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THE BASIC EQUATIONS OF FLUID MECHANICS IN FORM CHARACTERISTIC OF THE FINITE VOLUME METHOD

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A b s t r a c t

The article presents the derivation of the basic balance equations in form characteristic of the Finite Volume Method. In the following sections the origin of the balance equation of momentum, mass and energy is presented. Next, the conservation equations in vector form are described. The Finite Volume Method is used in Computational Fluid Mechanics for numerical modeling of fluid flows in wide range. The motivation for writing the article was the difficulty in finding short and concise articles on this subject in the literature. The current article is a first part of a cycle dedicated to the mathematical basis of the Finite Volume Method.

PODSTAWOWE RÓWNANIA MECHANIKI PŁYNÓW W FORMIE CHARAKTERYSTYCZNEJ DLA METODY OBJĘTOŚCI SKOŃCZONYCH

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S ł o w a k l u c z o w e: CFD, metoda objętości skończonych, równania bilansowe.

A b s t r a k t

W artykule przedstawiono wyprowadzenie podstawowych równań bilansowych w formie charakterystycznej dla metody objętości skończonych. Przedstawiono pochodzenie równania bilansu pędu, masy i energii, następnie równania zachowania w formie wektorowej. Metoda objętości skończonych jest stosowana w tzw. numerycznej mechanice płynów do numerycznego modelowania przepływów płynów w możliwie szerokim zakresie. Motywacją do napisania artykułu była trudność w znalezieniu podobnego rodzaju krótkich i zwięzłych opracowań na ten temat w literaturze. Artykuł jest pierwszą częścią cyklu poświęconego matematycznym podstawom metody objętości skończonych.

Introduction

The proliferation of computers at the end of the twentieth century allowed for the application of numerical methods, that is, methods of solving mathematical problems using operations on numbers rather than symbolic variables. Results obtained in this way are approximate but the accuracy of the calculations can be selected predetermined depending on the needs. The first consideration of numerical techniques was led by Carl Friedrich Gauss (1777–1855), who based on the works of Leonhard Euler (1707–1783), Joseph Louis Lagrange (1736–1813) and Isaac Newton (1643–1727). The work begun by Gauss resulted in the development of so-called Finite Difference Method (FDM), whose essence is replacing the derivatives occurring in differential equations by the corresponding differential quotients. FDM began to be used to solve various mathematical problems, including issues relating to the mechanics of solids and fluid mechanics. After some time it appeared that the FDM has some limitations that hinder its application to solve more complex physical problems. The search for new possibilities gave rise to the Finite Element Method (FEM), which is more favorable from the point of view of needs of solid mechanics and the Finite Volume Methods (FVM), much more appropriate to the specificity of fluid mechanics. It should be emphasized that in principle each of the methods listed here can be applied to solve the same differential equations; however, it is much more preferable to apply in various areas of physics methods specific to them.

The purpose of this article is to present the equations of fluid mechanics in form which is characteristic of the Finite Volume Methods. It should be stressed that the laws and principles of physics are the same as in the case of classical mechanics (non-numeric), but the way of their presentation is different, as is apparent from the method of reasoning specific to them. A general difference between FDM and the FVM is that the FVM does not apply to discretization of differential equations, but to the space in which they apply. Hence the basis for any action here is the so-called Finite Volume (or Control Volume). It is a virtual, closed by a surface, part of the space, in which then the mathematical considerations are carried out. It is important in that the Finite Volume must be large enough so that the matter inside the volume can be treated as a representative sample of the fluid and small enough so that it can also be regarded as a "discrete" point of the space. A consistent and very general system of balance equations, resulting from the analysis of the phenomena occurring in the Finite Volume, is the basis of computations in absolutely every case of the numerical flow analysis. This aspect distinguishes significantly the numerical fluid mechanics from the classical mechanics, in which, depending on the needs different "individual" laws or formulas are

used: the law of continuity, Bernoulli's law, Navier-Stokes equation, Fourier's law, Fick's law, Forhcheimer's law or other, often independent of one another. In the Computational Fluid Mechanics (CFD), these laws also exist but are written in another form. Some of them are a simplified version of one of the general balance equations, while others are merely a supplement (a closure). These issues will be discussed later.

