MODE SHAPES SUBTRACTION AND WAVELET ANALYSIS FOR DAMAGE DETECTION

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Summary

One of the modal parameters that indicate damage in a structure are mode shapes. They can be used both for detection and localization of damage, since their shape is locally disturbed by the potential damage. Their another advantage, comparing with natural frequencies or damping factors, is fact that they are less sensitive to environmental factors like ambient temperature changes. The problem is that for small damages the changes in shape of mode are difficult to be detected. To improve this detectability authors decided to subtract the mode shapes of the damage structure from analogous mode shape of the structure in reference state. Additionally to even more improve the sensitivity of the method and allow for some automation of damage detection, the wavelet analysis was introduced.

Keywords: mode shapes, wavelet transform, damage detection.

ODEJMOWANIE POSTACI DRGAŃ WŁASNYCH I ANALIZA FALKOWA DO WYKRYWANIA USZKODZEŃ

Streszczenie

Jednym z parametrów modalnych, które pozwalają na wykrycie uszkodzenia w obiekcie są postacie drgań własnych. Mogą być one stosowane zarówno do wykrywania jak i lokalizacji uszkodzenia, ponieważ są tylko lokalnie zaburzane przez potencjalne uszkodzenie. Ich kolejną przewagą w odniesieniu do częstości drgań własnych czy współczynników tłumienia modalnego jest ich znacznie mniejsza wrażliwość na czynniki środowiskowe takie jak zmiana temperatury otoczenia. Problemem natomiast jest fakt, że dla małych uszkodzeń zmiany są niewielkie i dlatego trudne do wykrycia. Aby poprawić tę wykrywalność autorzy zdecydowali się na odejmowanie postaci drgań własnych układu uszkodzonego od analogicznej postaci układu w stanie referencyjnym. Dodatkowo aby ułatwić interpretację wyników i pozwolić na pewnego rodzaju automatyzację działania metody zastosowano analizę falkową.

Słowa kluczowe: postacie drgań własnych, transformata falkowa, wykrywanie uszkodzeń.

1. INTRODUCTION

The most commonly met division of damage detection methods based on low-frequency vibration measurements is classification using the model type and the type of parameters applied for diagnostics. This division can roughly be presented as:
- methods based on modal models,
- methods based on non-parametric models,
- methods based on regressive parametric models,
- other methods.

The most convenient model, which can be applied in the damage detection, is a modal model, i.e. a set of natural frequencies, modal damping coefficients and modal vectors describing the dynamics of the tested object. The modal model is relatively easy to identify and, by means of operational modal analysis, may be identified only from response data; it is, therefore, very useful in diagnostics. The modal parameter perturbation (natural frequency, modal damping) is the simplest application of the modal model for damage detection. The results can be obtained from output-only data by means of operational modal analysis. In some cases it enables damage detection, but the detection strongly depends on object geometry, material and the nature of the damage itself. This technique does not provide information about the location of the damage, but the biggest disadvantage of the method is probably the fact that its effectiveness is dependent on environmental conditions. There is a serious problem distinguishing between the changes in parameters resulting from damage and those caused by environmental changes e.g. temperature or humidity. The changes in ambient temperature for civil engineering objects (bridges, viaducts, masts, tall buildings) may reach even tens of degrees in a relatively short period of time. It results in further changes in stiffness and finally in modal parameter variation. This effect is multiplied when the object is unevenly heated – e.g. when one side is exposed to the sun and the other is kept at a constant temperature due to its proximity to water. The influence of humidity variations is similar – concrete elements absorb moisture and this leads to increases in their mass and variations in modal parameters. Naturally, there are methods which enable the influence of environmental changes on the diagnostic procedure accuracy to be eliminated. In most applications, a lookup table is prepared in which modal parameters identified for different ambient temperatures and values of humidity are gathered together. Such a table is unique and independently prepared for every object as a result of a set of experiments. A more sophisticated method of eliminating the influence of weather conditions on the monitoring system's efficiency is application of an environmental filter [1]. It is generally an autoregressive model with a moving average (ARMA) identified from a set of experimental data. When environmental changes are eliminated in this way, any modification of the object (e.g. lining another asphalt layer on a bridge deck) causes the necessity to repeat the entire set of measurements, to update the lookup table or the environmental filter. There is however another modal parameter that allows not only for damage detection but also for location and what is also very important is much less sensitive for environmental changes [2]. These modal parameters are mode shapes.

