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MODEL FOR SELECTION OF INVOLUTE GEAR PARAMETERS USING THE AREA OF POSSIBLE SOLUTIONS METHOD

Sławomir Herma

University of Bielsko-Biała, Department of Production Engineering, Poland

Corresponding author:
Sławomir Herma
University of Bielsko-Biała
Department of Production Engineering
Willowa 2, 43-300 Bielsko-Biała, Poland
phone: +48 33 827-93-26
e-mail: sherma@ath.bielsko.pl

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Abstract
The paper presents a method of selecting the parameters of external involute spur gearing components with straight and helical teeth for general - purpose gears, using the method of searching an area of possible solutions, understood as a space for values that meet several geometric, strength and operating criteria. Possible effects of this method in the context of gear production process organization and optimization, were also described.

Keywords
mechanical engineering, gears, parameter optimization, mathematical models, production engineering.

Introduction

The gears structures and all kinds of power transmission systems are still the object of intense research, based primarily on the development of the existing design and analysis methods as well as creation of new methodologies, based on the latest developments in applied science. Most research in this area is very detailed and related to the geometry and strength of meshing, trying to create new, more precise mathematical models, and to ensure higher quality and reliability of gear boxes.

The contemporary literature is rich with studies on design methods for various types of propulsion systems, nevertheless, especially in terms of gearing, proposes the solutions based on static and sequential algorithms. Although, currently available algorithms allow the implementation of information technology, but they still have a discrete character, giving a limited analysis of meshing parameters. Descriptions of some graphical methods for visualizing the results of calculations can be found in a few publications [1–3], but an attempt to apply the no comprehensive approach to construction of toothed elements, later contributes to their fragmentary analysis.

The problem of gear design does not only boil down to the calculations of geometric parameters and meeting the related criteria. Existing discreet methods are insufficient when it is necessary to simultaneously take into account the many diverse groups of criteria (geometry, strength, exploitation, economy, organization of production process etc.) that are often in mutual opposition. Available scientific studies treat this sphere in a very selective way, since many items apply only to geometric problems [4–7], while a separate group of studies, related to the strength calculations [5, 8] and exploitation-oriented issues, are described only in a very narrow range of publications.

Trends in the ever wider use of computer techniques on design and analysis of the spur and helical gears, can be found in several publications of Litvin [9–11]. Also, the computational models of toothing strength analysis, especially in the context of the use of finite element method (FEM), seems to be interesting in the publications of Kramberger [12] and

Brauer [13]. The recent work, with emphasis on the creation of integrated models, such as Chong [14], Aberšek [15] and using genetic algorithms – Gogolu [16], confirm the thesis of the continuous research for uniform methods, combining many different aspects of the gear drives design.

There are, thus, no studies, characterized by a holistic approach to the subject, and indicating methods for rapid and effective gear design, taking into account all the aspects discussed, especially when it comes to connecting the design of gears and the organization of production process.

Therefore, the primary objective of this paper is an attempt to develop the basic assumptions for the generalized gear design model, allowing the development of complex software, intended to conduct a series of studies on dynamic optimization of gears parameters, including their strength and also the organization of the production process. Moreover, the presented model is intended to allow a combination of analytical algorithms with graphical methods for visualizing the effects of calculations, to extend the capabilities of the interpretation of final results, and thus facilitate the process of production preparation.

The basic assumptions of the model

Specifying the construction of general-purpose gear drives, the following assumptions should be taken into account [17–21]:

- each gear drive can be constructed as one or multi-stage system,
- each gear stage is made up of two toothed elements, forming a cooperating pair,
- each toothed element should have such parameters, that it would be possible to create multiple peer associations while maximizing the diversity of ratios obtained,
- any combination of pairs of toothed elements should comply with all restrictions imposed in terms of geometric, strength and performance accuracy, and as far as possible meet the accepted criteria of optimization,
- the degree of unification and standardization (the technological similarity of toothed elements) should provide on the one hand, low production costs and the other – the opportunity to obtain a wide range of products.

The model of parameters selection for toothed elements

The model concept for general-purpose gear drive model is shown in Fig. 1.

