00
2	4	5	2	10.00
3	1	1	4	20.00
3	1	3	4	15.00
3	1	5	4	5.00
3	2	1	2	10.00
3	2	3	2	10.00
3	2	5	2	10.00
3	4	1	2	17.00
3	4	3	2	10.00
3	4	5	2	10.00

s	j	d	Yb_{isd}
2	1	2	1.00
2	2	1	2.00
2	3	1	6.00
2	4	2	1.00
3	1	4	4.00
3	2	2	1.00
3	4	2	1.00

Example P2 $Fc^{opt} = 69000$

i	s	k	d	X_{iskd}
1	1	1	4	12.00
1	1	2	4	8.00
1	1	4	4	10.00
1	3	1	2	18.00
1	3	1	3	14.00
1	3	3	2	12.00
1	3	5	2	10.00
1	3	5	3	40.00
2	1	1	2	16.00
2	1	2	2	22.00
2	1	3	2	23.00
2	1	4	2	40.00

i	s	d	Xb_{isd}
1	1	4	2
1	3	2	1
1	3	3	1
2	1	2	2

s	j	k	d	Y_{iskd}
1	1	1	2	12.00
1	1	2	2	10.00
1	1	3	2	13.00
1	1	4	2	20.00
1	2	3	1	6.00
1	2	4	1	10.00
1	3	1	1	10.00
1	3	2	1	20.00
1	3	4	1	20.00
1	4	1	4	6.00
1	4	3	4	4.00
3	1	1	2	8.00
3	1	3	2	2.00
3	1	5	2	10.00
3	1	5	4	5.00
3	2	1	2	10.00
3	2	3	1	3.00
3	2	3	2	1.00
3	2	5	1	5.00
3	2	5	2	10.00
3	4	1	2	14.00
3	4	3	2	6.00
3	4	5	2	20.00

s	j	d	Yb_{isd}
1	1	2	1.00
1	2	1	2.00
1	3	1	6.00
1	4	4	1.00
3	1	2	1.00
3	1	4	2.00
3	2	1	2.00
3	2	2	1.00
3	4	2	2.00

Example P3 $Fc^{opt} = 75420$

i	s	k	d	X_{iskd}
1	2	1	3	60.00
1	2	2	3	30.00
1	2	4	3	40.00
1	2	5	3	18.00
1	3	3	3	31.00
1	3	5	3	32.00
2	2	4	1	10.00
2	3	3	4	4.00

i	s	d	Xb_{isd}
1	2	3	1
1	3	3	1
2	2	1	2
2	3	4	1

s	j	k	d	Y_{iskd}
2	1	1	2	20.00
2	1	1	2	20.00
2	1	2	2	10.00
2	1	4	2	20.00
2	1	5	2	3.000
2	2	1	1	10.00
2	2	4	1	10.00
2	2	5	1	5.000
2	3	1	1	10.00
2	3	2	1	20.00
2	3	4	1	20.00
2	4	1	2	20.00
2	4	5	2	10.00
3	1	3	4	15.00
3	1	5	4	12.00
3	2	3	2	10.00
3	2	5	2	10.00
3	4	3	2	10.00
3	4	5	2	10.00

s	j	d	Yb_{isd}
2	1	2	1.00
2	2	1	2.00
2	3	1	6.00
2	4	2	1.00
3	1	4	4.00
3	2	2	1.00
3	4	2	1.00

Table 5
The set of parts of tables with results for examples P1, P2, P3 – decision variables $Wx_{s,k}$, Vx_s.

P1			P2			P3		
s	k	Wx_{sk}	s	k	Wx_{sk}	s	k	Wx_{sk}
1	1	57	1	1	44	1	1	69
1	2	6	1	2	8	1	2	30
1	3	31	1	3	12	1	3	31
1	5	30	1	4	10	1	4	40
2	1	3	1	5	50	1	5	50
2	2	24	2	1	16	2	3	4
2	3	4	2	2	22	2	4	10
2	4	50	2	3	23			
2	5	20	2	4	40			

P1		P2		P3	
s	Vx_s	s	Vx_s	s	Vx_s
2	1580	1	1575	2	1910
3	1495	3	1500	3	1165

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