SINGULAR VALUES OF SYMPTOM OBSERVATION MATRIX OF A SYSTEM IN OPERATION AS INDICATORS OF SYSTEM DAMAGE

Czesław CEMPEL
Poznań University of Technology,
E-mail: czeslaw.cempel@put.poznan.pl

Summary

The last paper of the present author [19] was concerned with multidimensional condition monitoring of the machines and the application of singular value decomposition (SVD). It was shown there, the immunity of singular values against uncontrolled load change, and they are also some measures of damage intensity. Following this, a simple model of singular value evolution has been proposed here and tested by means of three cases of real diagnostics in industry. It was found that postulated linear growth of singular value is good approximation of its real behavior and the same concerns the exponential growth of singular values product. Moreover this measures are sensitive to the redundancy of observation space and can depict clearly a lifetime when real damage in a monitored system starts. These properties seem to be much wanted in condition monitoring, so further investigations are planned.

Key words: machine condition, symptom observation matrix, singular value decomposition, evolution of singular values.

WARTOŚCI SZCZEGÓLNE SYMPTOMOWEJ MACIERZY OBSERWACJI EKSPLOATOWANEGO SYSTEMU MECHANICZNEGO JAKO WSKAŹNIKI ZUŻYCIA

Ostatnia praca autora [19] pokazuje zastosowanie rozkładu wartości szczególnych symptomowej macierzy obserwacji w diagnostyce maszyn. Pokazano tam, że ewolucja wartości szczególnych rozkładu w czasie życia maszyny jest niewrażliwa na wahania obciążenia roboczego systemu. Zatem w obecnej pracy zaproponowano liniowy model ewolucji wartości szczególnych i ekspotencjalny model dla ich iloczynu. Porównania tych modeli z rzeczywistym przebiegiem wartości szczególnych eksploatowanych maszyn pokazują, że dla przypadku liniowego zużycia jest to dobry model, natomiast często widać skoki poziomu w ewolucji wartości szczególnej. Może to świadczyć o pojawieniu się dodatkowego uszkodzenia, bądź o przejściu zużycia do bardziej intensywnej fazy rozwoju. Planuje się, zatem przeprowadzić dalsze badania celem wyjaśnienia szczegółowego zachowania wartości szczególnych.

Słowa kluczowe: stan maszyny, symptomowa macierz obserwacji, dekompozycja SVD, ewolucja wartości szczególnych.

1. INTRODUCTION

The idea of symptom observation matrix (SOM) in multidimensional condition monitoring of machines is well established and brings several advantages, [1, 3, 15 - 19]. It is basing on $p > r$ rectangular symptom observation matrix, with (r) symptoms $\boldsymbol{S}_r$ in columns, measured along the system life θ, what gives p symptom readings in our passive diagnostic experiment [1]. This observation technique allows placing all physically different symptoms[1] measured in a phenomenal field of the machine in one SOM, and to process them in order to obtain projection of designed **observation space** to the **fault space** of machine, which we are looking for. Of course, at the beginning we usually observe more symptoms (*columns of SOM*), than there is expected number of essential faults[2] in a machine.

The preprocessing of SOM may be different (*see for example* [17]), but for condition monitoring it was found that normalization and extraction of symptom initial value is the best solution, bringing all symptoms to their dimensionless and most sensitive form. Then, the application of SVD to the dimensionless form of SOM gives needed projection of observation space (*symptoms*) to the fault space - described by the generalized fault symptoms and singular values. The resultant three matrices of SVD decomposition allow calculating the two

[1] Symptom, measurable quantity covariable (*or assumed to be*) with the system condition

[2] Essential fault can lead to machine breakdown or terminal damage, if not interrupted by machine renewal.

diagnostically important matrices. The first is ***SD*** matrix, which give us generalized fault symptoms SD_i, and in theory they are independent each other. From this matrix we can calculate so called total damage (*generalized*) symptom, as the sum of all SD_i generalized fault symptoms. This is mainly in order to calculate the symptom limit value S_l , or to make the forecast of the **total damage** symptom. The second ***AL*** matrix, allows us to assess the contribution of primary measured symptoms S_r to a newly formed generalized fault symptoms SD_i **.** In this way we can just say which of primary measured symptom is redundant, giving no substantial information contribution to the given SD_i, and as such can be rejected from further calculations and/or future measurements.

But should we use the lifetime evolution of generalized fault symptoms SD_i as the only measure of evolving faults in a machine? From the SVD decomposition we suppose that singular values σ_i may have some diagnostic meaning. They can be interpreted as the generalized fault intensities, describing the advancement of the given kind of damage in a wearing machine. And also, they can be calculated after each new observation of symptom vector, allowing in this way the step by step tracing of this measure of evolving fault [19]. We will try to show this new possibility of inference using singular value decomposition (***SVD***) technique, in the way as it was shown in our earlier papers. In these papers some new measures based on singular values has been defined, namely modified Frobenius norm of SOM, being a sum of all nonzero singular values - SV, and so called volume of fault space – being the product of all nonzero SV. It will be interesting to know how the evolution of these measures can be modeled mathematically, and how they do behave theoretically and in practical circumstances of machine condition monitoring. It was found in this way that the approximate model of individual singular value and Frobenius modified measure may have the linear growth, while the volume measure is exponential function of the system lifetime. The present paper shows these properties of singular values and indicates their possible applicability in a diagnostic inference.

2. SYMPTOM OBSERVATION MATRIX (SOM) PROCESSING AND GENERALIZED FAULT EVOLUTION

It was described earlier, our information on machine condition evolution is contained in p x r **SOM**, where r columns (*primary symptoms*) and p rows of successive readings of each symptom are located. Usually they are made at equidistant system lifetime moments θ_n, $n=1,2,\dots p$. In pre-processing operations, the columns of SOM are centered and normalized to the three point average of initial readings of every symptom. This is in order to make the SOM dimensionless, and to diminish starting disturbances of symptoms (*averaging*). This allows also to present the evolution range of every symptom from zero up to few times of its initial symptom value S_{0r}, (*measured in the vicinity of* $\theta = 0$). Also it was found in some earlier paper of the author, that the addition of linear growing system life symptom (***LS***) in the first column of SOM, give us new information concerning the intensity of use of the investigated machine.