2. OVERVIEW OF THE DAMAGE DETECTION METHODS BASED ON MODE SHAPES

Among methods based on changes in mode shapes, a few basic ones can be emphasised:
- tests of correlations between mode shapes in undamaged and current state (MAC or CoMAC),
- analysis of mode shapes curvature,
- analysis of mode shapes deformation energy.

The MAC (Modal Assurance Criterion) coefficient is defined as a scalar product of two modal vectors [3], from which the first is identified for an undamaged system, and the second is a mode shape for the object with damage. If the MAC coefficient is less than one, there is a change in the vibration mode. The MAC coefficient may be calculated both for one coordinate and for a certain area. It was applied to damage detection in the work [4]. For one selected coordinate, it is called *CoMAC* (*Coordinate MAC*). Designating it can additionally define in which area the damage is located. In practice, however, the method is not very sensitive and does not allow damage to be detected in the initial phases of development.

A development of these methods is the analysis of changes in mode shape together with changes in the mode participation factor. This method was described in the paper [5]. The authors divided the analytical model into sub-systems and analysed changes in higher mode shape in successive sub-systems. This method allows the localisation of damage, because higher mode shapes are changed only for sub-systems containing damage.

A slightly different use of mode shapes for damage detection was proposed by Ettouney [6]. He calculated the stiffness matrix or compliance matrix on the basis of knowledge of the modal model of undamaged and damaged objects. Changes in the calculated matrices indicated the presence of damage and its location.

In the place of modal vectors, a change in the curvature defined as the derivative or the second derivative of a modal vector are often analysed. This is more sensitive to changes than the mode itself. In particular, this concerns damage to objects which changes mode shapes locally. This method suffers from a relatively large error in cases where the number of measurement points is not sufficiently large to designate the following derivative of vibration modes with suitable accuracy. The derivative is calculated in points by linear approximation passing through the successive points or also through polynomial approximation of the deformation curve and derivative designation analytically. This second method is considerably less sensitive to measurement errors, however, it flattens the shape of modes, which may be a cause of damage being undetected. The effectiveness of applying these methods also includes the localisation of damage.

The first example of the application of mode shape curvature to NDT is the work of Maeck and DeRoeck from 1999 [7]. This method uses dependencies between the beam's bending stiffness and the bending moment divided by a suitable curve, being a second derivative of beam deformation. Changes in the stiffness matrix calculated on the basis of this dependency enable the detection of damage. This method was verified by the authors with the use of data from research conducted on the Z24 bridge in Switzerland.

The next application of modal curvature is the method described in the article [8]. The authors calculated the damage index as a relation of the modal curvature calculated for the damaged object to the analogical curvature of the undamaged object raised to a square. The curvature was counted as a second mode shape derivative. In the work, particular emphasis was placed on the influence of measurement errors on the accuracy of method. They showed that the higher mode shape derivatives are more sensitive to the presence of damage, but also cause multiplications of measurement noise and, due to this, their usefulness is doubtful. In their next publications [9], the same authors proposed a solution to the problem of increasing significance of measurement noise during analysis of changes in curvature. They presented another way of mode shape analysis, which had a high level of sensitivity to damage and low one to measurement error. As a symptom of damage, they proposed a change in the mode shape slope raised to a square. This slope was counted as the first mode shape derivative.

The damage index method from the work [9] was extended by Kim and others [10]. The novelty is based on the application of the method for objects where reference data (without damage) was not available. The authors showed a way to calculate the modal curvature of the object before damage on the basis of data coming only from measurements on the damaged object. In this method it was necessary to use the updated finite element model.

