EFFECT OF CAMSHAFT ECCENTRICITY AND FOLLOWER BACKLASH ON THE DYNAMIC BEHAVIOUR FLEXIBLE CAM MECHANISM

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Summary

A cam follower system is modelled in this work. The equation of motion and the eigenfrequencies are recovered. The computation of the dynamic response is made by mean of combined implicit Newmark-Newton Raphson algorithm. Moreover, dynamic behaviour of the follower train is analysed in presence of an eccentricity in the camshaft and backlash between the follower rods and its guide.

Key words: Cam, follower, dynamic response simulation, camshaft eccentricity, follower backlash.

INTRODUCTION

A cam follower system may be defined as a machine element having a curved outline or a curved groove, which, by its oscillation or rotation motion, gives a predetermined specified motion to another element called the follower. The cam has a very important function in the operation of many classes of machines, especially those of the automatic type, such as printing presses, shoe machinery, textile machinery, gear-cutting machines, screw machines, and automobiles. The performance of such cam follower systems can be evaluated by the precision of the follower motion. In high-speed cam follower mechanism, high precision of the follower motion become increasingly important especially in automatic machines. Several studies have yielded a vast literature on this topic [1-4]. Backlash errors or wear of the parts of the cam-follower system can cause an excessive vibration and noise [5, 6]. Dynamic analysis of cam-follower system have been of great interest by several researchers to investigate the possible factors which cause the deviation of the follower output from the desired one. Kim and Newcombe [7, 8] investigated the combined effects of three sources of errors: geometric inaccuracy, kinematic errors and dynamic effects. Grewal and Newcombe [9] studied the effect of machining errors of the contact surfaces on the motion of the follower. By an experimental study Norton [10, 11] investigated the effect of variation in manufacturing tolerance on the dynamic performance of eccentric and double dwell cams. Rao [12] developed relationships between the design parameters of the follower train for minimum flexibility error and optimum sensitivity synthesis of cam mechanisms. Wu and Chang [13, 14] presented an analytical method for analysing the mechanical errors of disk cam mechanisms with a flat-faced follower. In a recent work Wu and Chang [15] developed a computerized method based on the concept of the simulated higher-pair contact analysis to perform the tolerance analysis of disk cam mechanisms with roller follower.

All of geometric, kinematics errors and dynamic effects may influence the dynamic behaviour of the cam follower mechanism [16, 17, 18]. In this work, a dynamic analysis of a cam-follower mechanism with translating follower is performed. The system is modelled with seven-degrees-of-freedom model. The developed model includes the possibility of having a camshaft eccentricity error or a backlash between the follower rod and its guide. The paper is organized in the following manner: in section 2 the cam follower model is presented, the equation of motion and the parametric expression for the cam profile are developed. Section 3 is devoted to numerical simulations of the dynamic response with and without errors.

1 ANALYTICAL FORMULATION

1.1 The cam follower system model

In this work, the adopted model is derived from that developed by Kim and Newcomb [8]. This model with seven-degrees-of-freedom include nonlinear contact between cam and follower, flexible camshaft bearing and follower rod (figure 1). M_{eq} is the mass of the follower; x_i and y_i represent respectively the horizontal and vertical displacement of the camshaft and the follower. h, is the programmed motion machined on the cam. Suitable dampings are introduced in parallel with stiffness.

Applying Lagrange formulation yield to the equation of motion [8]:

$$M\ddot{q}+C\dot{q}+Kq=F(t) \tag{1}$$

with $q=\left\langle y_1, y_2, y_3, y_4, x_1, x_2, x_3\right\rangle^T$ is the vector of degrees of freedom

M is the mass matrix, K is the stiffness matrix, C is the damping matrix and F is the nonlinear time varying force vector.

