DECORRELATION OF INDICES FOR A SINGLE NUMBER ASSESSMENT OF THE ACOUSTIC QUALITY OF SACRAL OBJECTS

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Summary

A new approach for solving the problem of a single number index formula by creating the index from mutually-correlated indices by means of decorrelation of the Index Observation Matrix (IOM) was shown. The orthogonal singular vectors obtained from SVD were used in order to build a single number index. Application of the proposed formula for a local single number index of selected acoustic parameters of the interiors of six Roman Catholic churches was shown. Decorrelation of the indices for a single number assessment of the acoustic quality of sacral objects will be applied for complex global acoustic assessment of such interiors' index method, which is currently being improved.

Keywords: Singular Value Decomposition (SVD), decorrelation, index method of assessment

DEKORELACJA WSKAŹNIKÓW W JEDNOLICZBOWEJ OCENIE JAKOŚCI AKUSTYCZNEJ OBIEKTÓW SAKRALNYCH

Streszczenie

Pokazane w artykule nowe podejście do rozwiązania problemu opracowania wskaźnika jednoliczbowego, polega na utworzeniu tego wskaźnika ze wskaźników skorelowanych ze sobą, na drodze dekorelacji wskaźnikowej macierzy obserwacji. Do konstrukcji wskaźnika jednoliczbowego wykorzystano, uzyskane z rozkładu SVD, ortogonalne wektory szczególne. Weryfikację zaproponowanego wzoru na jednoliczbowy lokalny wskaźnik wybranych parametrów akustycznych obiektów sakralnych pokazano na sześciu rzeczywistych kościołach rzymsko-katolickich. Dekorelacja wskaźników w jednoliczbowej ocenie jakości akustycznej obiektów sakralnych będzie wykorzystana do kompleksowej globalnej oceny jakości akustycznej obiektów sakralnych - metody wskaźnikowej, która jest w dalszym ciągu udoskonalana.

Słowa kluczowe: rozkład względem wartości szczególnych, dekorelacja, wskaźnikowa metoda oceny.

1. INTRODUCTION

The state of object quality can be diagnosed by means of assessment indices. There are many acoustic parameters in interior acoustics, which are used to assess the acoustic quality of investigated objects. The measured values of those parameters and the comparison of their values to preferred ones are the basis of the assessment.

Long-term scientific research performed at the Department of Mechanics and Vibroacoustics has been carried out in order to elaborate a complex method of acoustic quality assessment of public buildings, such as sacral objects [3].

An index method for the acoustic quality of sacral objects proposed by Engel and Kosała in 2007 in [4] was based on the designed partial indices of assessment. The partial indices calculated from some relation are used in the assessment of particular acoustic properties of the object. An approximate assessment is conducted by means of single number global index. The index method is still being improved by using the Singular Value Decomposition (SVD) technique [9, 7].

An acoustic assessment by means of the index method (described in [5]) has one main drawback because certain information included in the correlated indices is duplicated. The application of singular vectors, obtained from SVD, is proposed to solve this problem. At the beginning, correlation research between partial indices is needed. When the partial index is not correlated (or weakly correlated) with the other ones, it can be used for the global assessment. However, a single local index of selected acoustic parameters, on the basis of strong correlated indices, should be created. A local index, as a single number, will be the next component of global assessment.

A single number index is obtained, by using the SVD technique, in two ways. One of them, which is shown in [8], is based on a single number index, and contains a new set of perfectly mutually-correlated indices, which is obtained from the Index Observation Matrix (IOM).

The next proposition for creating a local index is a solution based on the decorrelation of mutually-correlated indices, which is described in the article. A local index is based on uncorrelated singular vectors, which are obtained from the decomposition of the IOM by means the SVD technique.

2. DECORRELATION OF INDICES USING THE SVD TECHNIQUE

The IOM (Index Observation Matrix) was obtained from the values of mutually-correlated partial indices, which are described in [8]. The columns of the IOM are formed using the consecutive partial indices. The rows of the IOM are the sacral objects listed in Table 1 in [8].

The next step is decomposition of the IOM by the Singular Value Decomposition in, for example, Matlab environment.

The SVD is a calculation technique, which is commonly used in linear numerical algebra [6,10] and has applications in many fields of science such as diagnostics [1] and vibroacoustics [2],[4].

Due to the Singular Value Decomposition theory, three other matrices are obtained from the matrix $IOM \in R^{m \times n}$

$$IOM = U \cdot \Sigma \cdot V^T \qquad (1)$$

where: U – orthonormal matrix $m \times n$, Σ - diagonal matrix $n \times n$, V – orthonormal matrix $n \times n$.