After such preprocessing we obtain the dimensionless **SOM** in the form;

$$\mathbf{SOM} = \mathbf{O}_{pr} = \left[S'_{nm}\right], \quad S'_{nm} = \frac{S_{nm}}{S_{0m}} - 1, \qquad (1)$$

This additional symptom can not have to small values or to large values, because in this way it will, or will not, influence our calculation and final result. If machine observation starts from its good condition, than usually symptoms starts also from small values, and at the end of life we have maximal symptom values. Hence one way of scaling life symptom ***LS*** may include multiplying by the average of last readings of all observed primary symptoms. Let the counting of symptom readings in SOM will be $i = 1{:}n$, and for r symptoms one can write;

$$LS = (\, r\,)^{-1} . \sum_1^r S_{nm} \;.\; (i\,/\,n)\;,\; i = 1{:}n, \qquad (1a)$$

where S_{nm} means the last readings of symptom number m.

Now, adding ***LS*** symptom as a first column to the old **SOM** (1) we have a new appended $\mathbf{SOM_L}$, which includes explicit machine life information to our diagnostic calculations and decision. Having this, we can apply the Singular Value Decomposition (SVD) [10], [12l],[9], to our dimensionless SOM (1), to obtain singular components (*vectors*) and singular values (*numbers*) of SOM, in the form

$$\mathbf{O}_{pr} = \mathbf{U}_{pp} \cdot \mathbf{\Sigma}_{pr} \cdot \mathbf{V}_{rr}^{\,T}, \qquad (T - matrix\ transposition), \qquad (2)$$

where; U_{pp} is p dimensional orthonormal matrix of left hand side singular vectors, V_{rr} is r dimensional orthonormal matrix of right hand side singular vectors, and the diagonal matrix of singular values (*s. v*), Σ_{pr} is defined as below

$$\mathbf{\Sigma}_{pr} = diag\left(\sigma_{1,\dots,}\sigma_l\right),$$
$$whit\ nonzero\ s.v.: \quad \sigma_1 > \sigma_2 > \dots > \sigma_u > 0, \qquad (3)$$
$$and\ zero\ s.\ v.\ ;\ \sigma_{u+1} = \dots\ \sigma_l = 0; \quad l = \max(p, r),\quad u \le \min(p, r),\ \ u < r < p.$$

Going back to SVD itself it is worthwhile to say, that every non square matrix has such decomposition, and it may be interpreted also as the product of three matrices [12], namely

$$O_{pr} = (\text{Hanger}) \times (\text{Stretcher}) \times (\text{Aligner}^T) \qquad (4)$$

This is a very metaphorical description of SVD matrix transformation, but it seems to be a useful analogy for the inference and decision making in condition monitoring. The diagnostic interpretation of formulae (4) can be obtained very easily. Namely, using its left hand side part, we are stretching our SOM over the life (*observations*) dimension, obtaining the matrix of **generalized symptoms** ***SD*** as the columns of the matrix. And using the right hand side part of (4) we are stretching SOM over the observed (*primary*) symptoms dimension in the form of matrix ***AL***, assessing in this way the contribution of each primary symptom to the generalized fault symptom $\boldsymbol{SD}_i$ *,i=1,…u*. Hence

$$SD = O_{pr} \cdot V_{rr} = U_{pp} \cdot \sum pr;, \quad \text{and;} \quad AL = U^T_{pp} \cdot O_{pr} = \sum pr \cdot V^T_{rr} \qquad (5)$$

We will calculate the above matrices and use them for better interpretation of monitoring results (***SD***) and optimization of the dimension of the observation space (***AL***).

As the rows of SOM matrix are formed along the machine lifetime, so the columns of **SD** matrix have the same discrete argument of life time θ, and we can write their fault space interpretation as below;

$$\boldsymbol{SD}_t(\theta) \propto \boldsymbol{F}_t(\theta), \quad t=1,2,\dots,Norm\ (\boldsymbol{SD}_t) \equiv //\boldsymbol{SD}_t// = \sigma_t, \ t = 1, \dots, u \qquad (6)$$

For the assessment of total machine damage we can calculate the sum of all generalized symptoms

$$\mathbf{SumSD}_i(\theta) = \sum_{i=1}^{z} \mathbf{SD}_i(\theta) = \sum_{i=1}^{z} \sigma_i(\theta) \cdot \mathbf{u}_i(\theta) \propto \mathbf{F}(\theta) \qquad (7)$$

where; $\mathbf{u}_i$ is a column of U_{pp} matrix.

This concept of diagnostic inference, for individual fault $\boldsymbol{F_t(\theta)}$, (6), and total fault damage $\boldsymbol{F(\theta)}$ (7) has been proven in several papers [1, 3, 11], [13-19], and we will use it here in further consideration.

The above results, based on generalized fault symptoms, have been obtained only from the first matrix ***SD*** of (5). And the second matrix ***AL*** gives us the relative measure of information contribution to each generalized symptom, as given by particular primary symptom measured during the SOM gathering. This is one way of assessment of the primary symptom redundancy, but we need some other global indicators of rejection of the redundant symptom. In our previous papers we have used modified Frobenius norm of SOM and the generalized volume of the fault space created by SOM. What is important in such an approach, these two measures are based on singular values of SOM, which in turn can be treated as the faults advancement measure (*see (6-7)*). Hence we have;

$$\boldsymbol{Frob1} = \boldsymbol{\Sigma}\, \boldsymbol{\sigma_i}; \quad \text{and;} \quad \boldsymbol{Vol1} = \boldsymbol{\Pi}\, \boldsymbol{\sigma_i}, \quad i = 1, \dots u \qquad (8)$$

Looking for the way of value creation method of the above, one can infer that if some primary symptom will be really redundant (*small* σ_i) its rejection should give a small change to; *Frob1* measure, and in contrary it should increase much the fault space volume; *Vol1*. We will notice also how it behaves with real examples of symptom rejection and addition in SOM of diagnosed machines latter on.

Such way of diagnostic inference described in (8) has been used in last papers of present author. However there is question now; why do not treat these two measures (8) as evolving along system lifetime $\boldsymbol{\theta}$, together with the evolution of all generalized faults in a machine? So instead of (8) we can write down as below

$$\boldsymbol{Frob1(\theta)} = \boldsymbol{\Sigma}\, \boldsymbol{\sigma_i(\theta)}; \ \text{and;} \ \boldsymbol{Vol1(\theta)} = \Pi\, \sigma_i(\boldsymbol{\theta}), \quad i = , \dots u \qquad (9)$$

And of course the system lifetime will be the discrete variable in the above, the same as moment of symptom readings $\boldsymbol{\theta_n}$ ***n=1,…m***, in our primary SOM. We will see below what evolutional property have newly defined measures (9), and how much this can help in tracing the fault evolution (*development*) and reduction of observation redundancy in the real cases of machine diagnostics.