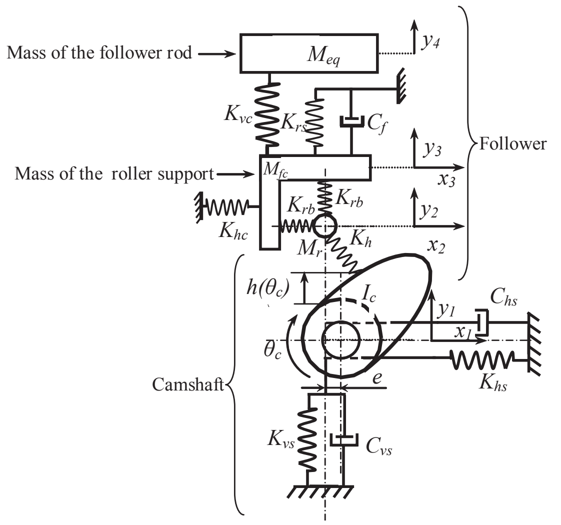


Fig. 1. Schematic representation of the cam-follower dynamic model

1.2 Parametric expression for the cam profile

In order to investigate how the manufacturing, assembling and wear errors affecting the dynamic behavior of the cam follower system, the analytical expressions for the theoretical cam profile should be determined. Figure 2 shows a disk cam mechanism with offset translating roller follower. The concept of velocity instant center is used to recover the cam profile [19-21].

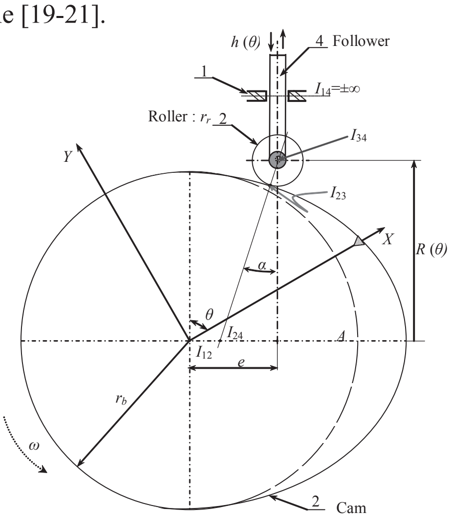


Fig. 2. Cam disk with an offset translating roller follower

The instant center between link i and j is denoted by I_{ij}, the theoretical offset reference point is denoted by A.

The links are numbered in the following way:

The ground link is numbered as 1, the cam as 2, the follower roller as 3 and the follower rod as 4.

The pressure angle, α, the nominal displacement of the follower, $h(\theta)$, the theoretical follower offset and the camshaft angular velocity, $\omega = {d\theta}/{dt}$, can be related from rigid body kinematic [21].

The velocity of instant center I_{24} is given by:

$$V_{I_{24}} = \frac{dR(\theta)}{d\theta}\omega \tag{1}$$

The displacement function of the follower can be expressed as:

$$R(\theta) = \sqrt{(r_b + r_r)^2 - e^2} + h(\theta) \tag{2}$$

Here, r_b is the radius of the base circle, r_r is the radius of the follower roller and e is the offset.
When the instant center I_{24} is assumed to be on link 4, this velocity is given as:

$$V_{I_{24}} = {dh}/{dt} \tag{3}$$

From the triangle $I_{24}AI_{34}$, the pressure angle α can be expressed as:

$$\alpha = tan^{-1}\left(\frac{I_{12}I_{24} - e}{R(\theta)}\right) \tag{4}$$

The parametric vector equations of the theoretical cam profile coordinates are [22]:

$$R_T(\theta) = \begin{Bmatrix} R_T(\theta) \\ R_T(\theta) \end{Bmatrix} = I_{12}I_{23}$$

$$I_{12}I_{23} = \begin{Bmatrix} R(\theta)cos\,\theta - e\,sin\,\theta - r_r\,cos(\theta - \alpha) \\ R(\theta)sin\,\theta + e\,cos\,\theta - r_r\,sin(\theta - \alpha) \end{Bmatrix} \tag{5}$$

1.3. Modeling of cam-follower defect

Parts cannot be made to ideal or design dimensions because inaccuracies are inherent in manufacturing process. In this section we recognize this imperfection as errors.

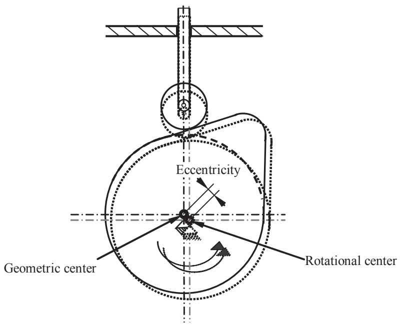
For the purpose of studying the effects of manufacturing or assembling errors, two defect cases are considered. Firstly, an eccentricity error between the camshaft and its bearing is considered. This analysis will be done by considering a disk cam with in line translating follower and an offset translating follower. Secondly, a backlash error between the follower rod and its support is considered.