Equation (1) can be expressed in the following way:

$$\begin{bmatrix} \\ IOM \\ \\ \end{bmatrix} = \begin{bmatrix} \\ U \\ \\ \end{bmatrix} [\Sigma][V^T] \qquad (2)$$

where: Σ - singular values of the matrix IOM, U – left singular vectors of the IOM - u_i, V^T - right singular vectors of the IOM - v_i.

Σ is the diagonal matrix:

$$\sigma_{ij} = \begin{cases} \sigma_i > 0 & for\ i = 1 \ldots n \\ \sigma_i = 0 & for\ i > n \end{cases} \qquad (3)$$

Diagonal elements fulfill the condition:

$$\sigma_1 \geq \sigma_2 \geq \ldots \sigma_n \qquad (4)$$

Decorrelation of mutually-correlated variables, such as indices - the components of the IOM, can be obtained by SVD. A new set of the left and right singular vectors in the matrices U and V^T suitably, are obtained. All the singular vectors in the matrix U are orthogonal to each other, as such they are entirely uncorrelated. The singular vectors of the matrix V^T have the same properties. Such properties were used to create a single number index of assessment, which contains a new set of the indices uncorrelated to each other.

It is proposed to construct the matrix with uncorrelated indices

$$B = u_1 v_1 + u_2 v_2 + \ldots + u_n v_n \qquad (5)$$

where: $u_1 \div u_n$ – singular vectors of the matrix U, obtained from SVD of the IOM,

$v_1 \div v_n$ – singular vectors of the matrix V^T, obtained from SVD of the IOM.

The matrix B is the sum of the products of orthogonal singular vectors. The columns of the matrix B create a new set of uncorrelated indices.

A single number index of assessment is calculated from the matrix B, according to the formula

$$W_{WPA\,j}^{uncor} = \sum_{i=1}^{n} b_{ji} \qquad (6)$$

where: $W_{WPA\,j}^{uncor}$ – a single number index to assess selected acoustic parameters of the j-th object, corresponding to the j-th row of the matrix B, b_{ji} – an uncorrelated partial index, corresponding to the j-th object (j-th row) of the matrix B and the i-th column of the matrix B.

A single number index W_{WPA}^{uncor} is the sum of the products of the singular vectors, corresponding respectively to the approximation rank from 1 to n, without taking into account the common components, such as approximations – the singular values $\sigma_1, .., \sigma_n$. It is proposed to use only such orthogonal (uncorrelated) singular values for assessment.

3. USE OF DECORRELATION OF INDICES IN ASSESSING THE ACOUSTIC QUALITY OF SACRAL OBJECTS

A graph with an IOM was shown in Fig. 1. The matrix was created on the basis of three mutually-correlated partial indices: the reverberation index – W_P, the music sound index W_M and the speech intelligibility index W_Z. The indices' computation procedures, applied to six sacral objects, were shown in [8]. The values of the coefficients of linear correlation between the partial indices are higher than 0.93 [8].

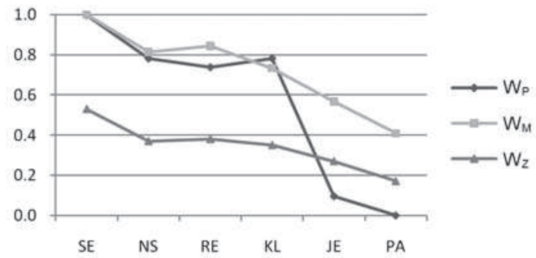

Fig. 1. Content of the Index Observation Matrix - IOM

The singular vectors and singular values were obtained by applying the Singular Value Decomposition to the IOM. The singular vectors were used for building the matrix B (Formula (5)).

Content of the matrix B with the uncorrelated indices as well as the singular vectors and the singular values, obtained from Singular Value Decomposition of the matrix B are shown in the Fig.2-5.

The singular values $\sigma_1=\sigma_2=\sigma_3=1$ (approximately) and graphs in Fig. 4. determine a lack of correlation between the columns of the matrix B. This is also determined by the condition number cond(B)=1.