The rest of this article will describe the derivation of balance equations of mass, momentum and energy, in form characteristic of the FVM. It is worth noting that this form replaces gradually the classic derivations based on a virtual cube placed in a Cartesian coordinate system (see the description of the Navier-Stokes equations in ORZECHOWSKI et al. (1997) and passes into the general literature related to the fluid mechanics (GRYBOŚ 1998, PUZYREWSKI, SAWICKI 2000). This follows from the fact that the balance equations specific for the FVM are characterized by extraordinary elegance and consistency, unachieved previously in the theory of fluid flows. The motivation to write this article was the observation that in the literature, the authors usually confine themselves to present the basic system of equations, with no explanation of its origin. In addition, comparing the various publications the lack of unified mathematic form can be noted. Sometimes, the basic equations look quite different and difficult to fit together (compare e.g. works (JAMESON, MAVRIPLIS 1985, MUZAFERIJA, GOSMAN 1997) and CUHNA, FERREIRA 2002). On the other hand, the authors who describe the basic equations of fluid mechanics (e.g. in academic books), do it usually in the classical way, without focusing on the Finite Volume Method. The lack of appropriate derivation was the motivation to create this article. The purpose of the article was to explain to the readers the basic approach to the theory in FVM, and to propose a universal and general form of balance equations.

The idea of balances in a Finite Volume

The mathematical description of fluid flow in FVM approach is created for a volume $V \in \Omega$ closed by a surface S. This surface is oriented by a vector $\vec{n}$, directed outwards from the volume (Fig. 1). For the Finite Volume it can define two main types of balances: the surface and the volumetric balance. The surface balance describes the possibility to exchange the value of Φ with the surroundings through the fluxes flowing through the surface:

$$\int_S (\Phi \vec{v} \cdot \vec{n})\, dS \tag{1}$$

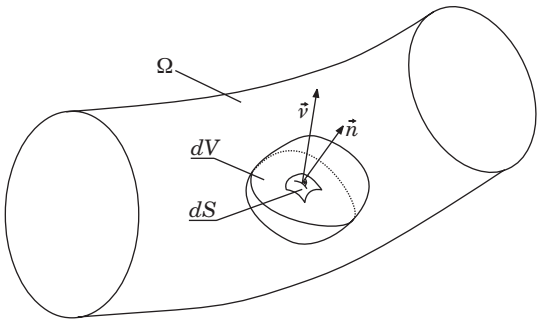

Fig. 1. The Finite Volume

Mathematically the flux is a vector defined as the product of any physical quantity Φ, e.g. mass, momentum or energy, and velocity $\vec{v}$ of flow in the direction perpendicular to the surface (Fig. 2).

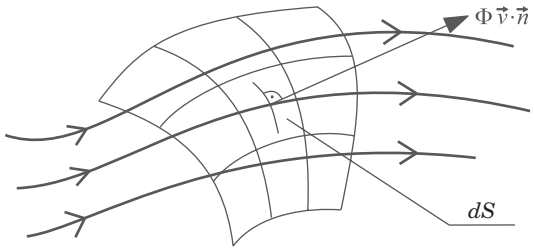

Fig. 2. Visualization of a flux

The volumetric balance describes the possibility to change of value Φ inside the volume V:

$$\int_V (\Phi) dV \tag{2}$$

To facilitate understanding of those balances we can use an example of heat balance in a room. The total heat inside the room can be changed by cooling or heating the walls (e.g. due to changes in outdoor temperature) or by heat sources like heater, various devices that produce heat and the heat generated by people in the room. The other parameters such as mass, momentum and energy can be treated analogically.