1.3.1. First case (Eccentricity error of the camshaft bearing)

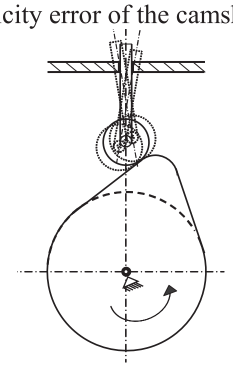
In this case we assume that we have an eccentricity on the camshaft. The eccentricity expresses the difference between the theoretical and the real rotational axis (figure 3a). This defect is introduced by adding a transmission error modeled as displacement on the line of action between the cam and the roller. Figure 4 show a schematic diagram of the eccentricity error of a disk cam with an offset translating roller follower. The introduction of this displacement in the equation of motion yields to an exciting force. As a result, an amplitude modulation of the contact force between the cam and the follower in the vertical direction is expected [23].

$$F(t) = \Delta e_c(t) K_h \tag{6}$$

$$\Delta e_c(t) = \overline{\Delta e_c}\, sin(\omega t + \zeta) \tag{7}$$



a- Eccentricity error of the camshaft bearing



b- Follower-rod error

Fig. 3. Schematic representation of the cam-follower errors

This eccentricity error provides a pressure angle and an offset variation ($\Delta\alpha$ and Δe). The instantaneous contact point varies from I_{23} to I^*_{23} with corresponding angle θ, (Here the deviation of eccentricity is exaggerated for clarity purposes). From the parametric vector $R_T(\theta)$ of the theoretical cam profile coordinates given in equations 5, the new position vector $R^*_T(\theta)$ of the contact point I^*_{23} in term of the cam rotation angle can be expressed as:

$$R^*_T(\theta) = \begin{Bmatrix} R^*_{T_x}(\theta) \\ R^*_{T_y}(\theta) \end{Bmatrix} = I^*_{12} I_{23}^*$$

$$I^*_{12} I_{23}^* = \begin{Bmatrix} R^*(\theta)\cos\theta - e^*\sin\theta - r_r\cos(\theta-\alpha^*) \\ R^*(\theta)\sin\theta + e^*\cos\theta - r_r\sin(\theta-\alpha^*) \end{Bmatrix} \quad (8)$$

Where

$$e^* = e \pm \Delta e_x \quad (9)$$

$$R^*(\theta) = \sqrt{(r_b + r_r)^2 - e^{*2}} + h(\theta) \quad (10)$$

The pressure angle which depends of the new position vector will be expressed as follow:

$$\alpha^* = \tan^{-1}\left(\frac{I^*_{12} I_{24}^* - e^*}{R^*(\theta)}\right) \quad (11)$$

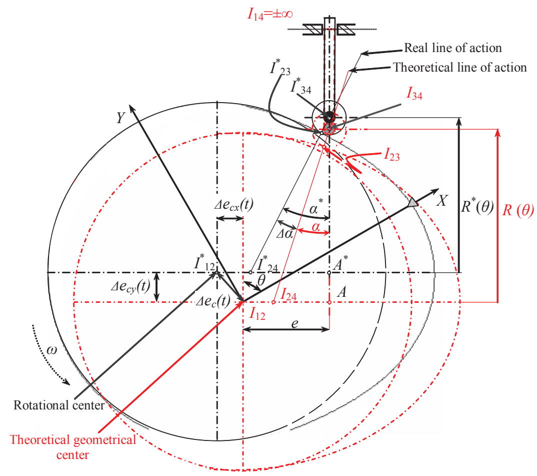


Fig. 4. Schematic diagram of the eccentricity error on the disk cam with an offset translating roller follower

1.3.2. Second case (Follower-rod error)

In this case we assume that we have a backlash error between the follower rod and the frame. This backlash error expresses the additional clearance caused by tolerances (figure 3b). Figure 5 show a schematic diagram of a backlash error between the follower rod and the frame on the disk cam with an offset translating roller follower in the two extreme cases. Because of a backlash error between the follower rod and the frame two defect positions of the follower can be observed depending on the period rising or return. These two defects position will provide follower direction change, a pressure angle and an offset variation ($\Delta\alpha$ and Δe).

This effect is introduced by adding a transmission error modeled as a displacement of the follower-roller-support in horizontal direction. Which yield to an exiting force in then horizontal direction of the bearing.