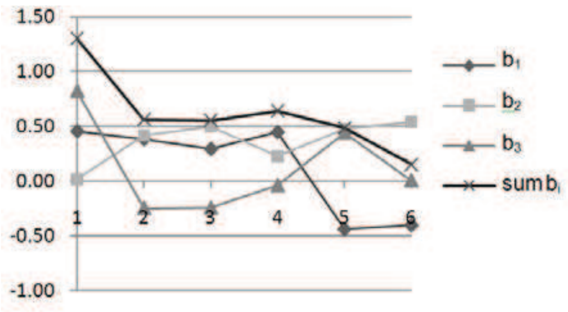

Fig. 2. Content of the matrix B with the uncorrelated indices

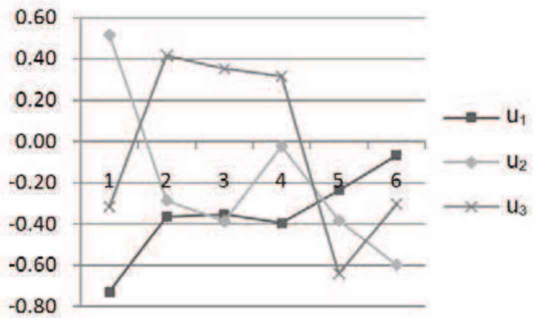

Fig. 3. Left singular vectors

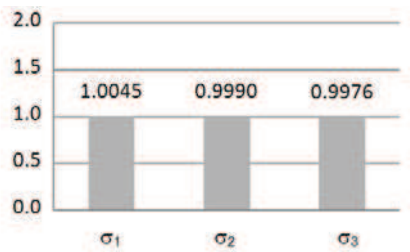

Fig. 4. Singular values

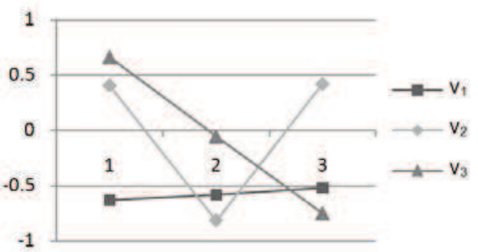

Fig. 5. Right singular vectors

The values of the index W_{WPA}^{uncor} within the range 0.15 to 1.3 from the formula (6) have been obtained. The maximum value of the index exceeded the assumed ones, equal to 1, therefore normalization in the shape of a transforming quotient was applied. All values of the indices W_{WPA}^{uncor} were divided by its maximum value. The result before (index W_{WPA}^{uncor}) and after (index W_{WPA}^{uncor*}) normalization are shown in Fig. 6.

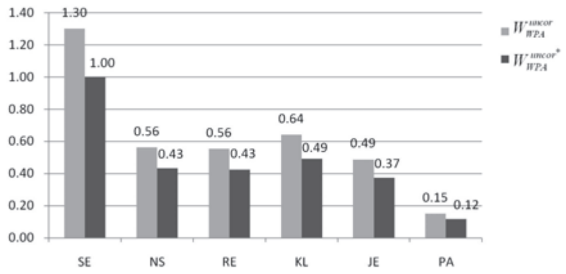

Fig. 6. The values of single number indices W_{WPA}^{uncor} and W_{WPA}^{uncor*}

The uncorrelated indices, which are written in the matrix B from the correlated partial indices – the components of the Index Observation Matrix, were obtained. They were obtained from the sum of the products of left and right singular vectors. However, the first index of the IOM - W_P and corresponding index b_1 of the matrix B, are correlated. The value of the coefficient of linear correlation between them is very strong and equals: 0.9820. Therefore, the proposed method, to obtain a single number index (the local index), which is complex and contains additional uncorrelated indices, seems to be proper.

According to Fig. 3, the best acoustic properties appear in the church of St. Sebastian in Strzelce Wielkie (W_{WPA}^{uncor*}=1), whereas acoustically the worst is a modern one, built on a circular lay-out: St. John Kanty's Church in Bochnia (W_{WPA}^{uncor*}=0.1). The assessment of those two objects is consistent with the subjective opinions of the churchgoers as well as with assessment by using the index method, proposed in 2007 [5].

4. CONLUSIONS

A new approach to the problem of a single number assessment of the acoustic quality of a sacral object, where a few mutually-correlated indices have been used, was shown in the paper. So far, a single number global assessment by means of correlated and uncorrelated indices has had one main drawback because certain information included in correlated indices was duplicated. A separation of correlated and uncorrelated indices was proposed. A local single number index of selected acoustic parameters from correlated indices was created. A single number index consists of the uncorrelated single vectors obtained from Index Observation Matrix decomposition. The proposed formula was applied to assess the real Roman-Catholic churches. The values of the index within the range 0.1 - for an object, which is acoustically bad, to 1 – for an object which has good acoustic properties, have been obtained. The assessment of those two objects is consistent with the subjective opinions of the churgoers.

More accurate subjective assessment of investigated objects will be the next stage of research and then more precise verification of obtained results will be conducted. The convolution functions for speech and music signals with impulse responses of the investigated churches will be used for subjective research.

The approach of the decorrelation of correlated indices, which was shown in the paper, together with different uncorrelated indices, will be used to create the global index of acoustic assessment of sacral objects.

Further investigation will be based on application of a single number assessment to other types of public building or different technical objects.

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