3. THE SIMPLIFIED THEORY OF SYSTEM DAMAGE AND SINGULAR VALUES EVOLUTION

Trying to build simple theory of singular value possible behavior, let us assume that our system in operation, machine or it element, has a constant working load, what assures the low level of external disturbances in symptom observation. Of course the constant working load is a source of constant wear of machine elements, giving a constant rise of each symptom value. In such case we can assume in a first approach, that our primary symptoms have also almost linear changes, as below.

$$\boldsymbol{S_j(\theta) = S_{jo} + \theta\, (b_{jo} + \varepsilon\, b_j(\theta)), \ \varepsilon << 1}, \ j = 1, \dots r; \quad \boldsymbol{\theta = \theta_1, \theta_2, \dots \theta_p} \qquad (10)$$

where $\boldsymbol{S_{jo}}$ and $\boldsymbol{b_{jo}}$ are the initial value and the slope of symptom number j, and ε is small number allowing the deflection of symptom from a linear law of behavior during consecutive symptom readings $\theta_1, \theta_2, \dots \theta_p$, and forming symptom observation matrix - SOM .

The decomposition of SOM give us the generalized fault symptoms $SD_t(\theta)$ and singular values $\sigma_t(\theta)$, $t = 1, \dots, u$, and we may assume also their similar almost linear behavior. This may be written as below.

$$SD_t(\theta) = SD_{t\,o} + \theta\,(c_{jo} + \varepsilon\, c_j(\theta)),\ \theta = \theta_m,\ m = 1,\dots p, \quad (11)$$

$$\sigma_t(\theta) = \sigma_{to} + \theta\,(a_{to} + \varepsilon\, a_t(\theta)),\ t = 1, \dots, u,$$

where u is the rank of SOM being usually the number of symptoms (*columns*) in SOM, and $c_j(\theta)),\ a_t(\theta)),\ b_j(\theta))$ are set to zero in a first approach to modeling their evolution.

The initial values of generalized fault symptom $SD_{t\,o}$ and singular value σ_{to} depends on the way of SOM pre-processing. In case where each primary symptom (10) is normalized and centred to the initial value, they become zero; $\sigma_{to} = 0$, and our decomposition results (11) starts their evolution from **zero value**. There is also the difference in SOM decomposition presentation along the life coordinate and the symptom dimension. In both cases the initial value is zero for θ_1, but later on the lifetime evolution generalized symptoms and singular values differ.

Let us see also how our measures of information content in SOM (9) will fit into our model (11) taken in a first approximation.

$$Frob1(\theta) = \Sigma\,\sigma_\tau(\theta) \approx \theta \cdot \sum a_{to},\ t = 1,\dots u. \quad (12)$$

and;

$$Vol1(\theta) = \Pi\,\sigma_\tau(\theta) \approx \theta^u \cdot \Pi\, a_{to} \approx \begin{cases} = 0, & t = 1,\dots m \\ \neq 0, & t = m+1,\dots u \end{cases}$$

As it is seen in a first approximation, both information content measures grows with system lifetime θ , and first ***Frob1*** grows linearly, while the second ***Vol1*** exponentially. Hence in reality of condition monitoring they must be non decreasing functions of lifetime, i.e. will grow with increased number of symptom readings, or the rows of the SOM.

The course of all generalized symptoms $SD_t(\theta)$ have nonzero values during the next life increments but singular values $\sigma_t(\theta)$ starts sequentially, the second life increment θ_m , $m=1$ gives nonzero $\sigma_t(\theta)$, for t=1 only, and each subsequent lifetime increment switches on the next nonzero singular value, up to the readings number $r+1$, (*r being the number of symptoms*), when all singular values have already nonzero values. We will see that this property is important for ***s.v*** product measure, when looking for processing results at the next point of the paper.

4. THE EVOLUTION OF SINGULAR VALUE OF SYSTEMS IN OPERATION -EXAMPLES

Starting our illustrative and validating examples let us take SOM of small ball bearing at durability testing stand under constant load during the whole test. Here the peak and rms amplitudes of acceleration and velocity of the bearing outer ring has been measured, together with acoustic emission energy, the temperature, and the stand driving power. Altogether the SOM has 7 symptoms and appended lifetime in the first column. During the SOM preprocessing the symptom normalization and centering to the initial value has been applied. Special Matlab® program **svdoptsvev.m** has been compiled from the author previous programs, and the processing result one can see in the figure below.

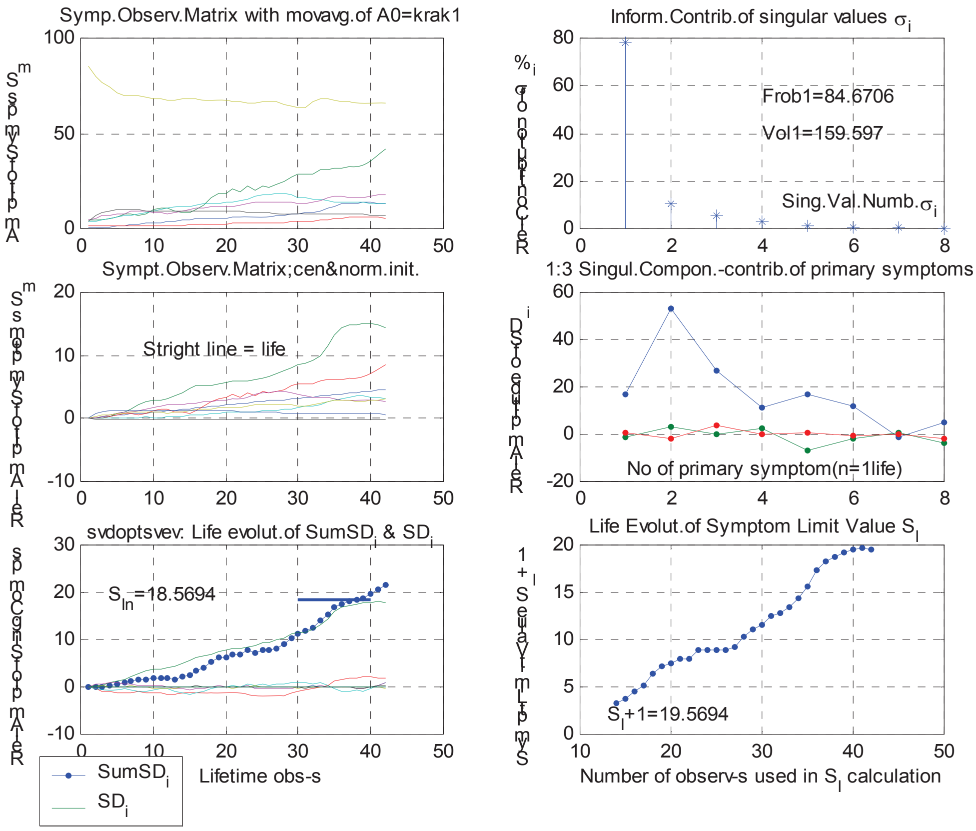

Fig. 1. Small ball bearing at durability testing stand and diagnostic decomposition of its SOM, as a sequence o pictures

The picture upper left presents graphically SOM as it was received and registered. As one can notice, one of the symptoms, the driving power of test stand is falling down, reciprocally to the deteriorating bearing condition. The other symptoms create the bunch of similar symptom life curves, much better seen after normalization and centering (*picture middle left*), where the straight line of bearing lifetime is also visible. We can notice that centering and normalization is right preprocessing method, allowing comparing the real diagnostic value and sensitivity of primary measured symptoms.