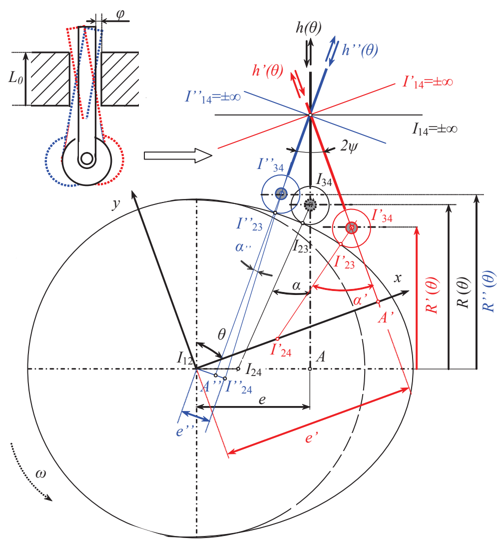


Fig. 5. Schematic diagram of a backlash error between the follower rod and the frame on the disk cam with an offset translating roller follower

The instantaneous contact point varies from point I_{23} to point I'_{23} or I''_{23} with corresponding angle θ. The new position vectors $R_T^{'}(\theta)$ and $R_T^{''}(\theta)$ in the two limit cases in term of the cam rotation angle can be expressed as:

$$R_T^{'}(\theta)=\begin{Bmatrix} R_{T_x}^{'}(\theta) \\ R_{T_y}^{'}(\theta) \end{Bmatrix}=I_{12}I_{23}^{'}$$

$$I_{12}I_{23}^{'}=\begin{Bmatrix} R^{'}(\theta)cos\,\theta-e'\,sin\,\theta-r_r\,cos\,(\theta-\alpha') \\ R^{'}(\theta)sin\,\theta+e'\,cos\,\theta-r_r\,sin\,(\theta-\alpha') \end{Bmatrix} \quad (12)$$

$$R_T^{''}(\theta)=\begin{Bmatrix} R_{T_x}^{''}(\theta) \\ R_{T_y}^{''}(\theta) \end{Bmatrix}=I_{12}I_{23}^{''}$$

$$I_{12}I_{23}^{''}=\begin{Bmatrix} R^{''}(\theta)cos\,\theta-e''\,sin\,\theta-r_r\,cos\,(\theta-\alpha'') \\ R^{''}(\theta)sin\,\theta+e''\,cos\,\theta-r_r\,sin\,(\theta-\alpha'') \end{Bmatrix} \quad (13)$$

$e'=e+\Delta e$ and $e''=e-\Delta e$

$$\psi=tan^{-1}\left(\frac{2\varphi}{l_0}\right) \quad (14)$$

Then the two positional vectors can be expressed by:

$$R^{'}(\theta)=\sqrt{(r_b+r_r)^2-e'^2}+h(\theta) \quad (15)$$

and,

$$R^{''}(\theta)=\sqrt{(r_b+r_r)^2-e''^2}+h(\theta) \quad (16)$$

The pressure angles in the two limit positions are:

$$\alpha'=tan^{-1}\left(\frac{I_{12}I_{24}^{'}-e'}{R^{'}(\theta)}\right) \quad (17)$$

and

$$\alpha''=tan^{-1}\left(\frac{I_{12}I_{24}^{''}-e''}{R^{''}(\theta)}\right) \quad (18)$$

2. NUMERICAL SIMULATIONS

In this section, results from simulation performed with healthy cam-follower system are compared with those computed with system having errors. The profile of the cam is computed by using modified trapezoidal curves (table 1). The parameter of the cam follower system are given in table 2 and the natural frequency are given in table 3. The rotational speed of the camshaft is 750 rpm.

Table 1. The cam's motion curves

	Follower motion			
	Dwell	Rise	Dwell	Return
Cam angle interval	0 ~ 30°	30° ~ 180°	180° ~ 210°	210° ~ 360°
Name of the curve		Modified trapezoidal		Modified trapezoidal

Table 2. The cam-follower system parameters

Mass (Kg)	M_{eq}=1.36, M_f=0.340, M_{fc}=0.07
Linear damping coefficients (N.sec/m)	C_{vs}= C_{hs}=750, C_f=0.08,
Linear stiffness (N/m)	K_{vs}=K_{hs} =2.6 10^8, K_{rb}=K_{hc}=K_f=2.5 10^8, K_{rs}=1.4 10^4,K_{vc}=1.7 10^8, K_h=1.9 10^8,
Spring initial deformation K_{rs} (mm)	δ_{rs}=13
Follower total rise (mm)	h_{max}=10
Diameter of cam base circle (mm)	$2R_a$= 60
Diameter of follower roller (mm)	$2R_1$=16
Follower offset (mm)	e= 10

Table 3. Natural frequency of the system (Hz)

f_{01}	f_{02}	f_{03}	f_{04}
2447	2447	5711	12777
f_{05}	f_{06}	f_{07}	
13990	17009	17009	

2.1. Healthy case

In this study a disk cam with an offset translating follower is considered. Fig. 6 and 7 show respectively the variation of the contact force and the effect of flexibility on the follower acceleration.