Mass balance equation

The total mass m [kg] contained in the volume V [m^3] filled by fluid with density ρ [kg/m^3] can be calculated as follows

$$m = \rho_{\text{average}} V = \int_V \rho \, dV \tag{3}$$

The change in mass in the time interval dt [s] is dependent on local fluctuations of density inside the fixed, rigid volume

$$\frac{d}{dt} m = \int_V \frac{\partial \rho}{\partial t} \, dV \tag{4}$$

In the general case of compressible fluid, the mass in volume V is subject to a compression. As a consequence inside the volume arise free space. To avoid a discontinuity in the flow, this free space must be supplemented by mass from the outside. This means that in time dt a resultant mass flux equal to the difference between mass fluxes flowing into and flowing out through the surface must get inside the volume V:

$$\int_V \frac{\partial \rho}{\partial t} \, dV = - \int_S (\rho \vec{v} \cdot \vec{n}) \, dS \tag{5}$$

where $(\rho \vec{v} \cdot \vec{n})$ is an elementary mass flux perpendicular to the surface of the Finite Volume. In the literature it is assumed that a positive sign applies to the effluent stream from the volume, hence the negative sign in the formula (since we consider the compression of mass in the volume). The surface integral on the right side can be converted to a volumetric integral using the Green-Gauss--Ostrogradzki Theorem (GGOT) (EVERSTINE 2010, SHARAFUTDINOV 1994, SAOUMA 2002) then:

$$\int_V \frac{\partial \rho}{\partial t} \, dV = - \int_V div(\rho \vec{v}) \, dV \tag{6}$$

or

$$\int_V \left(\frac{\partial \rho}{\partial t} + div(\rho \vec{v}) \right) dV = 0 \tag{7}$$

Above integral will be zero, only if the integrand expression to be zero

$$\frac{\partial \rho}{\partial t} + div(\rho \vec{v}) = 0 \tag{8}$$

Formula (8) represents the final form of mass balance equations. In fact it is a different form of well known and useful mass continuity equation.

Momentum balance equation

According to Newtonian mechanics, the momentum of the body $\vec{p}$ [kg · m/s] is equal to mass m [kg] times velocity $\vec{v}$ [m/s], which in the case of the Finite Volume must be expressed as a volumetric integral (see Eq. (3))

$$\vec{p} = m\vec{v} = \int_V (\rho \vec{v})\, dV \tag{9}$$

The change of momentum in time is only possible due to the forces (the change of momentum is equal to the force)

$$\frac{d\vec{p}}{dt} = \frac{d}{dt} \int_V (\rho \vec{v})\, dV = \vec{F}_T \tag{10}$$

where the total force $\vec{F}_T$ is a sum of the mass forces $\vec{F}_V$ [N] related to the mass contained in the volume V and the external forces $\vec{F}_S$ [N] from the impact of neighboring material on the surface S

$$\vec{F}_T = \vec{F}_V + \vec{F}_S \tag{11}$$

Equation (11) can add to the formula (10)

$$\frac{d}{dt} \int_V (\rho \vec{v})\, dV = \vec{F}_V + \vec{F}_S \tag{12}$$

The formula (12) shows that the momentum can be changed as a result of:
– mass forces (source forces) $\vec{F}_V$ operating inside the volume V. The mass forces are divided into external (e.g. gravity, electromagnetic force) and internal

forces (e.g. inertial forces). Mass forces act on each particle of fluid, therefore the resultant of forces acting on the Finite Volume V must be calculated as an integral of the volume (where $\vec{f}_V$ it the unit mass force [N]):

$$\vec{F}_V = \int_V (\rho \vec{f}_V)\, dV \tag{13}$$

– external surface forces $\vec{F}_S$ acting on the surface S of the volume V. Surface forces may be the hydrostatic and hydrodynamic pressure, internal friction or friction of fluid on the rigid walls (where $\vec{f}_S$ is unit surface force [N] and $\overleftrightarrow{T}$ is the total stress tensor acting on the surface S):

$$\vec{F}_S = \int_S \vec{f}_S\, dS = \int_S (\overleftrightarrow{T}\vec{n})\, dS = \int_V div\,(\overleftrightarrow{T})\, dV \tag{14}$$

Therefore, the change of momentum (12) is equal to the sum of the variations of forces acting on the volume V

$$\frac{d}{dt}\int_V (\rho\vec{v})\, dV = \int_V (\rho\vec{f}_V)\, dV + \int_V div\,(\overleftrightarrow{T})\, dV \tag{15}$$

