TECHNICAL SCIENCES
Abbrev.: Techn. Sc., No 14(2), Y 2011

RELATIONSHIPS BETWEEN GEOMETRIC PARAMETERS IN CONICAL ROTARY GRADERS

Waldemar Dudda

Department of Mechanical Engineering and Machine Construction
University of Warmia and Mazury in Olsztyn

K e y w o r d s: rotary grader, conical working surface, geometry.

A b s t r a c t

The objective of this study was to determine the formula for the radius describing the position of a point located on the conical working surface relative to the vertical axis of revolution in circular motion. Diagrams of conical working surface were presented, and a formula for the above radius was determined. The relationship was verified for randomly selected points on the conical surface, using a 3D model.

ZALEŻNOŚCI MIĘDZY WIELKOŚCIAMI GEOMETRYCZNYMI W STOŻKOWYCH TRYJERACH OBIEGOWYCH

Waldemar Dudda

Katedra Mechaniki i Podstaw Konstrukcji Maszyn
Uniwersytet Warmińsko-Mazurski w Olsztynie

S ł o w a k l u c z o w e: tryjer obiegowy, stożkowa powierzchnia robocza, geometria.

A b s t r a k t

Praca dotyczy wyznaczenia zależności na promień położenia punktu znajdującego się na stożkowej powierzchni roboczej względem pionowej osi obrotu w ruchu obiegowym. Przedstawiono schematy stożkowej powierzchni roboczej i wyprowadzono ścisłą zależność na wspomniany promień. Zweryfikowano uzyskaną zależność na modelu 3D dla dowolnie wybranych punktów leżących na powierzchni stożkowej.

Introduction

Cylinder graders, also known as trieurs, have been long used to remove impurities from grain. In the ongoing search for devices that deliver improved separating efficiency without a deterioration in separation quality, cylinders moving in circular motion were proposed (WIERZBICKI 1981, WIERZBICKI et al. 2000). As the structure of circular motion separators underwent further improvement, the cylindrical working surface was replaced with a horizontal surface in the shape of a beveled cone (JADWISIEŃCZAK 2007). This solution is illustrated in Figure 1. The conical surface revolves around its own axis ξ with angular velocity ω_1, and it also moves in circular motion in the horizontal plane around point O_2 (vertical axis z) with angular velocity ω_2. The kinematics and dynamics of a particle of matter (e.g. grain seed – points B and B^*, Fig. 1) on the conical working surface can be determined based on radius R_β which describes the element's position (in circular motion) relative to axis z.

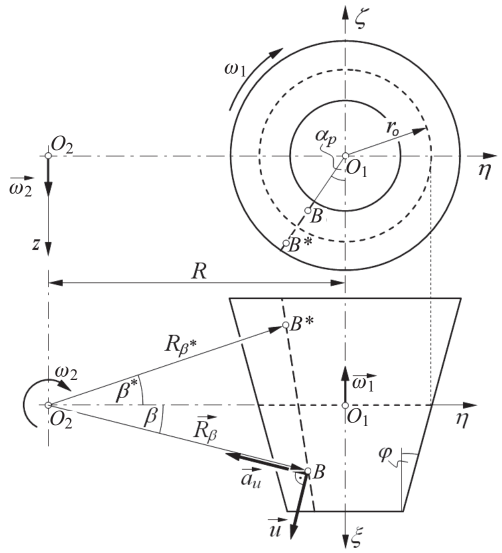

Fig. 1. Geometric parameters of a conical rotary grader

Point B moves in complex motion, therefore, its velocity $\vec{v}_B$ and acceleration $\vec{a}_B$ can be described by the following equations:

$$\vec{v}_B = \vec{u} + \vec{w}, \qquad \vec{a}_B = \vec{a}_u + \vec{a}_w + \vec{a}_c \tag{1}$$

where:
$\vec{u}$ – transport velocity,

$\vec{w}$ – relative velocity,
$\vec{a}_u$ – acceleration of transport,
$\vec{a}_w$ – relative acceleration,
$\vec{a}_c$ – Coriolis acceleration.

At given operating parameters (ω_2=const.), the velocity and acceleration of transport (Fig. 1) will take on the following form:

$$\vec{u} = \vec{\omega}_2 \times \vec{R}_\beta, \qquad \vec{a}_u = \vec{a}_u^{\,n} = \vec{\omega}_2 \times (\vec{\omega}_2 \times \vec{R}_\beta) \tag{2}$$

where:
$\vec{a}_u^{\,n}$ – normal acceleration of transport (centripetal).
If $\vec{\omega}_2 \perp \vec{R}_\beta$, the velocity and acceleration of transport can be calculated based on the following scalar dependencies:

$$u = \omega_2 \cdot R_\beta, \qquad a_u = (\omega_2)^2 \cdot R_\beta \tag{3}$$

In general, R_β should be a function of the grader's geometric parameters which define the position of point B on the conical surface. As shown in Figure 1, radius R_β should be determined by:
- radius R of cone's circular motion (distance between the cone's own axis of revolution ξ and axis z of circular motion),
- radius r_o of cone's intersection with plane $\eta\zeta$ w in mid-length,
- angle α_p at which a seed slides down the surface of the working element,
- angle φ between the cone's element and the cone's own axis of revolution ξ,
- angle β between the radius of cone's circular motion and radius R_β.