The most precise method based on modal vectors is presented in the work [11]. This method consists of comparing the deformation energy of vibration modes in systems without damage and systems with damage. In the described method a finite element model of the construction can be considered as a system without damage. In order to designate the SER_{ij} energy coefficient, the i-th vibration mode for the j-th element, one should possess the following data: mode shape ϕ_i, natural frequency ω_i, the global stiffness matrix of the finite element model K, as well as the stiffness matrix for the j-th finite element k_j:

$$SER_{ij} = \frac{\phi_i^T k_j \phi_i}{\phi_i^T K \phi_i} = \frac{\phi_i^T k_j \phi_i}{\omega_i^2} \tag{1}$$

The coefficient β_{ij} is named by the authors as a damage coefficient and is designated with the dependency:

$$\beta_{ij} = SER^u{}_{ij} - SER^d{}_{ij} \tag{2}$$

where: the index $_d$ represents data for constructions with damage, and $_u$ represents data for constructions without damage.

As simulation and experimental tests showed, this method is sensitive even to small stiffness changes in the construction (about 5%).

A similar approach can be found in the work [12]. The authors also calculated the damage indicator on the basis of the deformation energy of mode shapes. However, in this case, it was defined a little differently:

$$f_{ij} = \int_{a_{j-1}}^{a_j} \left(\frac{d^2\varphi_i}{dx^2} \right)^2 dx \Big/ \int_0^L \left(\frac{d^2\varphi_i}{dx^2} \right)^2 dx \tag{3}$$

where: i – mode shape number,
j – element number,
L – length of section on which the mode shape curvature is calculated,
φ – mode shape,
x – position on the section L,
a – integration limit.

The damage indicator was calculated as a relation of the sums f_{ij} along all mode shapes for the damaged object to analogical sums of the undamaged object.

Carrasco et al. [13] also applied mode shape deformation energy for damage detection and localisation. Their approach consists of dividing the tested object into sub-systems and the calculation of deformation energy separately for each sub-system. Changes in deformation energy in successive sub-systems allowed the authors to locate damage. Additionally, the authors showed that there is a close dependency between the size of the damage and the size of the change.

The mode shape deformation energy for damage detection and localisation was also applied by Choi and Stubbs [14]. Their work concerned damage detection in a two-dimensional element with the use of classic plate theory. As an example, they applied a finite element model to a rectangular plate.

A completely different approach for damage localisation on the basis of mode shapes was presented by Rucka and Wilde [15]. They analysed the mode shape of beam-like and plate-like objects in search of cracks. The tool which was used for this aim was the wavelet transform. The discovered discontinuities, where the geometry of the object was known, were the symptom of damage. This method requires very dense networks of measurement points. Its application to the mode shapes obtained through laser vibrometer measurements are presented in the work [16]. The main advantage of the approach is the fact, that it is a baseline free method. The disadvantages are: relatively small sensitivity to small sized damages and difficulties in unique interpretation of the obtained results.

3. FORMULATION OF THE METHOD

Generally the mode shapes are used for damage localization due to the fact that damage, in most cases, only disturbs the mode shapes locally. Unfortunately for the small sized damages the effect is hardly visible in the mode shapes (the change of mode shapes is very small). In many fields of application to show the small changes between two quantities their difference is presented. That was the main idea behind the proposed method. Instead of analyzing the mode shapes itself it is better to take into account their difference. One can find in the literature the example of mode shapes derivatives subtraction in order to better detect and localize damage [17]. To illustrate above statement in Figure 1 there are presented: 1st mode shape of the undamaged system, 1st mode shape of the damaged system and difference of these two modes. It is clearly visible that for such a small damage the mode shapes are almost identical and only their difference gives potential for damage localization.

