PREDICTION OF CHANGES IN THE TECHNICAL CONDITION USING DISCRIMINANT ANALYSIS

Paweł MIKOŁAJCZAK
Katedra Budowy, Eksploatacji Pojazdów i Maszyn. Wydział Nauk Technicznych. UWM w Olsztynie
pawel.mikolajczak@uwm.edu.pl

Summary

Discriminant analysis can be used for identification of variables which identify (discriminate) two or more naturally emerging groups of f.ex. diagnostic symptoms or factors influencing development of particular type of machine part use. The goal in this case is searching for rules for assignment of multidimensional objects to one of many populations of known parameters, at the lowest classification mistake level possible. The idea of discriminant analysis is definition if the groups differ on account of a mean of a variable, and using this variable for appurtenance to a group predicting (f.ex. new cases of diagnostic symptoms, factors influencing the use level). In the following paper, an example of discriminant analysis application to diagnostic parameters choosing and identification of external factors influencing intensity of rolling bearings use in rotary machines is introduced.

Keywords: mutual measures, symptoms, sensitivity analysis, discriminant analysis, technical diagnostics.

PREDYKCJA ZMIAN STANU TECHNICZNEGO Z WYKORZYSTANIEM ANALIZY DYSKRYMINACYJNEJ

Streszczenie

Analizę dyskryminacyjną stosuje się do rozstrzygania, które zmienne wyróżniają (dyskryminują) dwie lub więcej naturalnie wyłaniających się grup np. symptomów diagnostycznych lub czynników wpływających na rozwój danego rodzaju zużycia części maszyn. Stawianym celem w tym przypadku jest poszukiwanie reguł postępowania mającego na celu przyporządkowanie wielowymiarowych obiektów do jednej z wielu populacji o znanych parametrach przy możliwie minimalnych błędach klasyfikacji. Główna idea leżąca u podstaw analizy dyskryminacyjnej to rozstrzyganie, czy grupy różnią się ze względu na średnią pewnej zmiennej, a następnie wykorzystanie tej zmiennej do przewidywania przynależności do grupy (np. nowych przypadków symptomów diagnostycznych, czynników wpływających na wartość zużycia). W pracy przedstawiono przykład zastosowania analizy dyskryminacyjnej do wyboru parametrów diagnostycznych i predykcji stanu łożysk tocznych wentylatorów w zależności od czynników wymuszających zmianę tego stanu.

Słowa kluczowe: miary wzajemne, ocena wrażliwości symptomów, analiza dyskryminacyjna, diagnostyka techniczna.

1. MUTUAL MEASURES

Technical diagnostics of objects is, among others, comparing two states – current and standard. Hence, in quality or quantity assessment it is necessary to identify how much the signal analyzed is similar to standard signal. The problem is then creating signals estimators and checking for their mutual similarity. The problem can be solved by creating separate measures for the signal analyzed and the signal standard, and comparing the measures' values in diagnostic process [9].

Another solution is creating shared measures, definition of technical condition with shared measure is assessment of similarity of standard signal to the signal analyzed or comparing separate measures of both signals.

Shared measure describing relation between signal value *x(t)*in the moment *t* and the value of the second signal *y(t)* in the moment $t+\tau$ is the **function of mutual correlation (intercorrelation):**

$$R_{xy} = \lim_{T \to \infty} \frac{1}{T} \int_0^T x(t) \cdot y(t+\tau) dt \cong \hat{R}_{xy}(\tau) = \frac{1}{T} \int_0^T x(t) \cdot y(t+\tau) dt = \overline{x(t) \cdot y(t+\tau)} \quad (1)$$

where: $\hat{R}_{xy}(\tau)$ - estimator of mutual correlation function

Normalized mutual correlation function is described with the formula:

$$\rho_{xy}(\tau) = \frac{R_{xy}(\tau)}{\sqrt{R_x(0) \cdot R_y(0)}} = \frac{\overline{x(t) \cdot y(t+\tau)}}{\sqrt{\overline{x^2(t) \cdot y^2(t)}}} \quad (2)$$

Value of the function is in the range described with the formula $-1 \le \rho_{xy}(\tau) \le 1$ and it is equal to -1/1 when the signals are identical. Normalized correlation function can be used for localization of vibro-acoustic signal sources and for definition of signal track.