In JADWISIEŃCZAK (2007), R_β has been incorrectly determined, therefore, the objective of this study was to determine the correct relationship describing radius R_β.

Figure 1 presents radii R_β and R_β* and angles β and β* describing the position of points B and B* on the opposite sides of plane $\eta\zeta$ (intersecting the cone in mid-length). As demonstrated later, the relationships applicable to R_β and R_{β^*} will differ only in sign (+, –), therefore index (*) will not be used in successive parts of the study.

Geometric relationship

The input values were R, r_o, α_p, φ, β. The searched function will support the determination of the distance between point B and axis z, i.e. $R_\beta = f(R, r_o, \alpha_p, \varphi, \beta)$.

Figure 2 presents the geometric parameters required for determining radius R_β describing the position of point B situated in front of intersecting plane $\eta\zeta$.

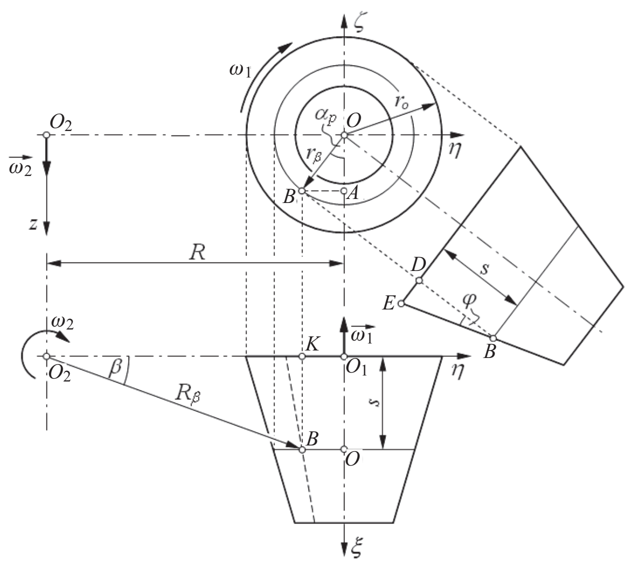

Fig. 2. Cone geometry in the part in front of intersecting plane $\eta\zeta$

Based on triangle EBD, we can deduce that:

$$\mathrm{tg}\varphi = \frac{r_o - r_\beta}{s} \tag{4}$$

therefore, radius r_β describing the location of point B relative to the cylinder's own axis of revolution ξ can be presented as:

$$r_\beta = r_o - s \cdot \mathrm{tg}\varphi \tag{5}$$

Triangle BKO_2 produces the following dependence:

$$s = R_\beta \sin \beta \tag{6}$$

When dependence (6) is substituted in equation (5), the result is:

$$r_\beta = r_o - R_\beta \sin \beta \,\mathrm{tg}\varphi \tag{7}$$

As demonstrated by Figure 2, distance R between axis z of cylinder's circular motion and the cylinder's own axis of revolution ξ is equal to:

$$R = \mathrm{KO}_2 + AB \tag{8}$$

Triangles BKO_2 and ABO produce the following equations:

$$\mathrm{KO}_2 = \mathrm{R}_\beta \cos \beta, \qquad AB = r_\beta \sin \alpha_p \tag{9}$$

therefore, when equations (9) are substituted in formula (8), the result is:

$$R = R_\beta \cos \beta + r_\beta \sin \alpha_p \tag{10}$$

Dependence (7) is substituted in equation (10) to produce:

$$R = R_\beta \cos \beta + r_\beta \sin \alpha_p - R_\beta \sin \beta \, \mathrm{tg}\varphi \sin \alpha_p \tag{11}$$

After simple transformation, the result is a relationship between radius R_β and point B situated behind intersecting plane $\eta\zeta$:

$$R_\beta = \frac{R - r_o \sin \alpha_p}{\cos \beta - \sin \beta \, \mathrm{tg}\varphi \sin \alpha_p} \tag{12}$$

Figure 3 presents geometric parameters required for the determination of radius R_β for point B situated behind intersecting plane $\eta\zeta$. For this part of the cone, the following dependence is derived from triangle EBD:

$$\mathrm{tg}\varphi = \frac{r_\beta - r_o}{s} \tag{13}$$

therefore, radius r_β describing the location of point B relative to the cylinder's own axis of revolution ξ can be presented in the following form:

$$r_\beta = r_o + s \cdot \mathrm{tg}\varphi \tag{14}$$

A comparison of Figure 2 and Figure 3 indicates that equation (6) has an identical form in both cases. When equation (6) is substituted in dependence (14), the result is:

$$r_\beta = r_o + R_\beta \sin \beta \, \mathrm{tg}\varphi \tag{15}$$

Equations (8) and (9) also have an identical form in both cases (comparison of Fig. 2 and Fig. 3). Therefore, dependence (10) will not change, and when dependence (15) is substituted, the result is:

$$R = R_\beta \cos\beta + r_\beta \sin\alpha_p + R_\beta \sin\beta \,\mathrm{tg}\varphi \sin\alpha_p \tag{16}$$