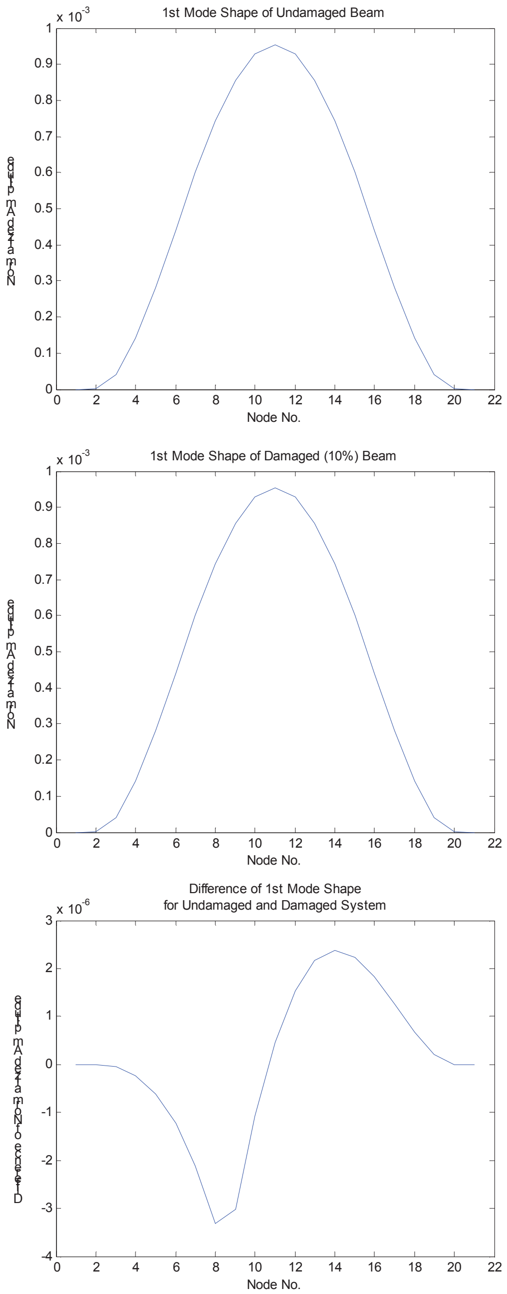


Fig. 1. Example of difference between mass normalized mode shapes for an undamaged structure and a structure with 10 % crack

The idea of the method proposed by authors can be summarized in two steps. In the first step the mode shapes of the structure in current stage are subtracted from the reference counterpart to emphasize eventual changes. As it is shown in Figure 1 the localization of damage based purely on mode shapes difference can be sometimes misleading. In the presented example there are maxima of the characteristic around Node 8 (where the crack was simulated) and 15. So simple amplitude analysis would give false results. One needs to look for discontinuities in the analyzed curve. The very good tool to do that is the wavelet transform. That is why in the next step for better results interpretation and some procedure automation the wavelet analysis is used in similar manner as in the work [15]. The scheme of diagnostic procedure based on proposed approach is presented in Figure 2.

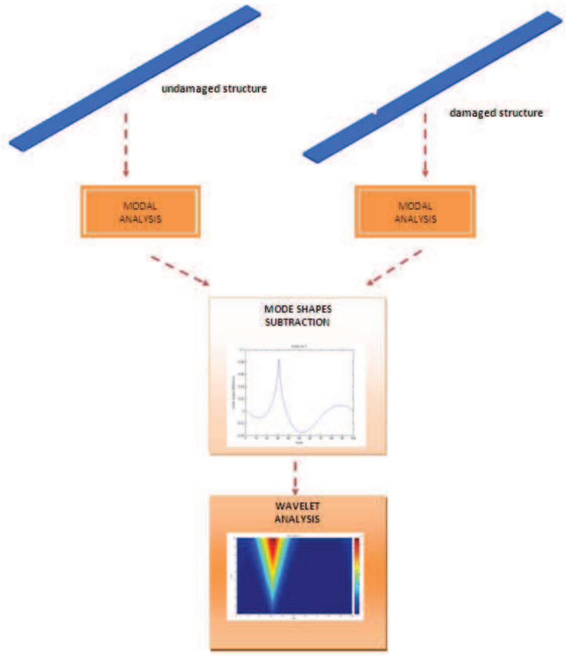


Fig. 2. The scheme of diagnostic procedure

Proposed approach can be characterized by following advantages: it is sensitive even for relatively small damages and provides the possibility for better results interpretation, even automatic crack detection, by usage of wavelet analysis. In opposite to that one can state the following cons: the method is no longer baseline free, and the computational cost is a little bit bigger.