Let a set of gearings be given:

$$R = \{r_r\}_{r=1,\dots,R} \tag{1}$$

and – a set of toothed elements (Fig. 1., item 1):

$$U = \{u_p\}_{p=1\dots,P}\,. \tag{2}$$

Let a binary matrix, determining the assignment of individual elements to the gearings be given:

$$Y = [y_{p,r}]\ \begin{matrix} p=1,\dots,P \\ r=1,\dots,R \end{matrix} \tag{3}$$

where

$$y_{p,r} = \begin{cases} 1, & \text{if } u_p \to r_r, \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

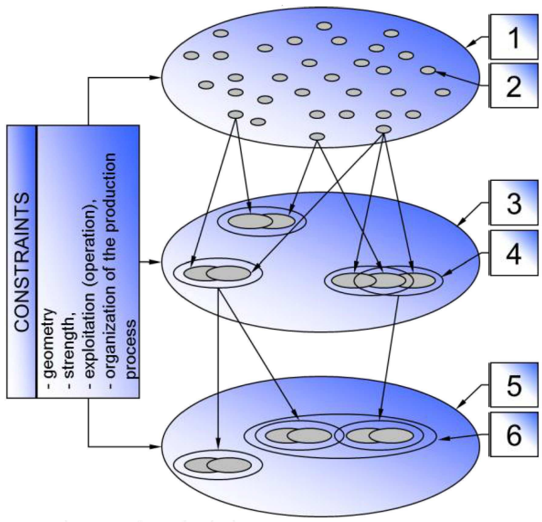
Moreover, let a matrix of gear mesh be given:

$$Z = [z_{i,j}]_{i,j=1,\dots,P} \tag{5}$$

corresponding to the set of meshing gears (Fig. 1., item 3) where:

$$z_{i,j} = \begin{cases} \Pi^p, \Pi^q, \Pi^{p,q}, & \text{if exists a gears mesh } u_p/u_q \\ \varnothing, & \text{otherwise} \end{cases} \tag{6}$$

and Π^p, Π^q represent a separate sets of attributes for u_p and u_q respectively, $\Pi^{p,q}$ represent a set of attributes for u_p and u_q arising out their meshing (Fig. 1, item 2,4).



1. The set of toothed elements;
2. The domain of existence for a single toothed element;
3. The set of meshing gears;
4. The domain of existence for each cooperating pair of gears;
5. The set of 1, 2, 3,..., n – stage gear drives;
6. The domain of existence for single gear drive;

Fig. 1. The model concept for general-purpose gear drive design.

Sets of attributes, from the standpoint of practical implementation of the model, can be regarded as the sum of subsets:

$$\Pi^p = \Pi^p_M \cup \Pi^p_K \cup \Pi^p_U$$
$$\Pi^q = \Pi^q_M \cup \Pi^q_K \cup \Pi^q_U \tag{7}$$

and

$$\Pi^{p,q} = \Pi^{p,q}_Z \cup \Pi^{p,q}_W \cup \Pi^{p,q}_E, \tag{8}$$

where

$$\Pi^p_M = \left\{\pi^p_{M_1}, ..., \pi^p_{M_m}, ..., \pi^p_{M_M}\right\}_{m\in\mathbb{N}}$$
$$\Pi^q_M = \left\{\pi^q_{M_1}, ..., \pi^q_{M_m}, ..., \pi^q_{M_M}\right\}_{m\in\mathbb{N}} \tag{9}$$

- are sets of material properties of elements u_p and u_q. These sets contain the basic parameters determining the surface layer and mechanical properties, also the chemical composition of material used to manufacture toothed elements u_p and u_q,

$$\Pi^p_K = \left\{\pi^p_{K_1}, ..., \pi^p_{K_k}, ..., \pi^p_{K_K}\right\}_{k\in\mathbb{N}}$$
$$\Pi^q_K = \left\{\pi^q_{K_1}, ..., \pi^q_{K_k}, ..., \pi^q_{K_K}\right\}_{k\in\mathbb{N}} \tag{10}$$

- are sets of design data of elements u_p and u_q. They define the basic overall parameters of gears. In order to standardize the data structure, an appropriate system of dimension classification may be adopted,

$$\Pi^p_U = \left\{\pi^p_{U_1}, ..., \pi^p_{U_u}, ..., \pi^p_{U_U}\right\}_{u\in\mathbb{N}}$$
$$\Pi^q_U = \left\{\pi^q_{U_1}, ..., \pi^q_{U_u}, ..., \pi^q_{U_U}\right\}_{u\in\mathbb{N}} \tag{11}$$

