DOUBLE SINGULAR VALUE DECOMPOSITION (SVD) OF SYMPTOM OBSERVATION MATRIX IN MACHINE CONDITION MONITORING

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Summary

Application of SVD to fault extraction from the machine symptom observation matrix (SOM) seems to be validated enough by means of data taken from real diagnostic cases. But sometimes the number of observations, i.e. rank of the SOM is low, what may influence obtained results and subsequent diagnostic decision. This was the reason to look for additional improvement by the second application of SVD to generalized fault matrix obtained by the first SVD. The result is strange, no accuracy increase flows from the application of the second SVD, independently of the SOM rank. This needs further deliberations and rethinking.

Keywords: multidimensionality, symptom observation matrix, singular value decomposition.

ZASTOSOWANIE PODWÓJNEGO ROZKŁADU WARTOŚCI SZCZEGÓLNYCH (SVD) DO SYMPTOMOWEJ MACIERZY OBSERWACJI W DIAGNOSTYCE MASZYN

Korzyści zastosowania SVD w wielowymiarowej diagnostyce maszyn są potwierdzone przez wielu autorów. Jednak dla małej ilości obserwacji, kiedy rząd symptomowej macierzy obserwacji jest niski, wyniki mogą wydawać się nieprecyzyjne, co może wpływać na wynikowa decyzje diagnostyczną. Zatem zastosowano podwójny rozkład SVD w skrajnych przypadkach wziętych z praktyki diagnostycznej, kilkunastu i kilkuset obserwacji. Otrzymany rezultat zaprzecza początkowej supozycji, dodatkowe zastosowanie SVD nie daje żadnego wzrostu dokładności obliczeń uogólnionych symptomów. Przy okazji tych badań podwójnego SVD łatwo było skonstruować nowy uogólniony symptom wskazujący na występowanie dwu liczących się uszkodzeń w obserwowanym obiekcie, co może być istotne w sytuacjach nadzory złożonych obiektów.

Słowa kluczowe: wielowymiarowość, macierz symptomowej obserwacji, rozkład wartości szczególnych.

1. INTRODUCTION

The idea of symptom observation matrix (SOM) in multidimensional condition monitoring of machines is well established and brings several advantages, [Cempel et al 07]. Usually it is $p>r$ rectangular matrix with (r) symptoms S_r measured along the system life θ (*p readings*) placed in separate columns. It allows placing all physically different symptoms[1] measured in a phenomenal field of the machine in a one SOM, and to process them in order to obtain projection of **observation space** to the **fault space** of machine. Of course we usually observe more symptoms (*columns of SOM*), than there is expected number of faults in a machine.

The preprocessing of SOM may be different, but for condition monitoring it was found that normalization and extraction of symptom initial value is the best solution, bringing all symptoms to their dimensionless form. Then, the application of SVD to the dimensionless form of SOM gives needed projection of observation space to the fault space. The resultant matrices of SVD decomposition allow calculating two important matrices. The first is ***SD*** matrix, which give us generalized fault symptoms $\boldsymbol{SD_i}$ of machine, and in theory they are independent each other. From this matrix we can calculate so called total damage (*generalized*) symptom, as the sum of all $\boldsymbol{SD_i}$ generalized fault symptoms. This is mainly in order to calculate the symptom limit value $\boldsymbol{S_l}$ or to make the forecast of the total damage symptom. The second ***AL*** matrix allows us to assess the contribution of primary measured symptoms to a newly formed generalized fault symptoms. In this way we can just say which of primary symptom is redundant, as it does not give substantial information contribution, and as such can be rejected from further calculations and/or future measurements.

But the column orthogonality of SD matrix is assured for sufficient size of SOM matrix; in reality the matrix of correlation coefficient of SD matrix gives sometimes quite big off-diagonal elements, some of order 0.5 and higher. So, may be some improvements in our diagnostic reasoning is possible by the application of another orthogonal decomposition to SD matrix? In reality there is no

[1] Symptom, measurable quantity covariable (*or assumed to be*) with the system condition

big choice of decomposition method; principal components analysis (PCA), which uses SVD as it can be shown [Golub et al 96], and both are well diagnostically interpretable [Tumer et al 02], [Jasinski 04], [Korbicz 04]. The well known QR decomposition seems to be not usable in diagnostics. According to unpublished study of present author, only the main diagonal of the upper triangular matrix ***R*** of this decomposition can be compared to the first generalized symptom SD_1, the higher upper diagonals are shortened and do not carry readable diagnostic information.