4. SIMULATION VERIFICATION

For the purpose of simulation verification of the described procedure, the following model was developed. This was a steel supported beam of length 10 m with the cross-section dimensions 0.6 x 0.1 m and consisted of 600 plate elements (quad4), with the size of each element 0.1 x 0.1 m. Such a dense mesh was used to allow for different sensor distribution testing. Next, the eigenvalue problem was solved for the model without damage to obtain its modal model parameters. As damage, a beam crack was modeled as node disconnectivity. The depth of the crack amounted to 10% of the entire beam cross-section area. Location of the crack was exactly 3 m from the right-hand end of the beam. For damaged model, the eigenvalue problem was also solved. Having both damaged and undamaged model results, the authors localized damage with the use of a different number of sensors (nodes). In Figure 3 the model is presented together with the damaged area designation.

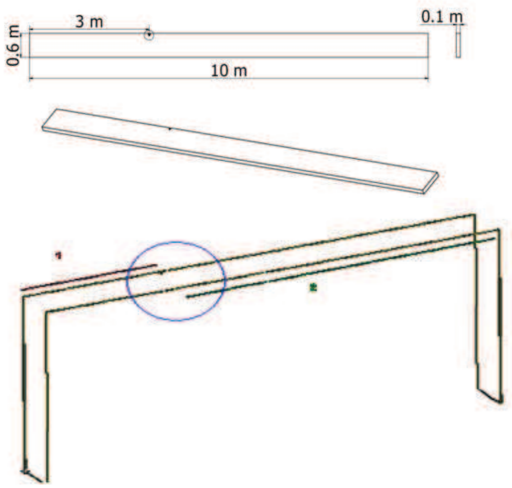


Fig. 3. Model prepared for the simulation verification

The preliminary investigation was focused on the selection of the wavelet that detects the discontinuity in shapes of the mode. The wide literature studies [18, 19, 20, 21, 22] allowed for preliminary selection of 4 types of wavelets:

- Gaussian wavelet of order 4,
- Mexican Hat wavelet,
- Complex Continues Gaussian wavelet of order 4,
- Daubechies wavelet of order 4.

In Figure 4 the results of damage localization with use of different wavelets are shown.

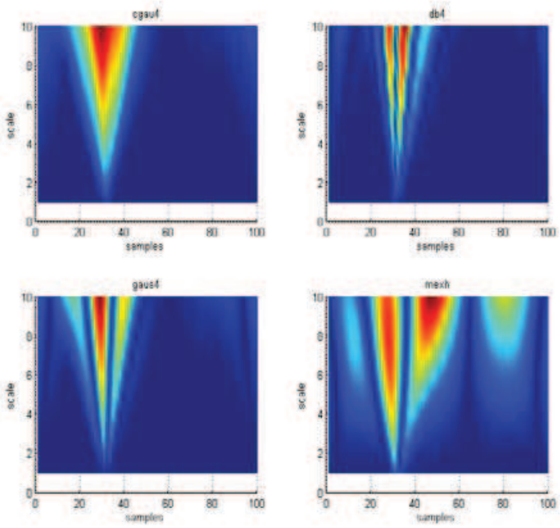


Fig. 4. damage localization with use of different wavelets

Although in case of low modes the Gaussian wavelet of order 4 and Mexican Hat wavelet gave good result, the Complex Continues Gaussian wavelet of order 4 was selected as it was able to detect discontinuities in the biggest number of mode differences and provided only one peak that points to a singularity in shape of mode.

In the next step the wavelet analysis was performed on the damaged mode shapes directly to prove that modes subtraction is necessary. In this work the Continuous Complex Gaussian wavelet with 4 and 6 vanishing moments was used. The type of wavelet was selected empirically. Result of this analysis is presented in Figure 4.