The left-hand side of equation (15) may be transformed using the Reynolds Transport Theorem (RTT) (RTT allows one to determine how a given quantity defined within a control volume changes with time as the control volume deforms) (WHITAKER 1976) and next GGOT to the form

$$\frac{d}{dt}\int_V (\rho\vec{v})\, dV = \int_V \frac{\partial(\rho\vec{v})}{\partial t}\, dV + \int_S (\rho\vec{v} \otimes \vec{v}\vec{n})\, dS = \int_V \frac{\partial(\rho\vec{v})}{dt}\, dV + \int_V div(\rho\vec{v} \otimes \vec{v})\, dV \tag{16}$$

then

$$\int_V \frac{\partial(\rho\vec{v})}{\partial t}\, dV + \int_V div(\rho\vec{v} \otimes \vec{v})\, dV = \int_V div\,(\vec{T})\, dV + \int_V (\rho\vec{f}_V)\, dV \tag{17}$$

Leaving signs of integrals we obtain

$$\frac{\partial(\rho\vec{v})}{\partial t} + div\ (\rho\vec{v} \otimes \vec{v}) = div\ (\overleftrightarrow{T}) + \rho\vec{s}_b \tag{18}$$

where $\rho\vec{s}_b = \rho\vec{f}_V$ is treated as the general record of the source forces.

The resultant of surface forces can be decomposed into two parts: normal and tangent.

$$\overleftrightarrow{T} = -p\overleftrightarrow{I} + \overleftrightarrow{\tau}^t \tag{19}$$

These forces are represented on the surface S, respectively, by the total shear stress tensor $\overleftrightarrow{\tau}^t$ and the spherical stress tensor (pressure) $-p\overleftrightarrow{I}$ (negative sign due to convention taken out by Cauchy). We then obtain:

$$\frac{\partial(\rho\vec{v})}{\partial t} + div\ (\rho\vec{v} \otimes \vec{v}) = div\ (-p\overleftrightarrow{I} + \overleftrightarrow{\tau}^t) + \rho\vec{s}_b \tag{20}$$

The total shear stress tensor $\overleftrightarrow{\tau}^t$ consists usually of the sum of viscous molecular stress tensor $\overleftrightarrow{\tau}^m$ and turbulent Reynolds stress tensor $\overleftrightarrow{\tau}^R$.

$$\frac{\partial(\rho\vec{v})}{\partial t} + div\ (\rho\vec{v} \otimes \vec{v}) = div\ (-p\overleftrightarrow{I} + \overleftrightarrow{\tau}^m + \overleftrightarrow{\tau}^R) + \rho\vec{s}_b \tag{21}$$

Sometimes it is needed to add other stress tensor, e.g. diffusion stress tensor $\overleftrightarrow{\tau}^{diff}$ or radiation stress tensor $\overleftrightarrow{\tau}^{rad}$. Equation (21) is the final form of momentum balance equation.

Energy balance equation

In the balancing of energy, one usually only takes into account the kinetic energy

$$E_k = \frac{m\vec{v}^2}{2} = \int_V \rho\,\frac{\vec{v}^2}{2}\,dV \tag{22}$$

and the internal energy

$$U = mu = \int_V (\rho u)\,dV \tag{23}$$

where u is the specific density of internal energy, which is the energy of all forms of molecular motion, especially the movement of heat. The sum of kinetic and internal energy gives the total energy E:

$$E = \int_V \rho\left(\frac{\vec{v}^2}{2} + u\right) dV = \int_V (\rho e)\, dV \tag{24}$$

where e is the sum of both components.