In principle, it is possible the second application of PCA to ***SD*** matrix, but as it is known eigen values of decomposition will be the squares of singular values of SVD, and singular vectors are equal to principal components. So there is no other solution like to apply SVD again, but for matrix ***SD*** only, not to ***AL*** matrix. This will transfer only part of SOM information content, but we will see the result. Such is the main idea of this paper, and as it is hope, it brings some advantages in condition assessment. It seems to, that the effect of this additional decomposition maybe data dependent and its real usability can be jugged for the given population of diagnosed objects.

2. OPTIMIZATION OF MULTI SYMPTOM MACHINE OBSERVATION

It was assumed earlier, our information about machine condition evolution is contained in p x r **SOM**, where in r columns and p rows of the successive readings of each symptom are presented. Usually they are made at equidistant system life time moments θ_n, $n=1,2,\ldots p$. In pre-processing operation the columns of SOM are centred and normalized to the three point average of three initial readings of every symptom. This is in order to make the SOM dimensionless, and to diminish starting disturbances of symptoms. This allows also to present the evolution range of every symptom from zero up to few times of the initial symptom value S_{0r}, (*measured in the vicinity of $\theta = 0$*).

After such preprocessing we obtain the dimensionless **SOM** in the form;

$$\mathbf{SOM} = \mathbf{O}_{pr} = \left[S'_{nm}\right], \quad S'_{nm} = \frac{S_{nm}}{S_{0m}} - 1, \tag{1}$$

Now we can apply the Singular Value Decomposition (SVD) [Golub 96], [Will 05], [Kielbasiński et al 92] to our dimensionless SOM (1), to obtain singular components (*vectors*) and singular values (*numbers*) of SOM, in the form

$$\mathbf{O}_{pr} = \mathbf{U}_{pp} \cdot \mathbf{\Sigma}_{pr} \cdot \mathbf{V}_{rr}^{T}, \ (T - matrix\ transposition), \tag{2}$$

where; U_{pp} is p dimensional orthonormal matrix of left hand side singular vectors, V_{rr} is r dimensional orthonormal matrix of right hand side singular vectors, and the diagonal matrix of singular values Σ_{pr} is defined as below

$$\mathbf{\Sigma}_{pr} = diag\left(\sigma_{1,\ldots,}\sigma_l\right), \quad whit\ nonzero\ s.v.: \quad \sigma_1 > \sigma_2 > \ldots > \sigma_u > 0, \tag{3}$$

and zero s. v. ; $\sigma_{u+1} = \ldots \sigma_l = 0;$ $l = \max(p, r),$ $u \le \min(p, r),$ $u < r < p.$

Going back to SVD itself it is worthwhile to say, that every non square matrix has such decomposition, and it may be interpreted also as the product of three matrices [Will 05], namely

$$O_{pr} = (\textbf{Hanger}) \times (\textbf{Stretcher}) \times (\textbf{Aligner}^T) \tag{4}$$

This is a very metaphorical description of SVD transformation, but it seems to be an useful analogy for the inference and decision making in condition monitoring. The diagnostic interpretation of formulae (4) can be obtained very easily. Namely, using its left hand side part, we are stretching our **SOM** over the life (*observations*) dimension, obtaining the matrix of **generalized symptoms *SD*** as the columns of the matrix . And using its right hand side part of (4) we are stretching **SOM** over the observed (*primary*) symptoms dimension, obtaining the assessment of contribution of every primary symptoms in the form of matrix ***AL***, assessing in this way the contribution of each primary symptom to the generalized fault symptom SD_i ,$i=1,\ldots u$.

$$SD = O_{pr} \cdot V_{rr} = U_{pp} \cdot \Sigma_{rr}\ ;$$
$$and; \quad AL = U^T_{pp} \cdot O_{pr} = \Sigma_{rr} \cdot V^T_{rr} \tag{5}$$

We will calculate the above matrices and use them for better interpretation of monitoring results (***SD***) and optimization of the dimension of the observation space (***AL***).

As the rows of **SOM** matrix were formed along the machine lifetime, so the columns of **SD** matrix have the discrete argument of life time θ , and we can write fault space interpretation as below;

$$\boldsymbol{SD}_t(\theta) \propto \boldsymbol{F}_t(\theta), \quad t=1,2,\ldots,$$

$$Norm\ (\boldsymbol{SD}_t) \equiv //\boldsymbol{SD}_t// = \sigma_t,\ t = 1, \ldots, u \qquad (6)$$

And for the total damage generalized symptom

$$\mathbf{SumSD}_i(\theta) = \sum_{i=1}^{z} \mathbf{SD}_i(\theta) = \sum_{i=1}^{z} \sigma_i(\theta) \cdot \mathbf{u}_i(\theta) \propto \mathbf{F}(\theta), \qquad (7)$$

where; $\mathbf{u}_i$ is a column of $\boldsymbol{U}_{pp}$.