The generalized fault symptoms obtained after SVD one can see in the picture bottom left, where two life curves are distinguishable only. These are, the total damage symptom $\boldsymbol{SumSD_i}$ and the generalized symptom of fault No 1 i.e. $\boldsymbol{SD_1}$. This means we have one type of damage in the bearing, what is confirmed from the picture upper right, where the succession of SOM singular values (***s.v***), $\boldsymbol{\sigma_i}$ is presented in descending order. There is also the assessment of information content of primary measured symptoms in the picture middle right, where one can see the low information contribution of last three primary symptoms, this means they maybe redundant. The last picture, the bottom right presents the step by step calculation of symptom limit value, needed in every diagnostic case, but not necessarily in our case.

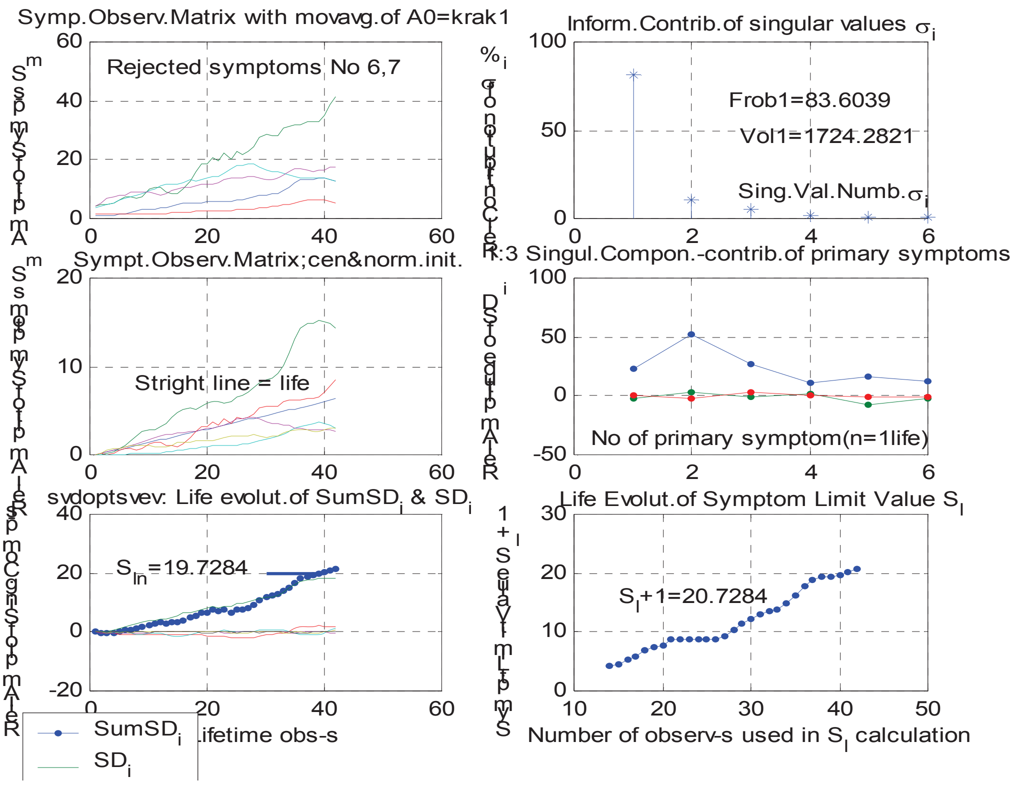

Fig. 2. SOM of the same bearing krak1 processed after rejection of 2 redundant symptoms

The presented software allows also the rejecting of some redundant symptoms from initial SOM, and the next Fig. 2 shows the result of such rejection of symptoms No 6 and 7, presented in the same mode of six pictures. One can say they are similar, but much smoother, and have different values of ***Frob1*** and ***Vol1*** information measures calculated from singular values of SOM. Hence it will of much interest to see the evolution of these measures and respective singular values as well.

The lifetime evolution of these quantities is shown in two pictures; Fig. 3 for the original SOM, and Fig. 4 for SOM with rejected symptom No 6, and 7. The left picture of each figure presents the evolution of individual singular values and sum of them, as the total damage symptom. We can notice that our simple theory with linear and exponential grows of these quantities (12) is here approximately true in both figures, i.e. for original SOM and for abbreviated SOM, after the rejection of two symptoms. The right hand side pictures of Fig. 3 and Fig. 4 shows us that the product of singular values is continuously growing exponential function, and is much more sensitive to detect wear changes of ball bearing when the redundancy of SOM has been reduced, (*Fig. 4*), where bearing deterioration is noticeable starting from 0.2 of dimensionless life of bearing.

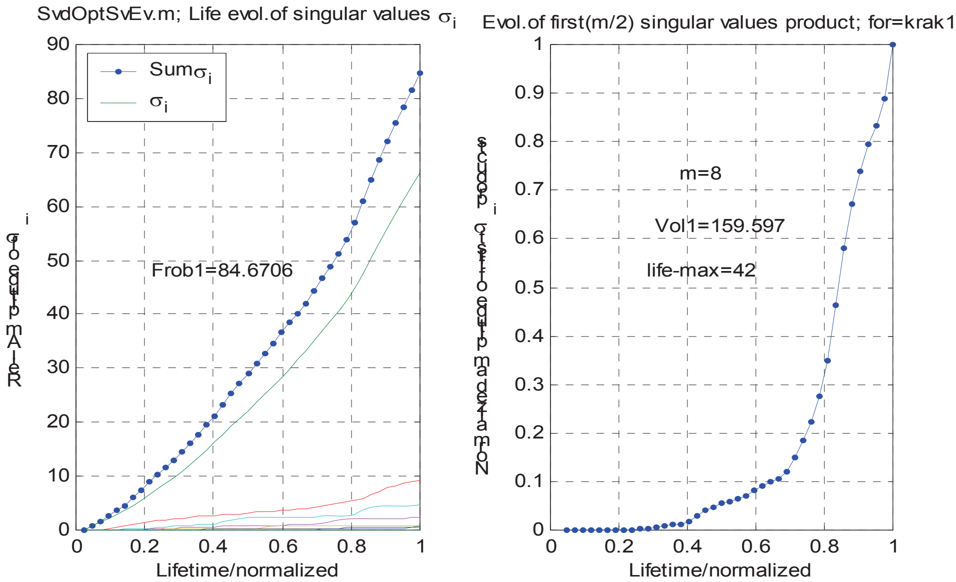

Fig.3. The lifetime evolution of singular values; their sum and product as the illustration of constant speed damage of the ball bearing krak1