- are sets of teeth geometric parameters for elements u_p and u_q respectively,

$$\Pi^{p,q}_Z = \left\{\pi^{p,q}_{Z_1}, ..., \pi^{p,q}_{Z_z}, ..., \pi^{p,q}_{Z_Z}\right\}_{z\in\mathbb{N}} \tag{12}$$

- is the set of geometric parameters that are the result of meshing between elements u_p and u_q,

$$\Pi^{p,q}_W = \left\{\pi^{p,q}_{W_1}, ..., \pi^{p,q}_{W_w}, ..., \pi^{p,q}_{W_W}\right\}_{w\in\mathbb{N}} \tag{13}$$

- is the set of strength characteristics of meshing u_p/u_q. It includes strength parameters such as various coefficients, design loads, breaking stress, nominal and working stress etc.

$$\Pi^{p,q}_E = \left\{\pi^{p,q}_{E_1}, ..., \pi^{p,q}_{E_e}, ..., \pi^{p,q}_{E_e}\right\}_{e\in\mathbb{N}} \tag{14}$$

- is a set of operating characteristics of the gear pair u_p/u_q. This set contains the parameters such as rotational and angular speed, the nature and characteristics of lubricating an cooling media, vibroacoustic and thermal parameters etc.

For each element of the setΠ^p, Π^q and $\Pi^{p,q}$ let it be known the boundary conditions on the admissibility of solutions as:

$$C^p = C^p_M \cup C^p_K \cup C^p_U,$$
$$C^q = C^q_M \cup C^q_K \cup C^q_U, \tag{15}$$
$$C^{p,q} = C^{p,q}_Z \cup C^{p,q}_W \cup C^{p,q}_E,$$

where each element c^p_i, c^q_i and $c^{p,q}_i$ is a logical expression, which may take the True (1) or False (0) depending on the argument π^p_i, π^q_i or $\pi^{p,q}_i$.

Let

$$D^p = \{d^p_1, ..., d^p_i, ..., d^p_I\}_{i\in\mathbb{N}}\,, \quad D^p \subset \Pi^p,$$
$$D^q = \{d^q_1, ..., d^q_i, ..., d^q_I\}_{i\in\mathbb{N}}\,, \quad D^q \subset \Pi^q, \tag{16}$$
$$D^{p,q} = \{d^{p,q}_1, ..., d^{p,q}_i, ..., d^{p,q}_I\}_{i\in\mathbb{N}}\,, \quad D^{p,q} \subset \Pi^{p,q},$$

are the sets containing the decision variables such that:

$$\underset{\pi^p_i\in\Pi^p\setminus D^p}{\forall} \quad \underset{F^p_i:\mathbb{R}^n\to\mathbb{R}^n}{\exists} F^p_i(D^p) = \pi^p_i, \;\; i\in\mathbb{N},$$
$$\underset{\pi^q_i\in\Pi^q\setminus D^q}{\forall} \quad \underset{F^q_i:\mathbb{R}^n\to\mathbb{R}^n}{\exists} F^q_i(D^q) = \pi^q_i, \;\; i\in\mathbb{N},$$
$$\underset{\pi^{p,q}_i\in\Pi^{p,q}\setminus D^{p,q}}{\forall} \quad \underset{F^{p,q}_i:\mathbb{R}^n\to\mathbb{R}^n}{\exists} F^{p,q}_i(D^{p,q}) = \pi^{p,q}_i, \;\; i\in\mathbb{N}, \tag{17}$$

where the functions:

$$F^p_i\left(D^p\right), \qquad F^q_i\left(D^q\right), \qquad F^{p,q}_i\left(D^{p,q}\right), \tag{18}$$

are known and result directly from the geometry of the involute meshing.

Moreover, let

$$Q^p = \{q^p_i(\pi^p_i)\}_{i\in\mathbb{N}}\,, \qquad Q^q = \{q^q_i(\pi^q_i)\}_{i\in\mathbb{N}}\,,$$
$$Q^{p,q} = \{q^{p,q}_i(\pi^{p,q}_i)\}_{i\in\mathbb{N}}\,, \tag{19}$$

the sets of functions which are the criteria of optimization be given.