This concept of diagnostic inference, for individual fault $\boldsymbol{F_t}$ (6), and total fault damage $\boldsymbol{F}$ (7) has been proved in several papers [Cempel 04], [Cempel at al 07], and we will use it in further consideration.

The above results, based on generalized fault symptoms, have been obtained only from the first matrix SD in (5). And the second matrix AL gives us the relative measure of information contribution to each generalized symptom given by particular primary symptom measured during the SOM gathering. This is one way of assessment of the primary symptom redundancy, but we need some other global indicators of rejection of the redundant symptom. In our previous papers we have used modified Frobenius norm of SOM and the generalized volume of the fault space created by SOM. What is important here, these two measures are based on singular values of SOM, which in turn can be treated as the fault advancement measures (*see (6)*).

$$Frob1 = \Sigma \sigma_i; \qquad (8)$$
$$and; \quad Vol1 = \Pi \sigma_i, \qquad i = 1, \ldots u.$$

Looking above for the value creation method, one can say that if some primary symptom will be really redundant its rejection should change *Frob1* measure only a little (*small* σ_I), and should much increase the fault space volume *Vol1*. We will notice how it behaves with real examples of symptom rejection in diagnosed machines.

3. TOTAL DAMAGE SYMPTOM AND DOMINATING SYMPTOM OF MACHINE, EXAMPLE

As a first example of application of our idea we will take a hard diagnostic case - a huge fan for coal milling from one of Polish thermo power station. Here the root mean square vibration velocity (V_{rms}) has been used as a symptom of condition, and initially altogether 11 symptoms at different places of fan mill aggregate structure were constantly monitored over 60 weeks lifetime θ. How unstable and noisy the fan running environment is, one can notice from the left top picture of the fig.1. It is seen further (*middle left picture*), that the symptom normalization and addition of life time symptom θ (*straight line*) do not change much the noisy behaviour of primary and generalized symptoms (*bottom left picture*).

Looking at the middle right picture of Fig.1, where matrix ***AL*** is presented, one can notice that symptoms No 7,8,9,10,11 do not give substantial contribution to the three dominating generalized symptoms, and probably can be rejected as redundant at the first approach. With this respect please note the value of Frobenius modified measure *Frob1* and the volume *Vol1* of the fault space at upper right picture. One can also note here, that there are two generalized symptoms with high information contents, and due to that two symptom limit values are assessed: S_{lc} for the total damage symptom, and S_{l1} for the first generalized symptom (*bottom pictures).*