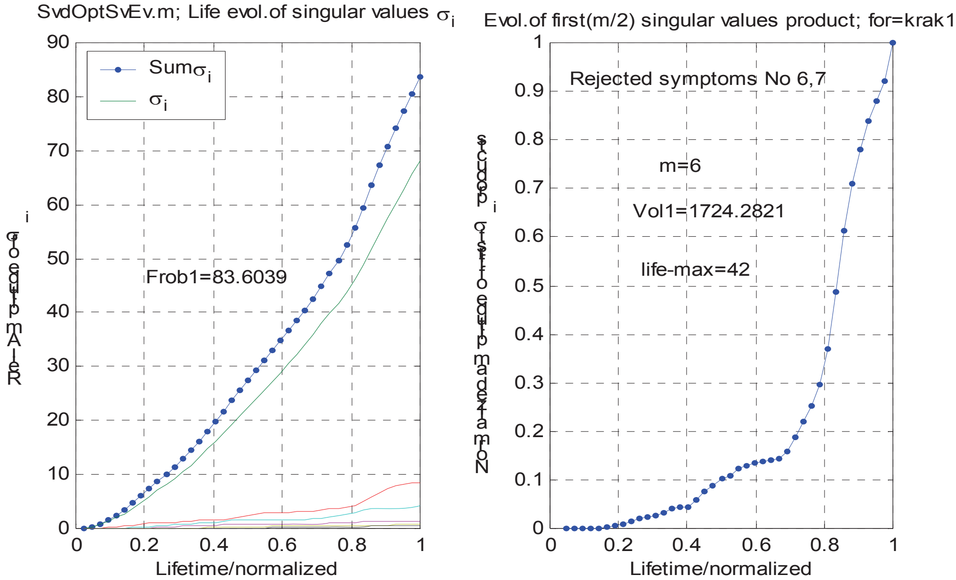

Fig.4. The lifetime evolution of singular values of ball bearing krak1, after rejection of two redundant symptoms

We can confirm again that the reduction of SOM redundancy causes small decrease of ***Frob1*** measure, but dramatic increase of the volume of generalized fault space ***Vol1*** is taking place. In conclusion one can say that ***Frob1*** is the good measure of overall system condition, and ***Vol1*** is sensitive to drop of the redundancy in SOM.

It will be of much interest to investigate the behavior of this measure at normal operating condition of the machine, not as it was just before, where ball bearing had the **constant** working load. Hence we present below two troublesome cases of real life diagnostics; ventilation fan in copper mines where the load control of the fan is impossible (*Fig. 5*), and the coal mill fan working with modulated load in very noisy surroundings (*Fig. 9*).

Fig. 5 presents the tragedy of uncontrolled load, where even after SVD decomposition; no one knows how to infer on fan condition. Here five vibrational symptoms have been monitored over 30 week's lifetime. But after the rejection of one load sensitive symptom (*No 4*), the situation was much more cleared and almost diagnosable, (*see Fig. 6*).

Looking now for the evolution of singular values (*Fig. 7 and 8*) we do not see any trouble with uncontrolled oscillating load, even with load sensitive symptom (*Fig. 7*). One can see almost linear grows of Frobenius SOM measure ***Frob1***, and damage sensitive measure of ***Vol1***, which starts rapidly exponentially, grows at **0.6** of dimensionless machine lifetime.

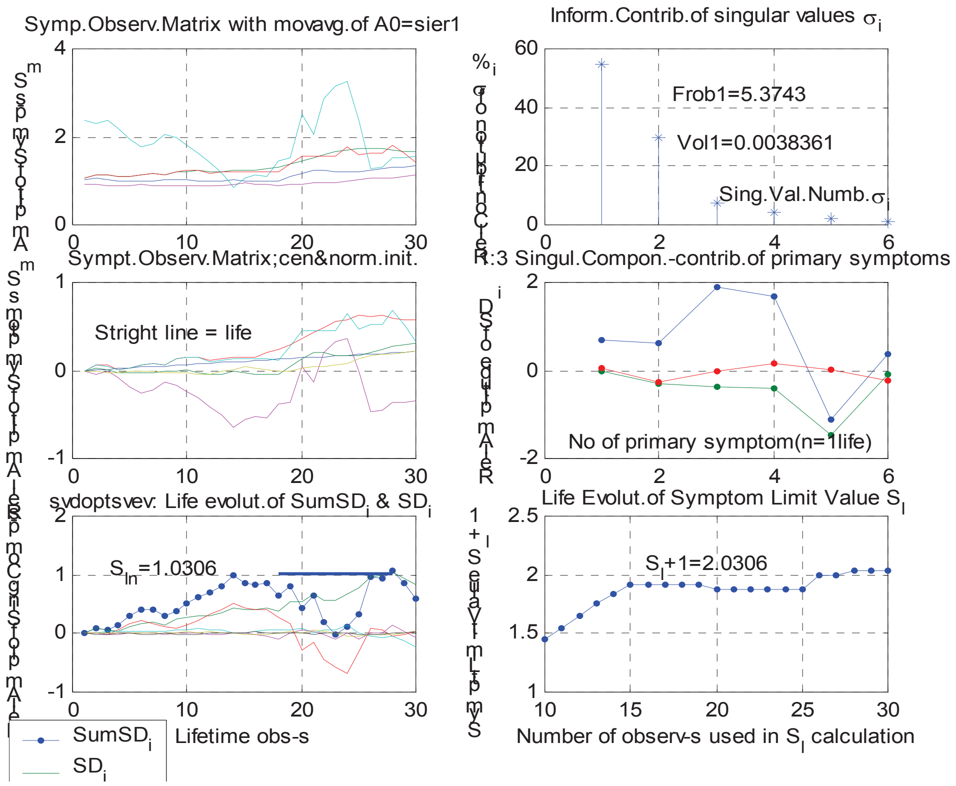

Fig. 5. Ventilation fan in one Polish copper mine with uncontrolled load, and its SOM decomposition