Consequently, the possible solutions sets for gear parameters are defined as:

$$S^p = \{s^p_n\}_{n\in\mathbb{N}}\,, \qquad S^q = \{s^q_n\}_{n\in\mathbb{N}}\,,$$
$$S^{p,q} = \{s^{p,q}_n\}_{n\in\mathbb{N}}\,, \tag{20}$$

where

$$s^p_n = \{\pi^p_i \in \Pi^p : c^p_i = 1\}_{i,n\in\mathbb{N}}\,,$$
$$s^q_n = \{\pi^q_i \in \Pi^q : c^q_i = 1\}_{i,n\in\mathbb{N}}\,, \tag{21}$$
$$s^{p,q}_n = \{\pi^{p,q}_i \in \Pi^{p,q} : c^{p,q}_i = 1\}_{i,n\in\mathbb{N}}$$

and $n \in \mathbb{N}$ is the number of iteration in the algorithm.

For all possible solutions $(S^p \cup S^q \cup S^{p,q})$ of meshing u_p and u_q (Fig. 1., item 5), the optimization procedure can be performed to obtain the optimum, according to the adopted criteria:

$$s^p_{n_{opt}} = \left\{ \pi^p_i \in \Pi^p : \underset{D^p \subset \Pi^p}{\exists} \underset{i,j \in \mathbb{N}}{\forall} q^p_j\left(\pi^p_i\right) = \pi^p_{i_{opt}} \right\}_{n \in \mathbb{N}},$$
$$s^q_{n_{opt}} = \left\{ \pi^q_i \in \Pi^q : \underset{D^q \subset \Pi^q}{\exists} \underset{i,j \in \mathbb{N}}{\forall} q^q_j\left(\pi^q_i\right) = \pi^q_{i_{opt}} \right\}_{n \in \mathbb{N}},$$
$$s^{p,q}_{n_{opt}} = \left\{ \pi^{p,q}_i \in \Pi^{p,q} : \underset{D^{p,q} \subset \Pi^{p,q}}{\exists} \underset{i,j \in \mathbb{N}}{\forall} q^{p,q}_j\left(\pi^{p,q}_i\right) = \pi^{p,q}_{i_{opt}} \right\}_{n \in \mathbb{N}}. \tag{22}$$

The area of possible solutions

Based on the presented mathematical model, computational algorithms have been defined and adopted the following parameters.

According to the formulas (6)–(8) and on the basis of the definition (9) the following parameters have been assigned to the sets of material properties:

$$\Pi^p_M = \{mat_1, E_1, \nu_1\}, \quad \Pi^q_M = \{mat_2, E_2, \nu_2\}, \tag{23}$$

where $E_{1(2)}$ is the Young's modulus, $\nu_{1(2)}$ is the Poisson's Ratio, and $mat_{1(2)}$ indicates a type of material used for the construction of toothed element. All the following symbols are used in accordance with the standards [22] and [23].

According to the definition (10), the following parameters have been assigned to the sets of overall dimensions:

$$\Pi^p_K = \{b_1\}, \qquad \Pi^q_K = \{b_2\}. \tag{24}$$

According to the definition (11) the following parameters have been assigned to the sets of teeth geometric values:

$$\Pi^p_U = \left\{ \begin{array}{l} d_1, d_{a1}, d_{b1}, d_{f1}, d_{h1}, d_{l1}, d_{p1}, d_{w1}, \\ h_{fP1}, j_{n1}, k_1, m_{n1}, m_{t1}, p_{t1}, p_{pet1}, \\ s_{n1}, s_{na1}, s_{t1}, s_{tw1}, W_{n1}, x_1, x_{gr1}, y_{n1}, \\ z_1, z_{gr1}, z_{z1}, \alpha_{n1}, \alpha_{t1}, \alpha_{nw1}, \alpha_{ta1}, \alpha_{tw1}, \\ \beta_1, \beta_{a1}, \beta_{b1}, \varepsilon_{\beta 1}, \rho_{fP1}, \rho_{ta1}, \rho_{tf1}, \rho_{tl1}, \rho_{tp1} \end{array} \right\}$$

Π^q_U– as before. (25)

According to the definition (12) the set of geometric parameters that are the result of meshing between elements u_p andu_q, has been determined as follows:

$$\Pi^{p,q}_Z = \left\{ \begin{array}{l} a, c, ZP, u, g_{\alpha t1,2}, \varepsilon_\alpha, c_{f2a1}, c_{f1a2}, \\ \rho_{ta1(d_{p2})}, \rho_{ta2(d_{p1})}, d_{a1(d_{p2})}, d_{a2(d_{p1})}, \\ \rho_{ta1(d_{f2})}, \rho_{ta2(d_{f1})}, d_{a1(d_{f2})}, d_{a2(d_{f1})}, \\ \rho_{ta1(d_{l2})}, \rho_{ta2(d_{l1})}, d_{a1(d_{l2})}, d_{a2(d_{l1})} \end{array} \right\}. \tag{26}$$