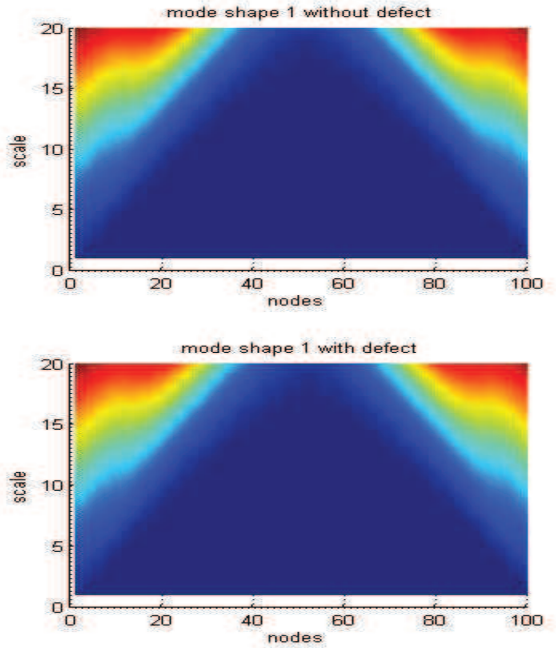


Fig. 4. Wavelet trans form of the 1st mode shape of the undamaged and damaged system

The results show that in case of so small crack (10%) the detection of the damage directly from the mode shapes analysis with use of wavelet transform is impossible.

Next the procedure formulated in the section 3 was launched for crack detection and localization. The results example is presented in Figure 5.

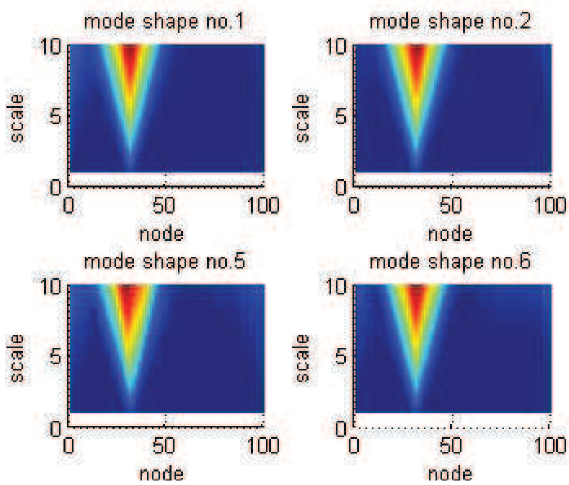


Fig. 5. Results of damage detection and localization

As it can be seen, the method easily detected changes in system’s modes and pointed the damage location. The positions of the detected crack in function of mode shape number are gathered together in Table 1.

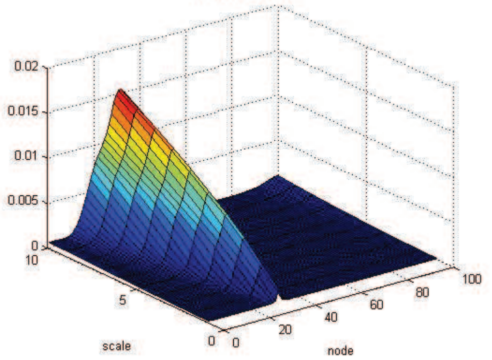
Table 1. Crack localization accuracy

MS No.	Recognized crack position [m]	Relative error [%]
1	3.11	3.7
2	3.19	6.3
3	3.11	3.7
4	No detection	-
5	3.06	2
6	3.19	6.3
7	No detection	-
8	3.11	3.7
9	No detection	-
10	3.19	6.3

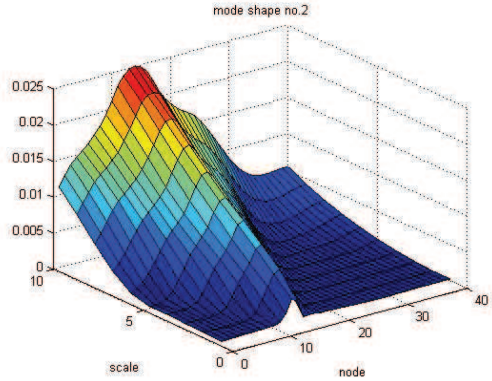
For the 100 sensors (nodes) uniformly spaced along the beam length used in the analysis the localization error was no bigger than 6.5 %. What is also important the analysis of scalograms is very unique and easy for some automation. For example in this case it is enough to look for maxima in the scalograms. The lack of detection for mode shapes no. 4, 7 and 9 results from the fact that in these mode shapes all the movement is in the tangent direction, while only the normal direction responses were taken to the analyses.