After a simple transformation of equation (16), the result is a dependence between radius R_β and point B situated behind intersecting plane $\eta\zeta$:

$$R_\beta = \frac{R - r_o \sin\alpha_p}{\cos\beta + \sin\beta \,\mathrm{tg}\varphi \sin\alpha_p} \tag{17}$$

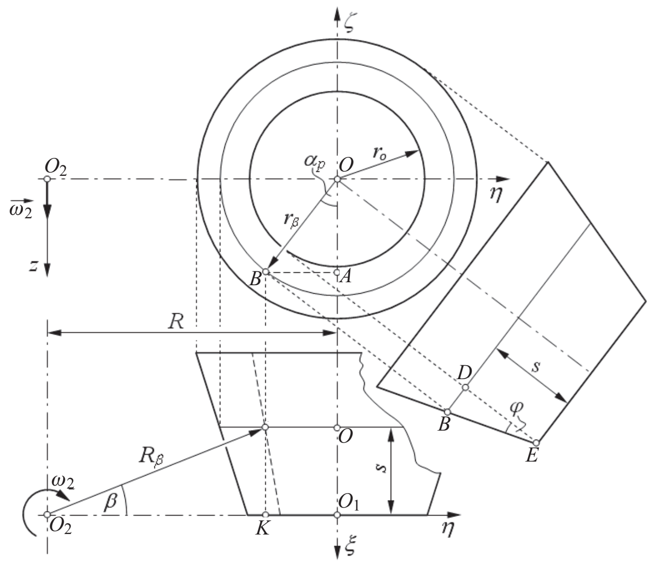

Fig. 3. Cone geometry in the part behind intersecting plane $\eta\zeta$

Equations (12) and (17) differ only in the sign (–, +) of the denominator. Therefore, both cases can be described by a shared dependence:

$$R_\beta = \frac{R - r_o \sin\alpha_p}{\cos\beta \pm \sin\beta \,\mathrm{tg}\varphi \sin\alpha_p} \tag{18}$$

Sign (–) applies to point B situated in front of intersecting plane $\eta\zeta$, whereas sign (+) applies to point B situated behind that plane.

Verification of relationship

A 3D model of the part of the cone in front of intersecting plane $\eta\zeta$ has been developed in the AutoCAD application (Fig. 2). The following model data were input: R=1000 mm, r_o=200 mm, height of beveled cone = 300 mm, radius of the smaller base = 100 mm (Fig. 4). Points B_1 and B_2 were mapped on the cone's lateral surface at two angles of α_p. Angles φ and β and the corresponding radii $R\beta$ were measured (Fig. 4), and the resulting values were presented in Table 1. The length of radii $R\beta$ determined based on dependence 12 is shown in the last column of Table 1.

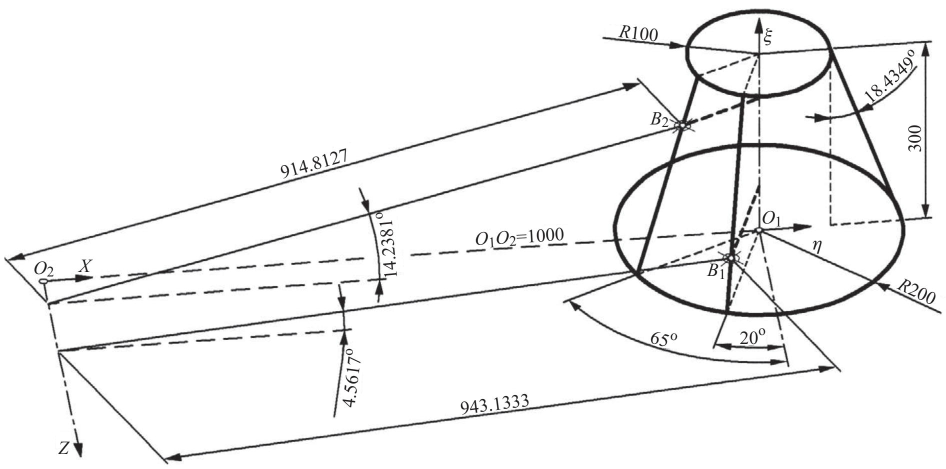

Fig. 4. A 3D model, radius and angle measurements

Table 1

Geometric parameters for point B

Point	Angle [°]			Radius R_β [mm]	
	α_p	φ	β	measured	based on (12)
B_1	20	18.4349	4.5611	943.1333	943.13326
B_2	65		14.2381	914.8127	914.81252

Conclusions

The relationship describing the distance between point B and axis z of cone's circular motion was determined in this study. In section 3, the formula (12) describing radius R_β was verified. The convergence between the measured values of R_β and the values of R_β derived from equation (12) is determined solely by the rounding-off of the values of trigonometric functions of angles α_p, φ and β. Therefore, it can be concluded that equations (12) and (17) have been formulated correctly.

Translated by Aleksandra Poprawska

Accepted for print 6.09.2011

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