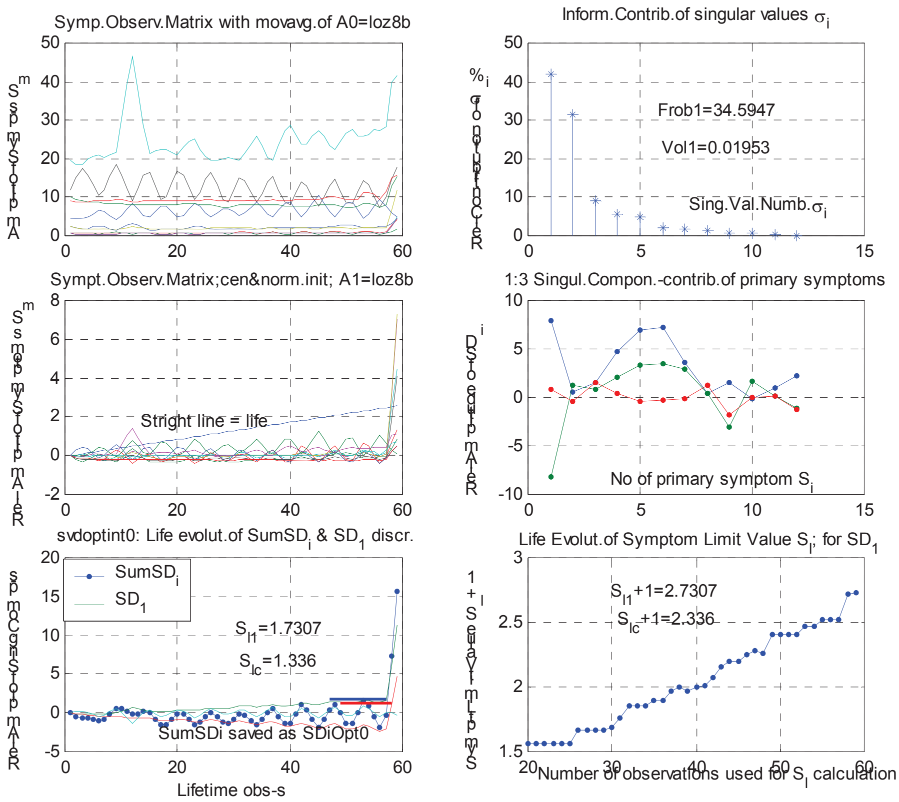

Fig. 1. Vibration condition monitoring (V_{rms}) of the coal mill fan observed at three bearings of fan and electric motor

Concerning symptom redundancy, let us look further at the Fig. 2 presenting the primary symptoms contributions measured in terms of correlation to SOM matrix, and to the total damage symptom ***SumSD$_i$***. Here one can come to similar conclusion for symptoms No 7, 8, 9, 10, 11 in particular. This is true with respect to the total damage symptom ***SumSD$_i$*** at the bottom picture and also partially to the overall information resource at the upper picture.

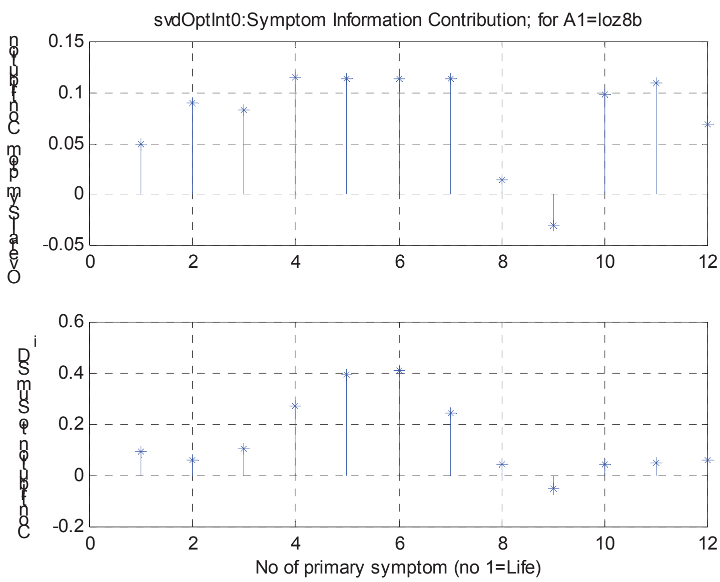

Fig. 2. Primary symptom contribution to the overall information resource and to the total damage symptom **SumSD**$_i$, for the fan coal mill aggregate (fig.1)

Following these guidelines of Fig. 2 five symptoms have been rejected from the primary number of eleven, and the result of new SOM decomposition is seen on the Fig. 3. As can be seen from the picture top left the most of troublesome un-diagnostic symptom was rejected. Due to this one can say we have new SOM with little increase of Frobenius modified measure, and dramatic increase of the fault space volume. Comparing pictures bottom left of Fig. 1 and Fig. 3 one can notice that most of generalized symptoms oscillation has been reduces, but they are still present there. Also it is worthwhile to notice the decrease of singular values contribution in a new SOM. There are still two dominating faults but amount of information they carry is increased now.

This is the result of SOM optimization by means of symptom rejection described already in some papers [Cempel 09]. Having done this one can now proceed to condition forecast, by means of neural nets [Tabaszewski 06] or by grey system theory [Cempel et al 07]. But we are interested now in some improvements of generalised fault symptoms, possible by new SVD processing, what will be done in a next point.