Normalized mutual correlation function is a local measure of similarity of signal in lag time, which means it shows similarity level between signals separately for each *τ*. Disadvantage normalized mutual correlation function is that it needs to be identified for signals generated concurrently, i.e. for the same value of object exploitation time Θ. Because of its local character diagnostic state assessment requires not a number but function or a set of numbers defining mutual correlation function [7].

Hence, there is also necessity to identify measures of signals similarity for the signals generated in various time of objects exploitation and given with a one number. The measure discussed is a **normalized mutual correlation function** described with the formula [1]:

$$K_{12}(\beta) = \frac{\int_{-\infty}^{\infty} R_{11}(\tau) \cdot R_{22}(\tau+\beta) d\tau}{\left[\int_{-\infty}^{\infty} R_{11}^2(\tau) d\tau \int_{-\infty}^{\infty} R_{22}^2(\tau) d\tau \right]^{\frac{1}{2}}} \tag{3}$$

Square of the formula presented above, for (β=0), gives the following result:

$$K_{12} = \frac{\left[\int_{-\infty}^{\infty} R_{11}(\tau) \cdot R_{22}(\tau) d\tau \right]^2}{\int_{-\infty}^{\infty} R_{11}^2(\tau) d\tau \int_{-\infty}^{\infty} R_{22}^2(\tau) d\tau} \le 1 \tag{4}$$

where: $R_{11}(\tau)$, $R_{22}(\tau)$ – functions of vibro-acoustic signals correlation in the time Θ_1 and Θ_2 of technical object exploitation.

The last formula presents **global measure of signals similarity for various time of exploitation**, thus it is a global measure of object exploitation state similarity.

Using plexus characteristics and Wiener-Chinczyn relation and applying a local kind of normalization in frequency range, normalized mutual correlation function can be described with the following formula [1]:

$$\gamma_{xy}^2(f) = \frac{\left| G_{xy}(f) \right|^2}{G_{xx}(f) \cdot G_{yy}(f)} \le 1 \tag{5}$$

where:

$G_{xx}(f)$, $G_{yy}(f)$ – functions of density of signals x(t) and y(t) power;

$G_{xy}(f)$ – function of mutual density of signals x(t) and y(t) power;

$\gamma_{xy}^2(f)$ - squared coherence function.

Value of coherence function may vary in the range given with the formula $0 \le \gamma_{xy}^2(f) \le 1$. For linear configurations with constant parameters $\gamma_{xy}^2(f) = 1$, which means signals $x(t)$ and $y(t)$ are completely coherent. In the case when for a given frequency $\gamma_{xy}^2(f) = 0$, signal $x(t)$ and $y(t)$ are incoherent. If signals $x(t)$ and $y(t)$ are stochastically independent, $\gamma_{xy}^2(f) = 0$ for all frequencies. When coherence function value is in the range given $0 \le \gamma_{xy}^2(f) \le 1$ measures results include disturbances, which means that the output signal is influenced not only by the input signal, but also other signals or a configuration combining signals $x(t)$ and $y(t)$ is not linear.

Damage in kinematic couple of technical object set causes changes in vibro-acoustic signal and consequently in coherence function. Thus coherence function measured for various time of object's life detects damage, changes in transmittance and relative changes of signal caused by damage. coherence function, like mutual correlation function, is a local measure of similarity of signals of vibration sources separately for each frequency.

It identifies relative quantity of information concerning input (primary) signal in output signal.

Global measure of vibro-acoustic signals generated in the same moment of object lifetime, in all frequency ranges **similarity** is so called sources similarity level S_{xy}^2, because at any transmittance of a configuration $S_{xy}^2 = 1$ [1]:

(3)

$$S_{xy}^2 = \frac{\int_{-\infty}^{\infty} \left| R_{xy}(\tau) \right|^2 d\tau}{\int_{-\infty}^{\infty} R_{xy}(\tau) R_{yy}(\tau) d\tau} = \frac{\int_{-\infty}^{\infty} \left| G_{xy}(f) \right| df}{\int_{-\infty}^{\infty} G_{xx}(f) G_{yy}(f) df} \tag{6}$$

Whereas when there are no input or output disturbances $S_{xy}^2 < 1$, these characteristics can be used in diagnostics for identification of vibration and noise sources.