According to the definition (13) the following parameters have been assigned to the set of strength characteristics of meshing u_p/u_q:

$$\Pi^{p,q}_W = \left\{ \begin{array}{l} K_v, K_{v\max}, K_{vF}, K_{vH}, K_A, K_{AF}, \\ K_{AH} K_{F\alpha}, K_{F\alpha\max}, K_{F\beta}, K_{F\beta\max}, \\ K_{H\alpha}, K_{H\alpha\max}, K_{H\beta}, K_{H\beta\max}, S_F, \\ S_{F\min}, S_H, S_{H\min}, Y_F, Y_N, Y_{RrelT}, \\ Y_S, Y_\beta, Y_{\delta relT}, Y_\varepsilon, Z_v, Z_{B,D}, Z_E, \\ Z_H, Z_L, Z_N, Z_R, Z_\beta, Z_\varepsilon, \sigma_F, \\ \sigma_{FE}, \sigma_{FES}, \sigma_{FG}\sigma_{FGN}, \sigma_{FGS}, \sigma_{F\lim}, \\ \sigma_{FP}, \sigma_{FPN}, \sigma_{FPS}, \sigma_{FS}, \sigma_{FO}, \\ \sigma_H, \sigma_{HG}, \sigma_{HGN}, \sigma_{HGS}, \sigma_{H\lim}, \\ \sigma_{H\lim S}, \sigma_{HP}, \sigma_{HPN}, \sigma_H, \sigma_{HS}, \sigma_{HO} \end{array} \right\}. \tag{27}$$

According to the definition (14) the following parameters have been assigned to the set of operating characteristics of the gear pair u_p/u_q:

$$\Pi^{p,q}_E = \{F_t, F_{t\max}, P, v_1, v_2\} \tag{28}$$

Sets of boundary conditions for determining the admissibility of solutions have been determined according to the definition (15) – as follows:

$$C^p_U = \left\{ \begin{array}{l} s_{na1} \geq 0.25 m_{n1}, \\ x_1 \geq x_{gr1}, \\ z_1 \geq z_{gr1}, \\ p_{nb1} = p_{nb2}, \\ r_{h1} \leq r_{f1} \leq r_{l1} \leq r_{p1} \leq r_{q1} \leq \\ \quad \leq r_1 \leq r_{w1} \leq r_{g1} \leq r_{k1} \leq r_{a1}, \end{array} \right. \tag{29}$$

C^q_U – as before,

$$C^{p,q}_Z = \left\{ \begin{array}{l} \tan\alpha_{A1} \geq \tan\alpha_{P1}, \\ \tan\alpha_{A2} \geq \tan\alpha_{P2}, \\ \varepsilon_\alpha > 1, \\ \varepsilon_\gamma = \varepsilon_\alpha + \varepsilon_\beta > 1, \\ a = \dfrac{d_{w1} + d_{w2}}{2} = \text{const}, \\ d_{p1} \triangleq d_{h2}, \\ d_{h1} \triangleq d_{p2}, \\ \pm\beta_1 \pm \beta_2 = 0, \\ i = \text{const}, \\ x_1 + x_2 = \text{const}, \end{array} \right. \tag{30}$$

$$C^{p,q}_W = \left\{ \begin{array}{l} (\sigma_H \leq \sigma_{HP}) \wedge (S_H \geq S_{H\min}), \\ (\sigma_{HS} \leq \sigma_{HPS}) \wedge (S_{HS} \geq S_{H\min}), \\ (\sigma_H \leq \sigma_{HPN}) \wedge (S_{HN} \geq S_{H\min}), \\ (\sigma_F \leq \sigma_{FP}) \wedge (S_F \geq S_{F\min}), \\ (\sigma_{FS} \leq \sigma_{FPS}) \wedge (S_{FS} \geq S_{F\min}), \\ (\sigma_F \leq \sigma_{FPN}) \wedge (S_{FN} \geq S_{F\min}), \end{array} \right. \tag{31}$$

$$C^{p,q}_E = \left\{ \begin{array}{l} (\varepsilon_\gamma = \varepsilon_\alpha + \varepsilon_\beta > 2.5) \vee (\varepsilon_\alpha > 2), \\ c_1 \geq 0.2 m_{n1}, \\ c_2 \geq 0.2 m_{n2}, \\ \gamma' \cong \gamma''. \end{array} \right. \tag{32}$$