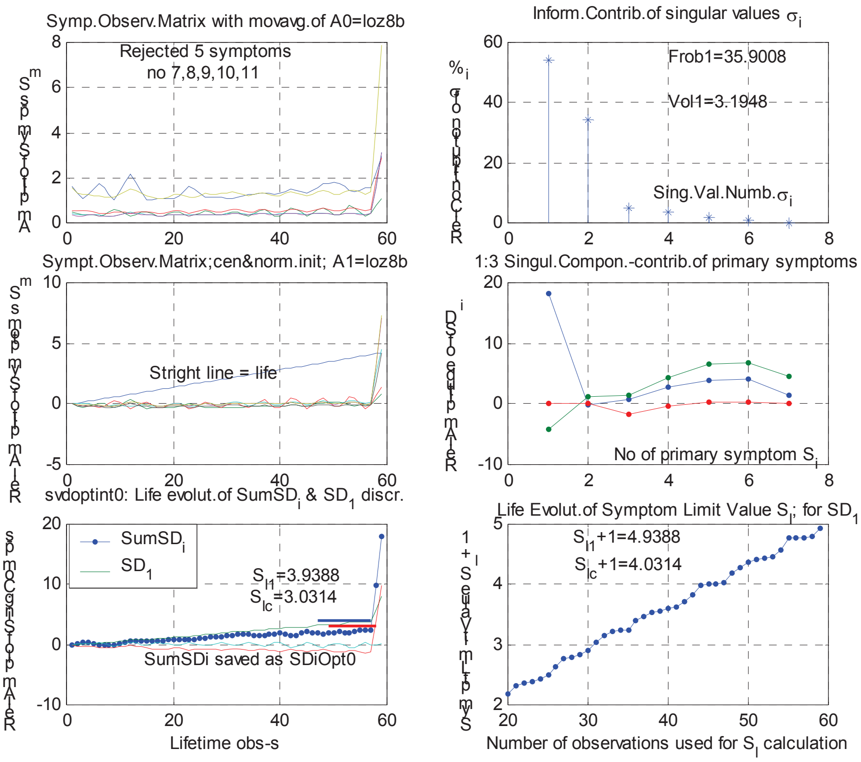

Fig. 3. Coal mill fan as on Fig.1 with five redundant symptoms rejected

4. THE SECOND DECOMPOSITION OF GENERALIZED SYMPTOM MATRIX SD

As it was said in the introduction the generalized symptoms theoretically have to be orthogonal, but usually it is not exactly true, depending on the type of SOM matrix. Hence let us recall partly the relation (5) creating the generalized symptom matrix SD

$$SD = O_{pr} \cdot V_{rr} = U_{pp} \cdot \Sigma_{rr} .$$

As one can see this is rectangular $p \bullet r$ matrix, but as relation (3) indicates only $u < r$ of singular values are different from zero, hence we should correct above to

$$SD = O_{pr} \cdot V_{uu} = U_{pp} \cdot \Sigma_{uu} . \quad (9)$$

Applying now SVD again to the above, analogously to (2) we will have

$$SD = U1_{pp} * \Sigma 1_{pu} * V1^{T}_{uu} . \quad (10)$$

And from the last decomposition we should pass to the new generalised fault symptom matrix in the same way as in (5), let's name it ***SD1***, as below;

$$SD1 = SD * V1_{uu} = U1_{pp} * \cdot \Sigma 1_{uu} . \quad (11)$$

It seems to, that after such double decomposition, the new generalized fault symptoms $SD1_i$ will be much more orthogonal, this means having less disturbances in the course of system life θ .

Let us turn our attention again to Fig. 3, picture top right. As we can notice from here it seems to be two faults in our machine; with relative strength close to 60% for the first and close to 40% for the second fault. It would be interesting if the double SVD confirm the presence o two faults and repeat their internal relation. Also for the inference process it would be helpful to create some new measure, or symptom, which confirms the presence of the second fault in a machine. This new symptom of the second fault ***(SF)*** can be simply the normalized product of the two first dominating symptoms as below;

$$SF = SD_1 * SD_2 * (|SD_1|)^{-1/2}, \qquad (12)$$

where normalization is acting only with respect of first dominating symptom SD_1 .

Such calculation as sketched above has been appended to the program **svdOptInt0.m** and the result one can see and analyze looking at the Fig.4 below.