Using the mutual correlation function defined as opposite Fourier's transformation of mutual spectra power of signals density and the definition of correlation function shifted by 90° for one of the signals, the following result can be obtained:

(4)

$$\rho_{xy}^2 = \frac{\left[\int_{-\infty}^{\infty} \left| G_{xy}(f) \right| \cos \vartheta df \right]^2 + \left[\int_{-\infty}^{\infty} \left| G_{xy}(f) \right| \sin \vartheta df \right]^2}{\int_{-\infty}^{\infty} G_{xx}(f) df \int_{-\infty}^{\infty} G_{yy}(f) df} \tag{7}$$

The formula presented above is a sum of square mutual power, active and passive, normalized to a product of x(t) and y(t) signals power.

Analysis of the characteristics of ρ_{xy}^2 needs to the conclusion that this is the global measure of signals similarity and it can be used in diagnostics for measuring similarity of processes performed in identical configuration.

Taking normalized function of mutual correlation into consideration, the global measure of similarity of vibro-acoustic signals in given frequency generated in various time of object exploitation K_{12}^2 can be defined as [1]:

$$K_{12}^2 = \frac{\left[\int_{-\infty}^{\infty} G_{11}(f) \cdot G_{22}(f) df\right]^2}{\int_{-\infty}^{\infty} G_{11}^2(f) df \cdot G_{22}^2(f) df} \quad (8)$$

where: $G_{11}(f)$, $G_{22}(f)$ – spectral density of power of vibro-acoustic signals defining technical states of an object in exploitation time Θ_1 and Θ_2. The measure enables assessment of changes in condition of the object in its exploitation time.

Mutual measures enable comparing two states or symptoms, analyzed and standard, however they do not facilitate choosing one of the diagnostic symptoms from a large group of symptoms. One can also apply the physical probabilistic models in task of decresing of uncertainty of reise and evolution of failure [10] or methods of automatic data classification [2],[6].

2. METHODS OF DIAGNOSTIC SYMPTOMS SENSITIVITY TO CHANGES OF TECHNICAL STATE ASSESSMENT

There are the following methods of diagnostics parameters choice presented in the literature [12], [9]:

- Maximum relative changes in diagnostic parameters method,
- Maximum variation of diagnostic parameter method,
- Maximum sensitivity of diagnostic parameter to changes in technical state method,
- Maximum information capacity of diagnostic parameter method.

Maximum relative changes in diagnostic parameters method – in this method, a diagnostic parameter which has the greatest value of k_j indicator is chosen. It takes average speed of parameters changes in time. It is calculated with the following formula:

$$k_j = \frac{b_j}{\sum_{j=1}^{m} b_j}$$

$$b_j = \frac{1}{K}\sum_{i=1}^{K} \frac{\left|y_j(v_{i+1}) - y_i(v_i)\right|}{(v_{i+1} - v_i)\left|y_i(v_i) - y_{j*g}\right|} \quad (9)$$

where: K – the number of elements of time series in (v_1, v_b) interval.

Diagnostic parameter y^* is chosen with the following formula:

$$y^* = y_{j^*}, j^* \in 1,...,m \Lambda k_{j^*} = \max(k_j), j = 1,...,m \quad (10)$$

Maximum variation of diagnostic parameter method – diagnostic parameters analyzed should presented a sufficient level of variability in machine exploitation time. The parameters with the greatest value of variability indicator g_j are chosen from the set of final results:

$$g_j = \frac{S_j}{\sum_{j=1}^{m} S_j} \quad (11)$$

$$S_j = \frac{1}{K}\sum_{i=1}^{K} \frac{\left|y_j(v_{i+1}) - y_j(v_i)\right|}{v_{i+1} - v_i} \quad (12)$$

Where: K– the number of elements of time series in (v_1, v_b) interval.