5. ANALYSIS OF THE METHOD ACCURACY

In the last step, the authors analyzed the relation between the number of sensors used for damage localization and the localization accuracy. Having both damaged and undamaged model results, the authors localized damage with the use of a different number of sensors (nodes) and their configurations. The following scenarios were tested:
- consecutively 10, 20, 40 and 100 sensors placed evenly along the length of the beam.
The results for the second mode with use of 100, 40, 20 and 10 sensors are consecutively shown in Figure 6.



a)



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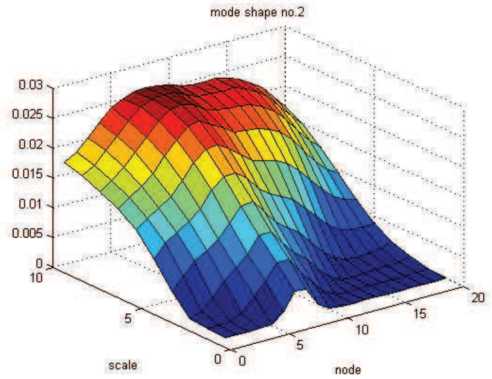
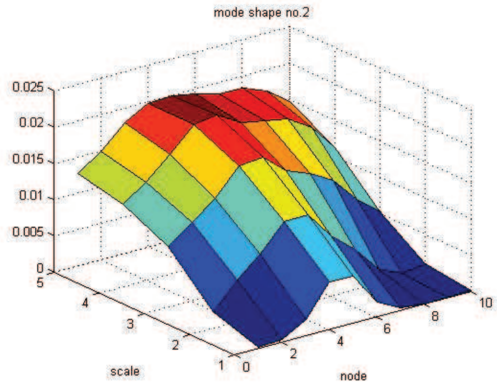


c)



d)

Fig. 6. Wavelet transform modulus of mode shapes difference for 2nd mode; (a) 100 sensors, (b) 40 sensors, (c) 20 sensors, (d) 10 sensors

In Table 2 the results of crack localization in function of sensors (nodes) number are presented.

Table 2. Crack localization accuracy in function of sensors number

No. of sensors	Recognized crack position [m]	Relative error [%]
100	3.145	4.8
40	3.45	15
20	3.94	31.6
10	4.71	57.1

In case of 40, 20 and 10 sensors (nodes) the wavelet of order 6 was used and additionally in case of 10 sensors the scale range was reduced to 1:5.

It was observed that detection of the crack was not possible only for case with 10 sensors for modes higher than 3. What is more, it was seen that the peak of the wavelet transform modulus becomes wider as the scale increases and for higher modes made it impossible to localize the damage properly. From the results of 40 sensors the modes up to 10^{th} was taken into consideration. The localization of the crack was identified as 3.45 m (15.1% relative error). In case of 20 measurement points the wavelet analysis is able detect crack at 3.5-4 m, but in case of mode higher than 3 the localization of crack can be done only according to values corresponding to scale from 1 to 5.

5. CONCLUSIONS

The presented investigations can be summarized by the following conclusions:

- the wavelet transform applied to mode shape difference instead of directly to mode shape of damaged structure provides better results,
- for analysis of mode shape differences the wavelet with at least 2 vanishing moments needs to be used, for wider sensor distribution the order of the wavelet should be increased,
- the best type of wavelet for these applications is the Complex Continues Gaussian wavelet,
- proposed method was able to detect 10 % crack for low mode using only 10 sensors,
- the crack localization relative error for 100 sensors was no bigger than 6.5 %,
- The accuracy of the results drops significantly with the smaller number of sensors used,
- The localization error is mainly produced by mode shape subtraction, as it is visible that the coordinate of the singularity in the signal is shifted in respect to damage localization.

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