The sets containing the decision variables have been determined according to the definitions (16), (17) and (18), such as:

$$D^p = \{\alpha_{n1}, \beta_1, z_1, m_{n1}, x_1, y_1, k^*_{a1}, b_1\},$$
$$D^q = \{\alpha_{n2}, \beta_2, z_2, m_{n2}, x_2, y_2, k^*_{a2}, b_2\}. \quad (33)$$

According to the proposed model, the selection of gear toothing parameters is to identify the sets of possible solutions (20), (21) and find the optimum (22), based on sets of criteria (34):

$$Q^{p,q} = \begin{cases} \sigma_H \to \min, \\ \sigma_{HS} \to \min, \\ \sigma_F \to \min, \\ \sigma_{FS} \to \min, \\ \varepsilon_\alpha \to \max, \\ \varepsilon_\beta \to \max, \\ v_{sa} \to \min. \end{cases} \quad (34)$$

The basic idea in the model is that the designer and the technologist had the opportunity to determine values of parameters characterizing the gear drive, to be at the same time met all the defined constraints and to the extent possible, met all the criteria of optimization. Moreover, it is desirable that in every moment of the gear drive process design it is possible to assess the fulfillment of constraints and criteria in the context of changing input parameters.

It is achievable trough the suitable construction of the possible solutions areas. To achieve this, it is necessary to identify such a decision variable, among the element of the sets D^p and D^q, which as far as possible influences on the behavior of other parameters, by which constraints and optimization criteria are formulated. For the general-purpose gear drives, it is most convenient to use the values of correction (shift profile) coefficients and analyze all the other parameters as a function of x_1 and x_2 (see Fig. 2, Fig. 3).

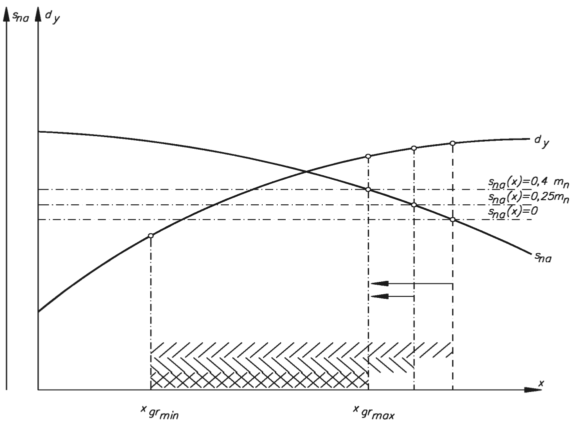


Fig. 2. The area of possible solutions for a single mesh, according to the following constraints: $x \geq x_{gr}$, $s_{na} \geq 0$, or $s_{na} \geq 0.25m_n$ or $s_{na} \geq 0.4m_n$.

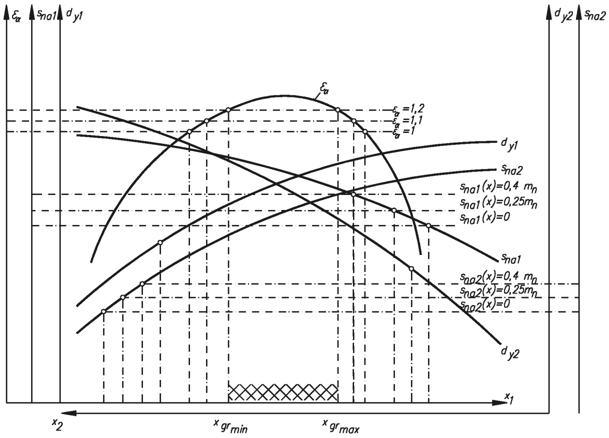


Fig. 3. The area of possible solutions for a single mesh, according to the following constraints: $x_1 \geq x_{gr1}$, $x_2 \geq x_{gr2}$, $x_1 + x_2 = 0$, $s_{na1} \geq 0$, $s_{na2} \geq 0$, $\varepsilon_{\alpha 12} \geq 1$, d_{y1}, $d_{y2} \in R^+$.