The parameter y^* is chosen with the following formula:

$$y^* = y_{j^*}, j^* \in 1,...,m \Lambda g_{j^*} = \max(g_j), j = 1,...,m \quad (13)$$

Maximum sensitivity of diagnostic parameter to changes in technical state method – the idea of the method is to choose, from the set of output parameters of a configuration or an object set, a parameter that has the greatest value of a_j indicator, which includes parameters dependence on machine state:

$$a_j = \sum_{i=1}^{k} M(i,j); \text{ i = 1,2,…,k; j = 1,…,m} \quad (14)$$

where: $M(i, j) \in [M(i, j)]_{kxm}$ – an element of binary diagnostic matrix of a technical object.

The parameter y^* then is chosen for the set of diagnostic parameters by choosing y_j with maximum value of a_j indicator:

$$y^* = y_{j^*}, j^* \in 1,...,m \Lambda a_{j^*} = \max(a_j), j = 1,...,m \quad (15)$$

Maximum information capacity of diagnostic parameter method – based on the choice of the parameter that provides the most information on machine's technical state. The diagnostic parameter is the more important for definition of object's technical state, the more it is connected to the object and the less it is connected to the other diagnostics parameters.

The relation discussed above is represented by the indicator of integral capacity of diagnostics parameter h_j introduced with the formula:

$$h_j = \left| \frac{r_j^2}{1 + \sum_{i,j=1}^{m} \left| r_{i,j} \right|} \right| \qquad (16)$$

where: $r_j = r(W,y_j);\ j = 1,\dots,m$ – linear correlation between variables coefficient, W – state of an object set, $r_{i,j}$ - linear correlation between y_i and y_j variables coefficient.

The parameter y^* is chosen to the set of diagnostics parameters by maximization of h_j indication with the following formula:

$$y^* = y_{j^*}, j^* \in 1,\dots,m \Lambda h_{j^*} = \max(h_j), j = 1,\dots,m \qquad (17)$$

Each of the methods diagnostic parameters choice presented above considers changes in parameters' values in technical state of an object analyzed function on a different level, however, they do not enable simultaneous assessment of numerous diagnostic symptoms and exploitation conditions influence on track of technical object use value. In the next chapter the example of discriminant analysis use for forecasting changes in rolling bearings state with respect to the set of diagnostic symptoms and external enforcements given is presented.

3. DISCRIMINANT ANALYSIS

Discriminant analysis is a statistical analysis which allows to identify differences between two or more groups, analyzing several variables at the same time. Variables used for groups distinguishing are called **discriminant variables**. Practically, discriminant analysis is a general term referring to several related statistical procedures. Simplifying, the procedures can be distinguished into [5], [7], [11]:

- Descriptive procedures and inter-group differences identifying procedures. The procedures explain: if it is possible, with a set of several variables, distinguish one group from another? How well discriminant variables distinguish groups given? Which of the variables are the best in discriminating?
- Procedures of cases classification, which is definition of characteristics' values, based on observation or experience, that enable decision to which of the groups a new case should be included. It is connected with definition of one or more functions classifying the cases analyzed to the right groups.

The example of discriminant analysis to be presented is based on exploitation data concerning the group of fans introduced in the table 1. The purpose of the analysis is showing, how the use progress of rolling bearings can be predicted with the diagnostics parameters analyzed.

Table. 1. Comparison of diagnostics and exploiation parameters for the analyzed group of fans

Fan number	v(fo) [mm/s]	v(2fo) [mm/s]	T [^{0}C]	Choking [%]	State of the rolling bearings
w1	3,6	5,2	65	5	inadmissible
w2	7,5	3	70	20	inadmissible
w3	4	4	75	25	inadmissible
w4	8,5	2,6	60	10	inadmissible
w5	4,2	3,2	84	30	inadmissible
w6	3,5	4,2	65	40	sufficient
w7	5,1	2,2	63	20	sufficient
w8	3,9	3,5	62	30	sufficient
w9	5,5	1,5	55	15	sufficient
w10	4	3,7	70	30	sufficient
w11	4,3	1,2	61	30	good
w12	2,5	0,9	74	10	good
w13	1,8	0,3	56	15	good
w14	1,5	2,6	61	20	good
w15	2	1,1	73	35	good

Symbols used in the table: v(fo) – the first harmonica of vibrations,
v(2fo) – the second harmonica of vibrations, T – temperature of rolling bearing cover,
Choking – percentage of flow choking.