Change in total energy of a fluid may occur as a result of acting (GRYBOŚ 1998):

– work done by the mass forces

$$A_V = \int_V (\rho \vec{f}_V \cdot \vec{v})\, dV \tag{25}$$

and surface forces (normal and tangential)

$$A_S = \int_V (\vec{f}_S \cdot \vec{v}) dS = \int_V (\overleftrightarrow{T}\vec{v} \cdot \vec{n}) dS \tag{26}$$

– internal heat (sources of energy)

$$Q_V = \int_V (\rho q_V)\, dV \tag{27}$$

and external heat, supplied to the volume V through the surface S at time dt:

$$Q_S = \int_V (\vec{q}_s^{\,t} \cdot \vec{n})\, dS \tag{28}$$

The total energy change can be written as

$$\frac{d}{dt}\int_V (\rho e)\, dV = \int_V (\rho \vec{f}_V \cdot \vec{v}) dV + \int_S (\overleftrightarrow{T}\vec{v} \cdot \vec{n}) dS + \int_S (\vec{q}_s^{\,t} \cdot \vec{n})\, dS + \int_V (\rho q_V)\, dV \tag{29}$$

After comparison of all integrals

$$\frac{d}{dt}\int_V (\rho e)\, dV = \int_V (\rho \vec{f}_V \cdot \vec{v}) dV + \int_V div\,(\overleftrightarrow{T}\vec{v}) dV + \int_V div\,(\vec{q}_s^{\,t})\, dV + \int_V (\rho q_V)\, dV \tag{30}$$

The left-hand site of the formula (30) can be transformed using the RTT and GGOT to the form

$$\frac{d}{dt}\int_V (\rho e)\,dV = \int_V \frac{\partial(\rho e)}{\partial t}\,dV + \int_S (\rho e\vec{v}\cdot\vec{n})\,dS = \int_V \frac{\partial(\rho e)}{\partial t}\,dV + \int_V div\,(\rho e\vec{v})\,dV \quad (31)$$

After substituting (31) to (30) we obtain

$$\int_V \frac{\partial(\rho e)}{\partial t}\,dV + \int_V div\,(\rho e\vec{v})dV = \int_V (\rho\vec{f_V}\cdot\vec{v})dV + \int_V div(\overleftrightarrow{T}\vec{v})dV + \int_V div(\vec{q_s^t})dV + \int_V (\rho q_V)\,dV \quad (32)$$

This equation is equivalent to

$$\frac{\partial(\rho e)}{\partial t} + div\,(\rho e\vec{v}) = \rho\vec{f_V}\cdot\vec{v} + div\,[(-p\overleftrightarrow{I} + \overleftrightarrow{\tau^t})\,\vec{v}] + div(\vec{q_s^t}) + \rho q_V \quad (33)$$

Energy change caused by the work of the mass forces and energy sources can be saved together in the form of total energy source $\rho s_e = \rho\vec{f_V}\cdot\vec{v} + \rho q_V$, then:

$$\frac{\partial(\rho e)}{\partial t} + div\,(\rho e\vec{v}) = div\,[(-p\overleftrightarrow{I} + \overleftrightarrow{\tau^t})\,\vec{v}] + div(\vec{q_s^t}) + \rho s_e \quad (34)$$

Both members under divergence signs on the left side can be linked to the formulation

$$\frac{\partial(\rho e)}{\partial t} + div\,(\rho e\vec{v}) = div\,[(-p\overleftrightarrow{I} + \overleftrightarrow{\tau^t})\,\vec{v} + \vec{q_s^t}] + \rho s_e \quad (35)$$

To simplify the notation, the bottom index s in the external heat flux will be omitted in further considerations. The total heat flux $\vec{q^t}$ is usually the sum of the molecular heat flux $\vec{q^m}$ and turbulent heat flux $\vec{q^R}$, then

$$\frac{\partial(\rho e)}{\partial t} + div\,(\rho e\vec{v}) = div\,[(-p\overleftrightarrow{I} + \overleftrightarrow{\tau^m} + \overleftrightarrow{\tau^R})\,\vec{v} + \vec{q^m} + \vec{q^R}] + \rho s_e \quad (36)$$

Sometimes it is needed to add other fluxes, e.g. diffusive heat flux $\vec{q^{diff}}$ or radiative heat flux $\vec{q^{rad}}$.