Selecting the profile shift coefficient as the main determinant of the possible solutions area size is also based on practical reasons on the effects on:
1. meshing correctness in the sense of geometry,
2. strength (eg. bending stress, surface durability),
3. operation (exploitation).

Due to the fact, that a gearing analysis is performed in the context of the profile shift coefficient, it is necessary to take into consideration both the P-0 correction (which implies a symmetrical distribution of profile shift, ie. $x_1 + x_2 = 0$) and P (where $x_1 + x_2 = \text{const}$).

The orientation of the x-axis of both coordinate systems, in case of P-0 correction, is shown in Fig.4.

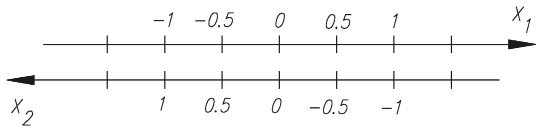


Fig. 4. Axis orientation in the analysis of meshing with correction P-0.

In the case of P – correction (where $x_{n1} + x_{n2} = \text{const}$), the x-axes are shifted relative to each other by the value corresponding to a fixed sum $x_{n1} + x_{n2}$ (see Fig. 5).

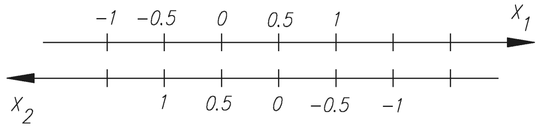


Fig. 5. Axis orientation in the analysis of meshing with correction P.

With a simple transformation, the analyzed mesh can be represented in a form which permits establishing an area of possible solutions in terms of active contact range (ie. between the diameters $d_{p1(2)}$ and $d_{h1(2)}$), as shown in Fig. 6.

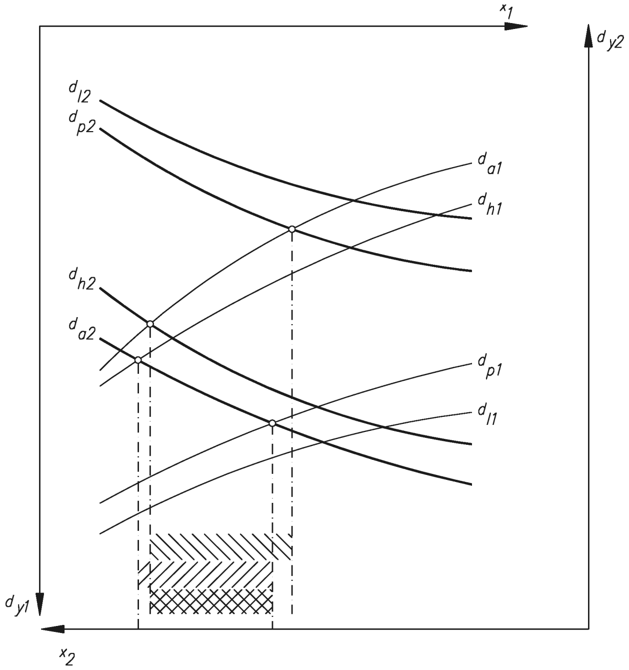


Fig. 6. The area of possible solutions concerning the active control range correctness.

Given the fact, that most geometric characteristics of the mesh are presented as a functions of x_{n1} and x_{n2}, it is possible to identify and analyze – in a common coordinate system – more than one mesh at the same time, with all the possible constraints arising from the conditions of geometric accuracy.

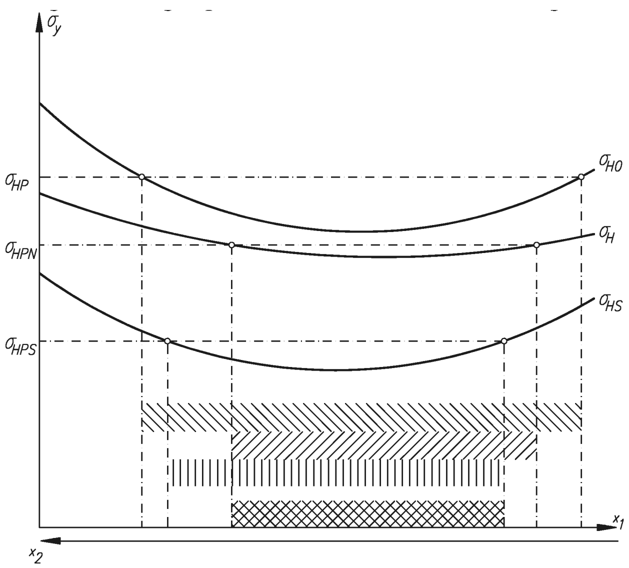


Fig. 7. An example of an area of possible solutions for the gear strength parameters.