Analysis starts with definition of **canonical discriminant function** distinguishing the groups analyzed. Any function can be used for the definition, however, the most commonly used are linear functions. The functions to be used are described with the following formula [5], [7], [11]:

$$D_{kj} = \beta_0 + \beta_1 x_{1kj} + ... + \beta_p x_{pkj} \qquad (18)$$

where: p – number of dicriminant variables (in the following example p = 4), n – size of the sample (in the following example n = 15), g – number of groups (in the following example g = 3), D_{kj} - canonical discriminant function for k-*th* case in j-th group value, k=1, …,n and j=1,…,g, x_{ikj} – value of i-th discriminant variable for k-th case in j-th group, i=1,...,p, β_i – coefficients of canonical discriminant functions defined with characteristics of the function.

The problem of β_i coefficients definition is definition is solving matrix equation given with the formula [3], [11]:

$$(\mathbf{M}-\lambda\mathbf{W})\beta=0 \qquad (19)$$

where: **W** – intragroup matrix of squares and mixed products, **M**- intergroup matrix of squares and mixed products, β - vector of canonical discriminant functions coefficients, λ - so called matrix value.

In the table 2 there are calculated, raw and standardized values of discriminant functions coefficients. The calculations were performed with STATISTICA PL software, version 9.0.

Table 2. Coefficients of discriminant functions

	Function 1 (raw coefficients)	Function 2 (raw coefficients)	Function 1 (standardized coefficients)	Function 2 (standardized coefficients)
v(fo) [mm/s]	-0,84973	-0,07620	-1,30723	-0,117224
v(2fo) [mm/s]	-1,21239	-0,32739	-1,22125	-0,329777
T [^{0}C]	-0,09550	0,12583	-0,73908	0,973896
Choking [%]	0,05088	-0,09304	0,51721	-0,945767
constant	11,86678	-5,09074	8,31713	0,625460
value	8,31713	0,62546	0,93006	1,000000
cumulated ratio of cleared variation	0,93006	1,00000	-1,30723	-0,117224

Canonical discriminant functions are described with the following formulas:

D_1 = 11,86678 - 0,84973v(fo) - 1,21239v(2fo) - 0,09550T + 0,05088Choking
D_2 = - 5,09074 - 0,07620(fo) - 0,32739V(2fo) + 0,12583 - 0,0930Choking

The first function is responsible for 93% of cleared variance (while the second for 7% only), which means the first function is the most important.

Raw coefficients of discriminant function can be used with the data from observation mainly for calculating function value, but they cannot be used for interpretation. One of the reasons is the fact that coordinates beginning is not equal to the central point represented by average values of discriminant variables for all the points. To eliminate this problem standardization of coefficients is used [3], [4], [5]:

$$\hat{\beta}_i = \beta_i \sqrt{n-g} \ \text{ i } \ \hat{\beta}_0 = -\sum_{i=1}^{p} \hat{\beta}_i \bar{x}_i \qquad (20)$$

where: $\bar{x}_i$ - average value of i discriminant variable, $\hat{\beta}_i$ - standardized coefficients of canonical discriminant function, the rest as described above.

To identify how each discriminant function distinguishes the groups, the diagram presented in the figure 1 can be used.