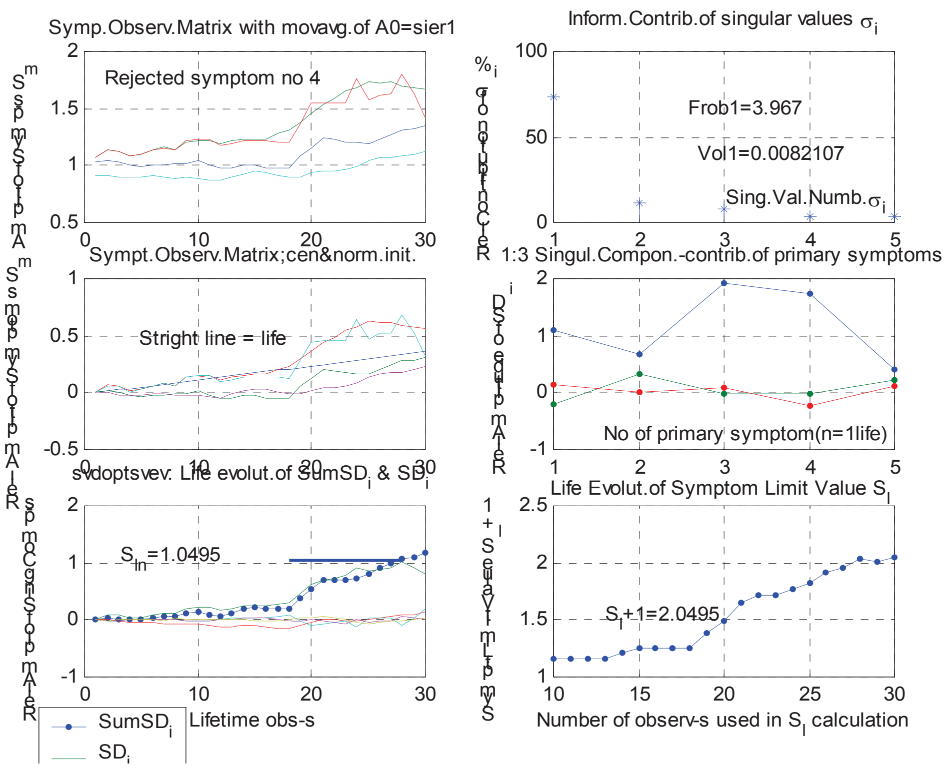

Fig. 6. SOM of the same fan as in fig.5 but without symptom no 4

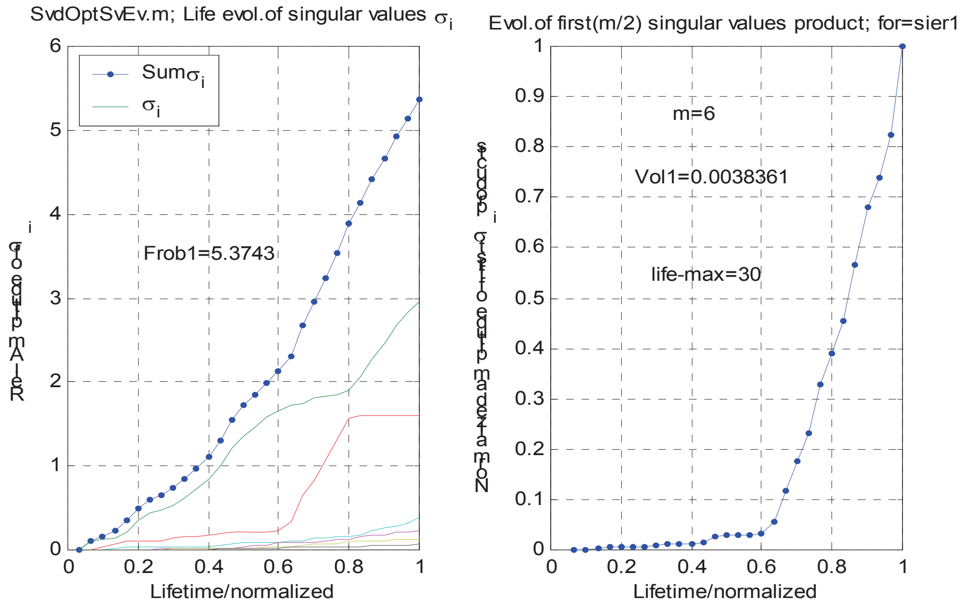

Fig. 7. The evolution of singular values for the ventilation fan with SOM presented in Fig. 5

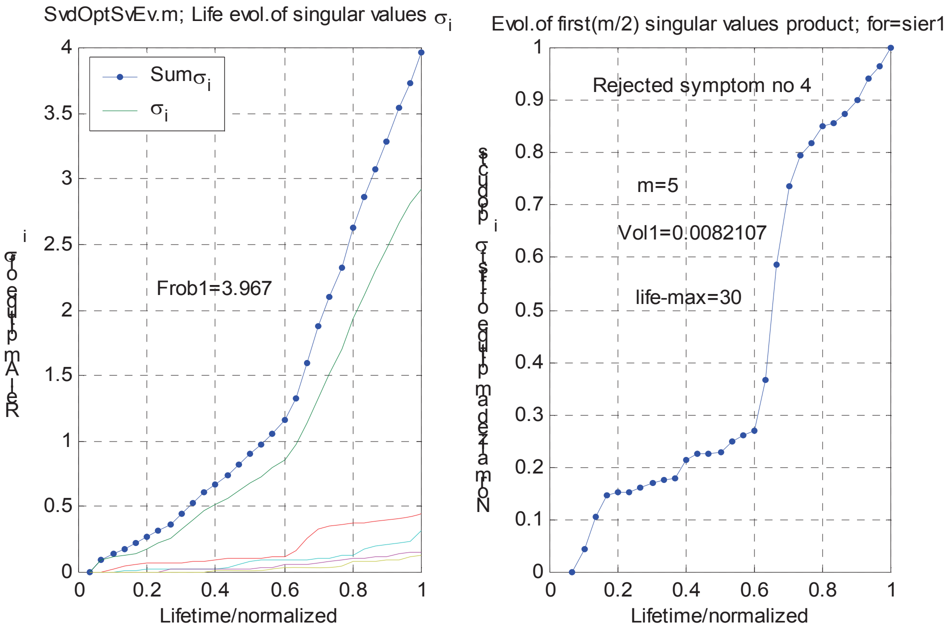

Fig. 8. Ventilation fan as before but with rejected one symptom

But when we reject the load sensitive symptom (*no 4, Fig. 8*), the coarse of both measure is improving much, especially ***Vol1***, which shows system degradation starting much earlier, exactly at **0.2** of machine dimensionless lifetime, having damage acceleration point at 0.6 lifetime as before.