It is well observed that contact force is time varying and periodic at the cam rotational period T_r =0.08 s. Also there are fluctuations much more observed on the rise period than in the return period because of the greater acceleration fluctuation produced by the modified trapezoidal motion. Furthermore the contact force between the cam and the roller is variable according to the position of the follower and depends on the particular combinations of K_{rs} spring force and positive or negative inertia forces that occur. Figure 8 show the camshaft bearing amplitude acceleration, this acceleration reaches its maximum amplitude around f_{01} and at 2/5 f_{01}. We think that 2/5 f_{01} is induced by the parametric resonance caused by the time varying Hertzian stiffness whose spectrum is presented in fig 9.

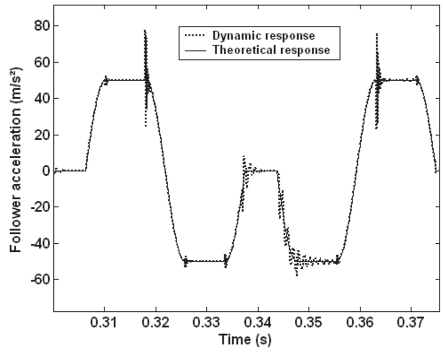


Fig. 6. Dynamic and theoretical response of the follower acceleration over one period

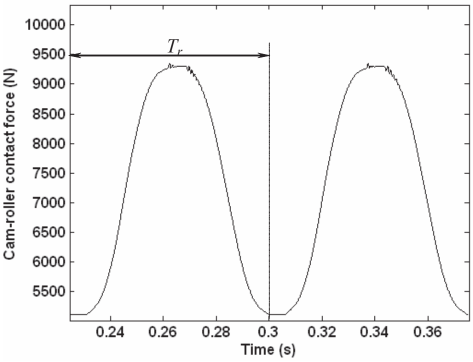


Fig. 7. Variation of the contact force

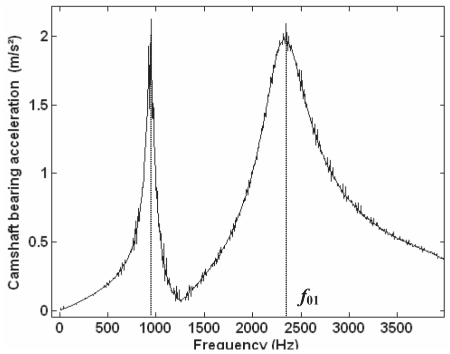


Fig. 8. Frequency response of the camshaft bearing acceleration

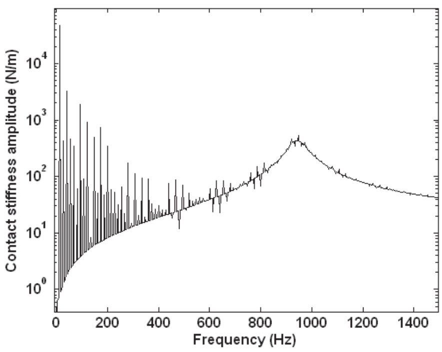


Fig. 9. Frequency response of the contact stiffness cam-roller

2.2. Defect cases

In this study two defect cases are considered. In the first one, we will consider an eccentricity of 10 µm on the camshaft. In the second case, a backlash of 10 µm between the follower rod and its frame is considered.

2.2.1. First case (eccentricity error)

Figure 10 shows the effect of the eccentricity error on follower acceleration. It is well noticed that the amplitude of the follower acceleration has increased especially on the dwell period where the follower reach its maximum value. As for the contact force (figure 11) we can observe the same phenomena. On the other hand, the camshaft bearing acceleration presents an amplitude modulation compared with the healthy case (figure 12).