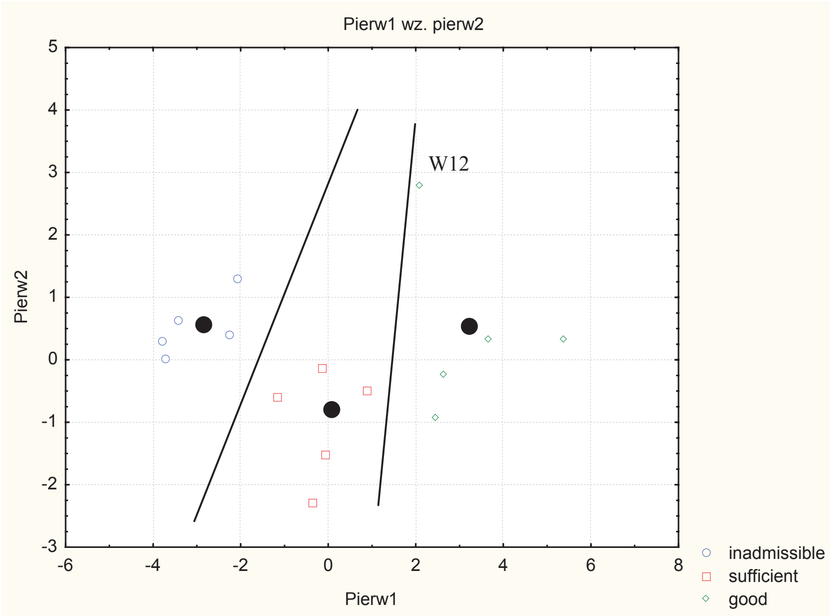

Fig. 1. The diagram of canonical values dispersion

Fig. 1 verifies the results of variation cleared. In the diagram, the fans with bearings in good, sufficient and inadmissible are distinguished with the first discriminant function (the vertical lines can be drawn to identify these groups – 93% of cleared variation). The one exception is the point identified for the fan number 12. The reason for that is the fact that despite good dynamic condition (low vales of the first and the second harmonica) there is a great interaction and a loadof the fan. Thus, there is a risk of changing the condition, from the "good" level to the "sufficient" level (which means increasing the distance from "good" group centroid, marked with the black point).

Another measure in multidimensional variables space can be used – **Mahalanobis distance** [4], [7], [11]. It is a distance between each case and a group centroid. It is worth mentioning, that the smaller Mahalanobis distance, the more certain appurtenance to the group. In the figure 2, square Mahalanobis distance for the case analyzed is presented.

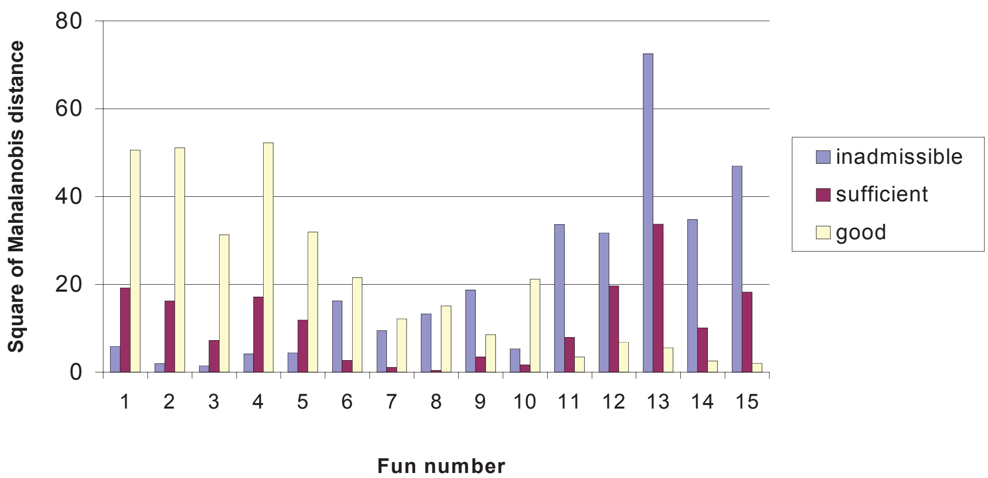

Fig. 2. Mahalanobis distance column graph

Analysis of the figure 2 leads to the conclusion, that predefined functions discriminate three groups of rolling bearings states analyzed very well. In the first five fans, condition of rolling bearings was inappropriate, and in these cases columns labeled with "inadmissible" are the smallest. The group of five fans with rolling bearings of sufficient condition is the one in the middle, in this group the smallest values are represented by the columns labeled "sufficient". The last group is also easy to be distinguished, columns labeled as „good" are the smallest.