Let us apply now the same methodology to the special case of ball bearing testing with specially modulated load, as shown in Fig. 9. One can see here that such load modulation is visible in every stage of signal decomposition (*left picctures*); after normalization and centering, and after SVD decomposition as well. However the symptom limit vale S_l is determined easily, what can be seen in the picture bottom right of Fig. 9. And now the important question is, what will be the life behavior of singular vales and their derivative measures; ***Frob1*** , ***Vol1*** in this case? This can be seen in detail in Fig.10, where one can notice some small modulation imposed on the steady growth of all life dependent curves. This concerns only to the singular values and their sum, but not on the product of singular values, and such behavior is independent of redundancy reduction. However, after the rejection of three redundant symptoms, the course of ***Vol1*** curve is much more life sensitive, as it was reported before (*see Fig. 11*).

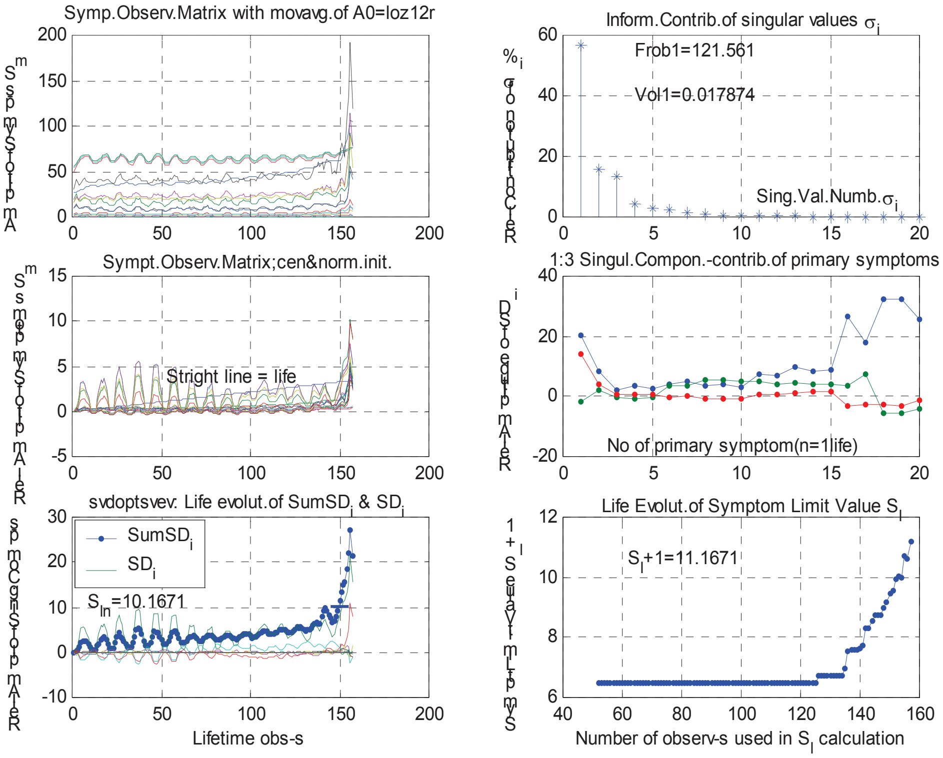

Fig. 9. The decomposition of SOM of a ball bearing at the durability testing stand with an modulated load

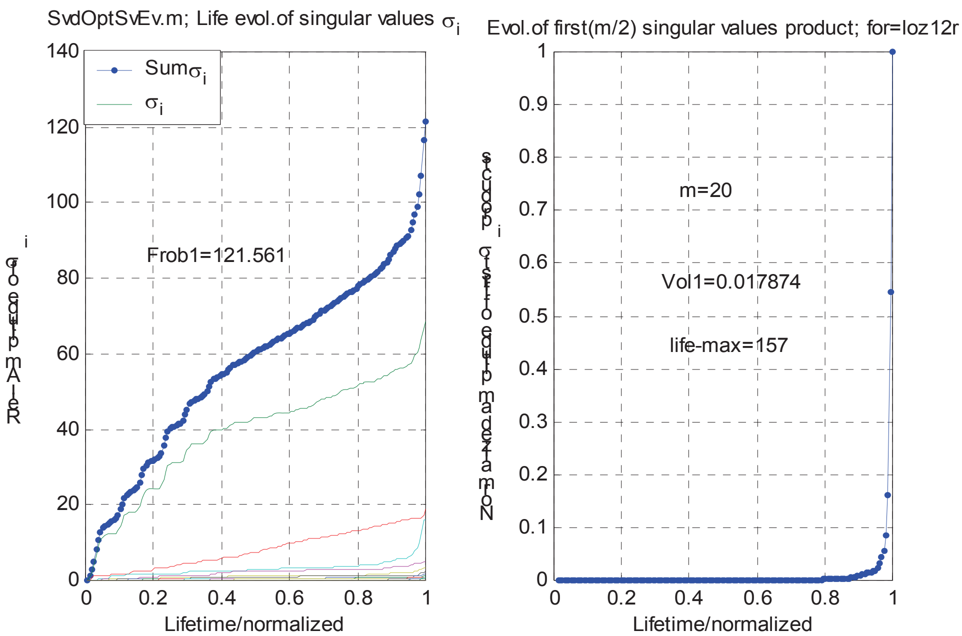

Fig. 10. The evolution of singular values and their derivative measures for the case of Fig. 9