The formula (36) can be treated as the final version of the energy equation. Sometimes, however, this equation is further transformed until a form containing the enthalpy. To achieve this, the expression $-p\overleftrightarrow{I}$ under the sign of divergence's can be moved to the left side of the equation (34), then

$$\frac{\partial(\rho e)}{\partial t} + div\ (\rho e\vec{v} + \rho\vec{v}) = div\ [(\overleftrightarrow{\tau}^m + \overleftrightarrow{\tau}^R)\ \vec{v} + \vec{q}^m + \vec{q}^R] + \rho s_e \qquad (37)$$

or in other form

$$\frac{\partial(\rho e)}{\partial t} + div\left[\rho\left(e + \frac{p}{\rho}\right)\vec{v}\right] = div\ [(\overleftrightarrow{\tau}^m + \overleftrightarrow{\tau}^R)\ \vec{v} + \vec{q}^m + \vec{q}^R] + \rho s_e \qquad (38)$$

Note that the expression $e + \frac{p}{\rho}$ is the enthalpy. Denoting the symbol of the enthalpy h, we get the alternative version of the energy balance equation.

The vector form of balance equations

In the introduction it was said that the essence of the FVM are volume and mass balances, described by integrals (1) and (2). However, in the next part of the article the integrals were omitted, and the final shape of the obtained equations has a differential form. This is due to the specific needs of the FVM. Note that the mass, momentum and energy balances can be saved in one system of equations

$$\begin{cases} \dfrac{\partial\rho}{\partial t} + div\ (\rho\vec{v}) = 0 \\ \dfrac{\partial(\rho\vec{v})}{\partial t} + div\ (\rho\vec{v}\otimes\vec{v}) = div\ (-p\overleftrightarrow{I} + \overleftrightarrow{\tau}^m + \overleftrightarrow{\tau}^R) + \rho\vec{s}_b \\ \dfrac{\partial(\rho e)}{\partial t} + div\ (\rho e\vec{v}) = div\ [(-p\overleftrightarrow{I} + \overleftrightarrow{\tau}^m + \overleftrightarrow{\tau}^R)\ \vec{v} + \vec{q}^m + \vec{q}^R] + \rho s_e \end{cases} \qquad (39)$$

in which successive terms of individual equations have the same logical structure. It turns out that those terms can be treated together, and the entire system of balance equations can be solved by "columns" using the same numerical technique for all elements of a column. It is one of the most

important features of FVM, allowing, inter alia, relatively easy to add new equations, as long as they have a common structure which matches the template. In practice, which is in numeric codes, the balance equations are solved together in one loop calculation. The number of repetitions of main numerical schemes on n^{th} time level is equal to the total number of balance equations and equations of evolution (this term is explained later).

In the equation (39), the pressure member can be transferred to the left site:

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + div\,(\rho\vec{v}) = 0 \\ \dfrac{\partial(\rho\vec{v})}{\partial t} + div\,(\rho\vec{v} \otimes \vec{v} + p\overleftrightarrow{I}) = div\,(\overleftrightarrow{\tau}^{m} + \overleftrightarrow{\tau}^{R}) + \rho\vec{s}_b \\ \dfrac{\partial(\rho e)}{\partial t} + div\,(\rho e\vec{v} + p\overleftrightarrow{I}) = div\,[(\overleftrightarrow{\tau}^{m} + \overleftrightarrow{\tau}^{R})\,\vec{v} + \vec{q}^{m} + \vec{q}^{R}] + \rho s_e \end{cases} \tag{40}$$

Now in divergence on the left there are only convective parts, which can convert without energy losses. In the divergence on the right there are only parts connected with energy losses: viscosity, turbulence etc. This division is convenient for implementation in numerical codes: other numerical techniques are used to the reversible and other to the dissipative terms. Therefore, it is better to save the system of equations (40) in the so-called vector form (CHMIELNIAK 1996, KUDRYŃSKI et al. 1997, LOBO 1997, WRÓBLEWSKI 2000)

$$\frac{\partial}{\partial t} U + div\,(F^{c} + F^{e}) = div\,(F^{v}) + S \tag{41}$$

where: U is vector of conservative variables, F^{c} is the convection flux vector, F^{e} is recoverable flux vector, F^{v} is the diffusive flux vector and S is the vector of sources:

$$U = \begin{Bmatrix} \rho \\ \rho\vec{v} \\ \rho e \end{Bmatrix},\ F^{c} = \begin{Bmatrix} \rho \\ \rho\vec{v}\otimes\vec{v} \\ \rho e\vec{v} \end{Bmatrix},\ F^{e} = \begin{Bmatrix} 0 \\ p\overleftrightarrow{I} \\ p\overleftrightarrow{I}\,\vec{v} \end{Bmatrix}\ F^{v} = \begin{Bmatrix} 0 \\ \overleftrightarrow{\tau}^{m} + \overleftrightarrow{\tau}^{R} \\ (\overleftrightarrow{\tau}^{m} + \overleftrightarrow{\tau}^{R})\vec{v} + \vec{q}^{m} + \vec{q}^{R} \end{Bmatrix},\ S = \begin{Bmatrix} 0 \\ \rho s_b \\ \rho s_e \end{Bmatrix} \tag{42}$$

Only after such a transformation returning to the main idea of balances in the Control Volume is possible. This return gives the so-called integral form

of the conservation equations, which constitutes a form of output for the creation of numerical codes:

$$\frac{\partial}{\partial t}\int_V U dV + \int_V div(F^c + F^e)\, dV = \int_V div\ (F^v)\ dV + \int_V S dV \tag{43}$$

or (after using the GGOT and RTT):

$$\frac{\partial}{\partial t}\int_V U dV + \int_S (F^c + F^e)\, \vec{n} dS = \int_S (F^v)\ \vec{n} dS + \int_V S dV \tag{44}$$

Summary

In the area of the CFD several classes of equations can be distinguished:

- balance (or transport) equations – describe the balance of a certain size in the macrostructure level (single cell in the grid). Examples are the transport equations of mass, momentum, energy.
- evolution equations – describe the balance of a certain size of the microstructure (in the area of a single grid cell) and are usually a supplement to the basic system of equations. The role of a supplement is to add to the next equations the same structure as in formula (40). The vectors in conservation equations will consist of four, five or more elements, depending on the number of added evolution equations. Examples are the evolution equations of turbulence energy, the degree of dissipation ($k - \varepsilon$) or ($k - \varpi$) turbulence models), vorticity, intermittency and other. The equation of evolution is used generally to describe the phenomenon of turbulence and the number of equations defines a model class. For example $k - \varepsilon$ or $k - \varpi$ turbulence models are 2-equations models.
- kinematics equation – describes the kinematics properties of the fluid. There is only one equation describing the velocity of fluid element deformation. This equation is needed to calculate the viscous molecular stress tensor $\overleftrightarrow{\tau}^m$.
- constitutive equations – describe the properties of the fluid. Generally there are four types of constitutive equations:
  - equation of state (connecting the basic variables, e.g. Clapeyron's equation),
  - closure on the stress tensors (usually fluid model and turbulence model),
  - closures of the sources (e.g. the gravity, additional heat sources),
  - closures of heat transfer (e.g. Fourier law).

The set of equations (40) is the main and basic level of equations in CFD. Note that the system (40) is not complete and needs to be supplemented by many "closures", that is models describing the individual issues. Depending on these "closures" the equation (40) can be appropriate for a wide range of fluid flows: Newtonian, non-Newtonian, laminar, turbulent, with or without heat transfer or other. It is worth noting that the set of equations (40) applies only to single-phase flow. Issues concerning "closures" and multiphase flows will be discussed on another occasion.

It should be pointed out again that the forms of equations presented in this article are not inconsistent with generally applicable laws of fluid mechanics, although it has a specific character. As already mentioned, the mass balance equation is a generalized version of the classical equation of continuity. Similarly, the momentum balance equation has its classical counterpart: it can be treated as the Navier-Stokes equation or as the reaction equation. The energy balance is in turn a generalized form of the Daniel Bernoulli equation. These compounds can be demonstrated, but it is not the purpose of the article.

The balance equations presented in this paper are very general. They should be treated as a "template" which in each case will undergo modifications depending on the specifics of the flow. It turns out that many of the mathematical models from the various literature sources can be fitted through a relatively simple transformation to the set of equations (40). This issue is very important in practice because it creates a common basis for many different fluid flow issues.

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