4. SUMMARY

1. Application of mutual measures allows to identify and measure the difference between the signal analyzed and the standard. Mutual measures are the methods applicable for identification of signals which can be described as changing their values significantly, with respect to changes of sate, exploitation time and enforcements a technical object is exposed to.
2. Signals sensitivity assessment methods should be used when choosing the best diagnostic parameter from the set of parameters given. It can be done with the following criterion: relative parameter change, maximum variance, maximum sensitivity of diagnostic parameter to technical condition changes or maximum information capacity.
3. Discriminant analysis allows to identify which variables are characteristic for diagnostic symptoms groups or for factors influencing development of considered type of machine parts use. Thus, a set of diagnostic symptoms can be used even though one of the symptoms does not identify technical condition of the object analyzed, because multidimensional analysis of the set as a whole is diagnostically useful.
4. Because of the limited size of the following chapter, the issues concerning additional methods of discriminant functions developed assessment are not included. However, the methods listed below are worth mentioning:

- Wilks's lambda and multidimensional F statistics, both based on the level of density of points around the centroid;
- Partial Wilks's lambda – identifying influence of a variable on group discrimination,
- V Rao statictics, measuring groups' cetroids dispersion;
- *A posteriori* probability of identification if a case analyzed belongs to a given group.

5. The examples of diagnostics parameters presented in the chapter are not directly connected with rolling bearings condition. They introduce various states of unworthiness (including unbalancing, clutch failure, wrong smearing or assembly of rolling bearings) [6], [10], however, thanks to discriminant analysis influence of these unworthiness on rolling bearings use can be forecasted. With other words, discriminant analysis allows to identify influence of particular causes of rolling bearings failures.

REFERENCES

[1] Cempel C. *Diagnostyka wibroakustyczna maszyn.* PWN, Warszawa 1989.
[2] Gibiec M., Barszcz T., Bielecka M.: *Selection of clustering methods for wind turbines operational data*. Diagnostyka 4(56)/20100 p. 37-42.
[3] Giri N. C. *Multivariant Statistical Data Analysis of Multivariate Observations.* New York, Wiley 1996.
[4] Huberty C. J. *Applied discriminant analysis*. New York, Wiley 1967.
[5] Klecka W.R. *Discriminant analysis.* Sage, London 1980.
[6] Mikołajczak P. *Klasyfikacja zbiorów symptomów diagnostycznych z wykorzy-staniem metody Dattoli.* Diagnostyka 4(40)/2006, p.185-190.
[7] Morrison D. F. *Wielowymiarowa analiza statystyczna. PWN*, Warszawa 1990.
[8] Morel J.: *Drgania maszyn i diagnostyka ich stanu technicznego*. PTDT,Warszawa 1992.
[9] Niziński S., Michalski R.: *Diagnostyka obiektów technicznych. I*TE Radom 2002.
[10] Radkowski S., Zawisza M. *Failure oriented diagnostic models in condition monitoring*. Diagnostyka 4(52)/2009, p. 93-97.
[11] Staniszkin A. *Przystępny kurs statystyki z zastosowaniem STATISTICA PL*. Tom 3. Analizy wielowymiarowe. StatSof Kraków 2007.
[12] Tylicki H.: *Optymalizacja procesu prognozowania stanu technicznego pojazdów mechanicznych.* Rozprawa nr. 86, ATR, Bydgoszcz 1998.
[13] Żółtowski B.: *Podstawy diagnostyki maszyn*. ART, Bydgoszcz 1996.

Dr inż. **Paweł MIKOŁAJCZAK** pracuje na stanowisku adiunkta w Katedrze Budowy, Eksploatacji Pojazdów i Maszyn Wydziału Nauk Technicznych UWM w Olsztynie. Od wielu lat zajmuje się diagnostyką wibroakustyczną, systemami informatycznymi w utrzymaniu maszyn jest autorem wielu artykułów i ekspertyz z tego zakresu. Członek PTDT, redaktor „Diagnostyki".