There is also the possibility of finding such values of arguments, both graphical and analytical, for which the optimization criteria (as defined in sets Q^p and $Q^{p,q}$) take the optimum (eg. the face contact ratio $\varepsilon_{\alpha 12}$).

An example of an area of possible solutions for the gear strength parameters is shown in the Fig. 7.

In the chart above many other characteristics concerning strength parameters can be also embed (bending stress and surface durability).

Software implementation

Based on the presented method, a computer aided gear design system for general-purpose gear drives, has been developed.

The basic functions of this system relate primarily to the visualization of geometric and strength gear parameters and identification of possible solutions areas, taking into account all restrictions (constraints) on the meshing correctness.

The system was built with several related modules. Figure 8 shows an example of geometric parameters for wheel and pinion. Basic input data for the calculation are presented in Table 1.

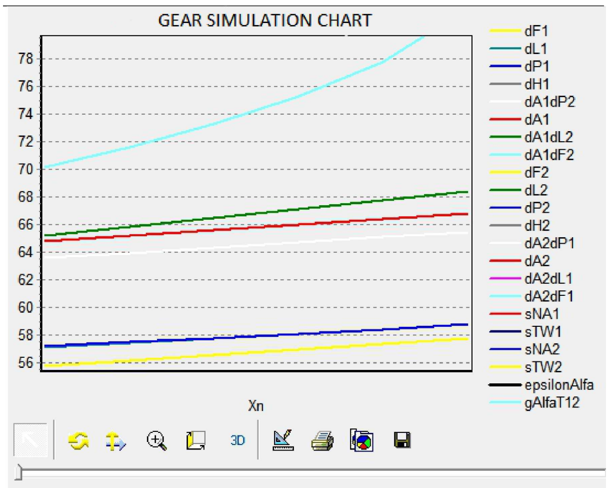


Fig. 8. Application user interface.

Table 1
Exemplary input data for calculations.

Parameter	Symbol	Pinion	Wheel
Number of teeth	$z_{1,2}$	29	69
Normal module	$m_{n1,2}$	2	
Normal pressure angle	$\alpha_{n1,2}$	20°	
Helix angle	β_2	11.478°	
Direction	–	left-hand	right-hand
Center distance	a	100.0	
Clearance	c	0.25	

The main emphasis in the design phase of the application was placed on the possibility to dynamically change the values of selected decision variables (eg. k_a^*, α_n, β) and study the impact of these changes on the possible solution areas, as well as the behavior of all remaining parameters presented in the simulation chart. The use of the object-oriented programming (not only in the user interface, but also in the same calculation algorithm) resulted in the possibility of real-time visualization of any change in the possible solution area.

Conclusions

The possibility of visualizing the characteristics of selected gear parameters and consideration of all known constraints (geometry, strength, exploitation) allows for a gradual narrowing the common area of possible solutions and consequently obtaining the interval of the arguments (x_1 and x_2), for which a solution of meshing really exists.

The last stage of the design process is to identify such values of decision variables, for which the adopted criteria of optimization will be met most satisfactorily.

The combination of graphical and analytical methods for determining areas of possible solutions, and practical application of the methodology described, also provides the following benefits:

- the ability to design custom gears, with not standardized parameters ($\alpha_{n1,2}$, $\beta_{1,2}$, $m_{n1,2}$, etc.),
- the ability to efficiently and quickly recover gear elements, that do not have the original technical documentation,
- the ability to create series of gear drives,
- the possibility of an effective classification of gear components, taking into account the technological similarity,
- the ability to use any criteria for creating groups of technological similarity,
- the possibility to improve the manufacturing process organization by simplifying the technological routes for similar products,
- the ability to differentiate the gear elements in terms of the type of materials (testing the suitability of material alternatives while maintaining the same geometrical parameters),
- the possibility to simplify and accelerate the production process through the unification of product range.
- the gear design process automation with the use of computer algorithms,
- reduce the time and costs of technical production preparation stage.

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