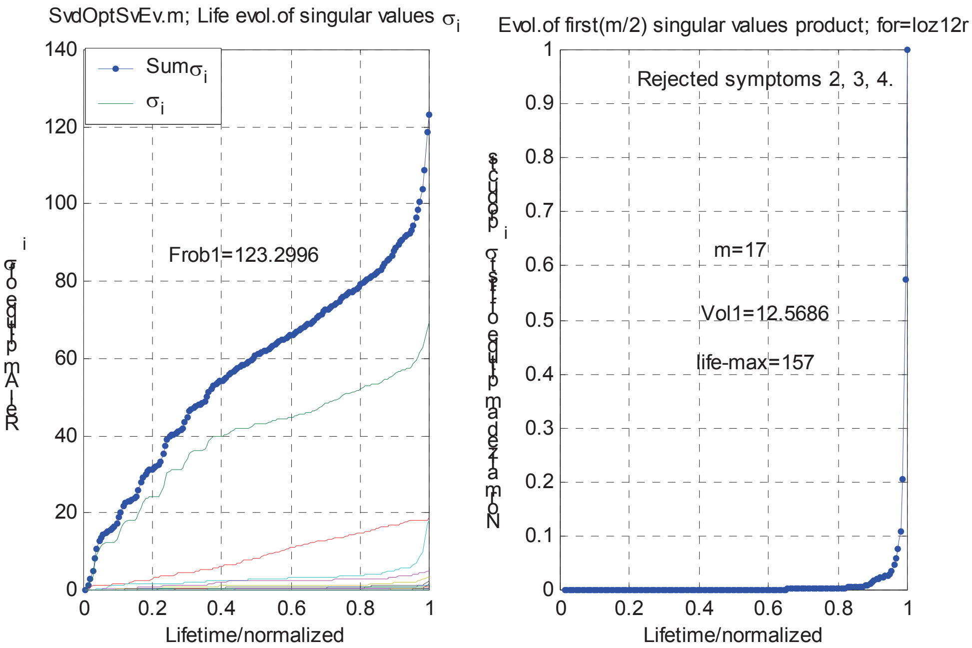

Fig.11. The same as in Fig. 10, but with rejected three redundant symptoms

When comparing the course of total damage symptom of the machine (*curve with dots in Fig. 9 picture bottom left*) with total damage symptoms obtained as modified Frobenius measure (*Fig. 10 an 11*) one can note that ongoing breakdown of the machine is much more visible when looking for the new measure ***Frob1***.

It seems to the author that enough conclusive evidences have been presented above, concerning some new unknown property of singular values, when SVD is applied to SOM of system in operation. This is particularly well seen when singular values and their derivatives are presented along system lifetime. This conclusion needs more rethinking and condition monitoring verification, but up to now the presented results seems to be sound. In future investigations one should verify if the linear growths property (12) is connected with system damage or with the number of rows in a SOM matrix. This seems to be essential for a further diagnostic application of newly detected properties of singular vales.

5. CONCLUSIONS

In a recent papers [11, 13 -18] of the present author, the problem of diagnostic decision in case of load sensitive machines has been considered. And the last paper [19] brings some new useful property of SOM singular values; namely their immunity against machine load change. It s very important property, hence following this, a simple model of singular values evolution has been proposed and validated by means of several diagnostic cases; first with constant load and with unstable machine load latter on. It was found that proposed linear growth model of individual singular value fits well to the real cases of machinery diagnostic, the same as exponential model for the product measure of singular values ***Vol1***. It was also found, that this measure is sensitive to the redundancy in observation space and can depict well the beginning of the damage in monitored system.

Acknowledgment:

This paper was partially financed by research grant No 1751/B/T02/2009/37, given by Polish Ministry of Science and Higher Education.

6. REFERENCES

[1] Cempel C., (1999), *Innovative developments in systems condition monitoring*, Keynote Lecture, Proceedings of DAMAS'99, Dublin, Key Engineering Materials, Vols. 167-168, Trans Tech. Public., Switzerland, 1999, pp172-188.

[2] Cempel C., Natke H. G., Yao J. P. T., (2000), *Symptom Reliability and Hazard for Systems Condition Monitoring,* Mechanical Systems and Signal Processing, Vol. 14, No 3, 2000, pp 495-505.

[3] Cempel C., Tabaszewski M., (2005*), Multidimensional vibration condition monitoring of non-stationary systems in operation*, Mechanical Systems and Signal Processing*,* 2007, 21 (3), pp. 1233-1241).

[4] Korbicz J., et al, (*eds*.), (2004), Fault Diagnosis – Models, Artificial Intelligence, Applications, Springer Verlag, Berlin - Heidelberg, 2004, p828.

[5] Tumer I. Y., Huff E. M., (2002), *Principal component analysis of tri-axial vibration data from helicopter transmission,* 56th Meeting of the Society of Machine Failure Prevention Technology, 2002.

[6] Jasiński M. (2004), *Empirical models in gearbox diagnostics* (in Polish), PhD Thesis, Warsaw University of Technology, Warsaw, December 2004.

[7] Cempel C., (1991), *Vibroacoustic Condition Monitoring*, Ellis Horwood Press, New York, 1991, p212.
[8] Cempel C., (1987), *Simple condition forecasting techniques in vibroacoustical diagnostics*, Mechanical Systems and Signal Processing, 1987, pp 75 – 82.
[9] Kielbasinski A., Schwetlik H., *Numeric Linear Algebra*, WNT Press Warsaw, 1992, p502.
[10] Golub G. H., VanLoan Ch. F., *Matrix Computation*, III-rd edit, J Hopkins Univ. Press, Baltimore, 1996, p694.
[11] Cempel C., Tabaszewski M., (2007), *Multidimensional condition monitoring of the machines in non-stationary operation*, Mechanical Systems and Signal Processing, Vol. 21, 2007, pp 1233-1247.
[12] Will T., (2005), *Hanger matrix, two-thirds theorem*, Internet; http://www.uwlax.edu/faculty/will/svd/svd/index.html, June 2005; (*see also: SVD ingredients,* Mathematica, *April 2004*).
[13] Cempel C., *Optimization of Symptom Observation Matrix in Vibration Condition Monitoring*, Proceeding of ICRMS09, Chengdu-China, July 2009, pp 944-960.
[14] Tabaszewski M., (2006), *Forecasting of residual life of the fan mill by means of neural nets* (in Polish), Diagnostyka, Vol. 3, (39), 2006, pp 149-156.
[15] Cempel C., Tabaszewski M., (2007), *Application of grey system theory in multidimensional machine condition monitoring* (in Polish), Diagnostyka, No 2, (47) 2007, pp11-18.
[16] Cempel C., Double singular value decomposition (SVD) of symptom observation matrix in machine condition monitoring, Diagnostyka, No 1(53), 2010, pp3-12.
[17] Cempel C., Decomposition of Symptom Observation Matrix of some Population of Applications as the Help in the Decision Process of Reviewers Board, (in Polish), Ergonomia, Technika i Technologia - Zarządzanie, M. Fertch (edit.), Poznań University of Technology Press, Poznań 2009, pp245-253.
[18] Cempel C., Tabaszewski M., *Singular Spectrum Analysis as a smoothing method of machine load variability*, Diagnostyka, No 4(56), 2010, p3-8.
[19] Cempel C., *The evolution of generalized fault symptoms and fault intensities as indicators of observation redundancy and coming system breakdown*, Mechanical Systems and Signal Processing, 25 (2011), p3116-3124.