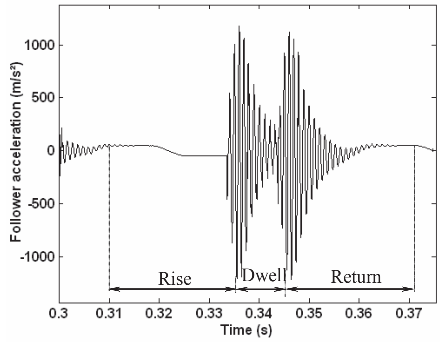


Fig. 10. Effect of an eccentricity error on the follower acceleration

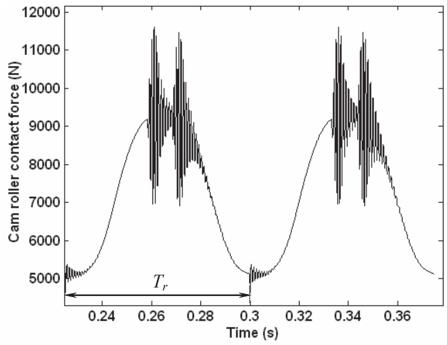


Fig. 11. Effect of the eccentricity error on the contact force

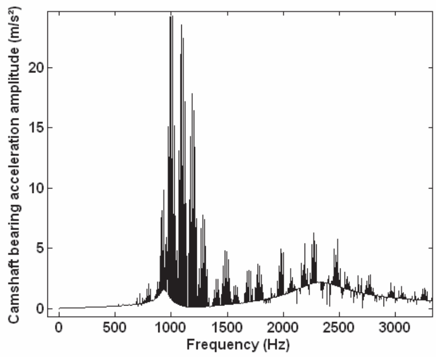


Fig. 12. Frequency response of the camshaft bearing acceleration for an eccentricity error

2.2.2. Second case (follower backlash error)

In this case amplitude of the follower acceleration (figure13) and the contact force (figure 14) have increased especially on the two dwell periods when the follower reaches its maximum ($h = h_{max}$) and its minimum value ($h = 0$). This can be explained by the fact that the existence of the backlash allow the follower rod to oscillate in the two lateral directions.

Figure 15 show the camshaft bearing acceleration which present an amplitude modulation compared with the healthy case, however this modulation is less important than for the eccentricity error case.

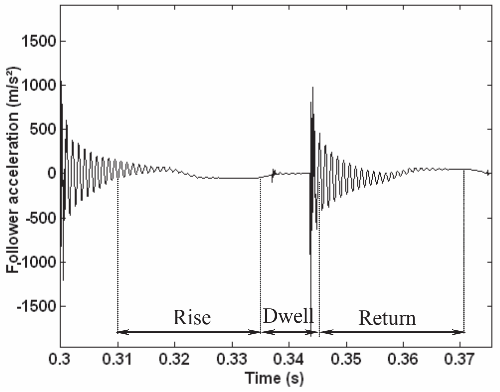


Fig. 13. Effect of backlash follower error on the follower acceleration

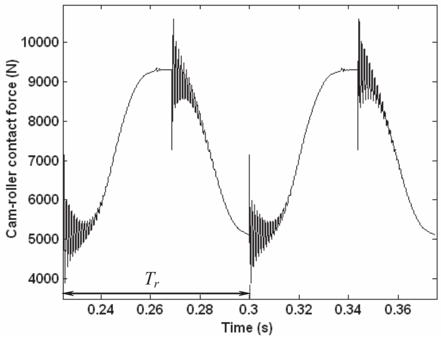


Fig. 14. Effect of a follower backlash error on the contact force

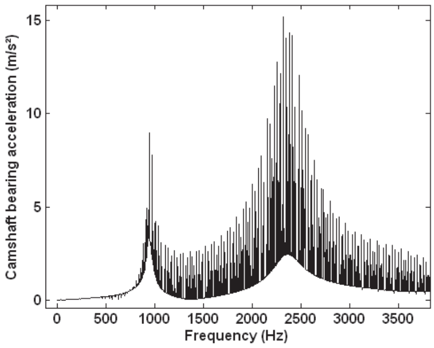


Fig. 15. Frequency response of the camshaft bearing acceleration for a follower backlash error

3. Conclusion

In this paper, seven degrees of freedom of a cam follower system is modeled and simulated in order to look at its dynamic behavior in presence of an eccentricity and a follower backlash errors induced by manufacturing imprecision and wear of the part of the cam follower system. It was shown that the flexibility of the system affects the dynamic behavior of the system. In fact, fluctuations are observed in the follower accelerations: when the errors are introduced, the fluctuations of the follower acceleration become greater and this will produce higher vibration levels included by amplitude modulation phenomenon.

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