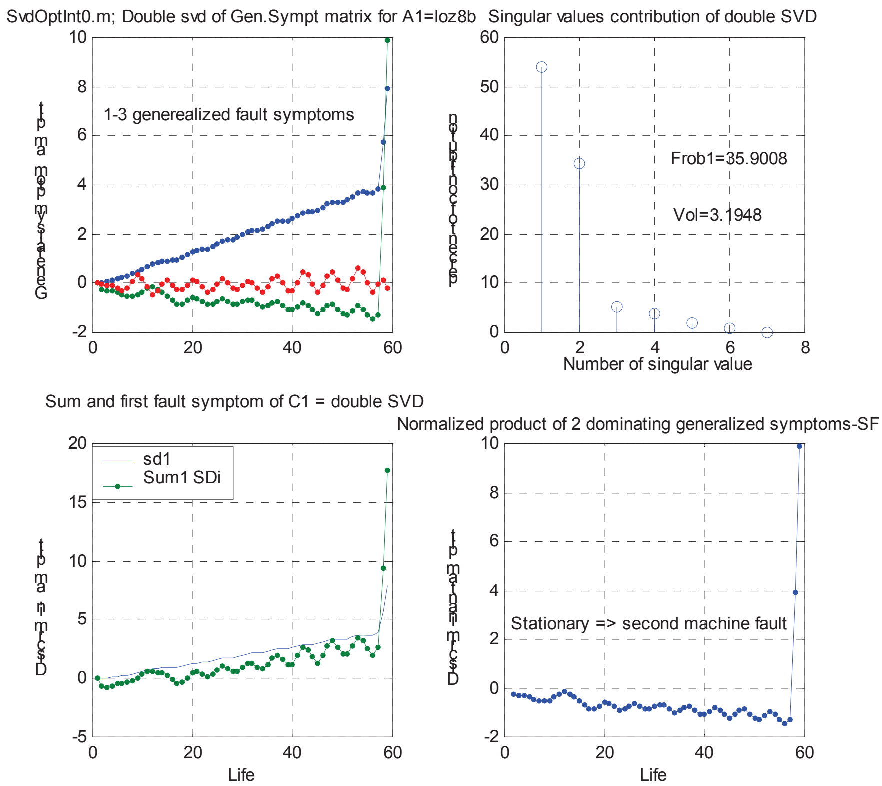

Fig. 4. The second SVD of generalized fault symptom matrix SD, and the new symptom of second fault presence, for the data of Fig.3 of a fan mill

Fig. 4 and Fig. 3 (*bottom left and top right*) we can notice their identity with the respect of shape of the curves, their values, as well as the values of Frobenius measure and Volume of the generalized fault space. This identity is shocking result. Moreover, if we calculate correlation coefficient matrix for generalized fault matrices SD and SD1 after second SVD, their results are also identical, even with respect of values of the off-diagonal elements.

What does it mean? We know that SD matrix, created according relation (5), does not carry all the information of primary SOM. The same is with SD1 matrix according to relation (10) and (11). Until now, this fact can not be interpreted correctly using author's understanding only.

But not all information contained in a Fig. 4 brings us to confusion. Looking for the picture bottom and top right, we can notice that in this case we have independent confirmation of second fault existence in our machine. So, the calculation of product of first two generalized faults (12) gives us independent confirmation of second fault existence or its not existence.

But may be this identity of results after second SVD application is data oriented? Let us take another example. This time it comes from ball bearings durability testing stand, where slowly pulsating load was applied additionally[2] and 19 symptoms has been measured initially. Fig. 5 present here optimized SOM of the ball bearing experiment, after rejection of 3 redundant symptoms, and Fig. 6 presents the results of second SVD application to the SD matrix.

And again comparing bottom left and top right pictures of Fig. 5, with the top pictures of Fig. 6 one can find their identity, the same as in previous case. So this is a rule of data processing, and identity is not data oriented. But there is a god news in Fig. 6,

[2] Author is obliged here to Dr M. Tabaszewski for providing the data.

it is confirmation of the existence of the second fault, emerging during the life testing of the bearing (*picture bottom right*). Of course there are some oscillations on the course of SF due to oscillation of bearing load, but the mean course of SF measure strictly indicate second fault presence and its evolution.

In previous examples we had rather big data base; sixty or one hundred sixty rows in SOM matrices (*readings*). Hence let us change the dimension of SOM to very short, of order of twenty symptom readings. This can be the case of railroad diesel engine condition monitoring[3], where at the top of one cylinder all vibrational quantities has been measured, each ten thousand kilometers of its mileage up to the breakdown. Figures 7 and 8 present this case in the same manner as it was done previously. Although Fig. 7 is already optimized with 3 symptoms rejected, one can see that last four singular values are very small, giving the small value of the volume of generalized fault space of order 10^{-7}. We have here situation where only one fault is developed, what is clearly seen from the pictures top right and bottom left. The rest of singular values and generalized symptoms as well, do not give substantial contribution to the total damage symptom ***Sum SD$_i$***. And the same is seen from the Fig. 8 where the second SVD have been applied to the generalized fault symptom matrix ***SD***, picture left bottom of Fig. 7. As there is no second fault visible from the distribution of singular values (*top right pictures*), the second fault measure SF behaves quite strangely on the Fig. 8 bottom right showing us that there is no second fault. This confirms the relevance of the SF measure introduced here first time.

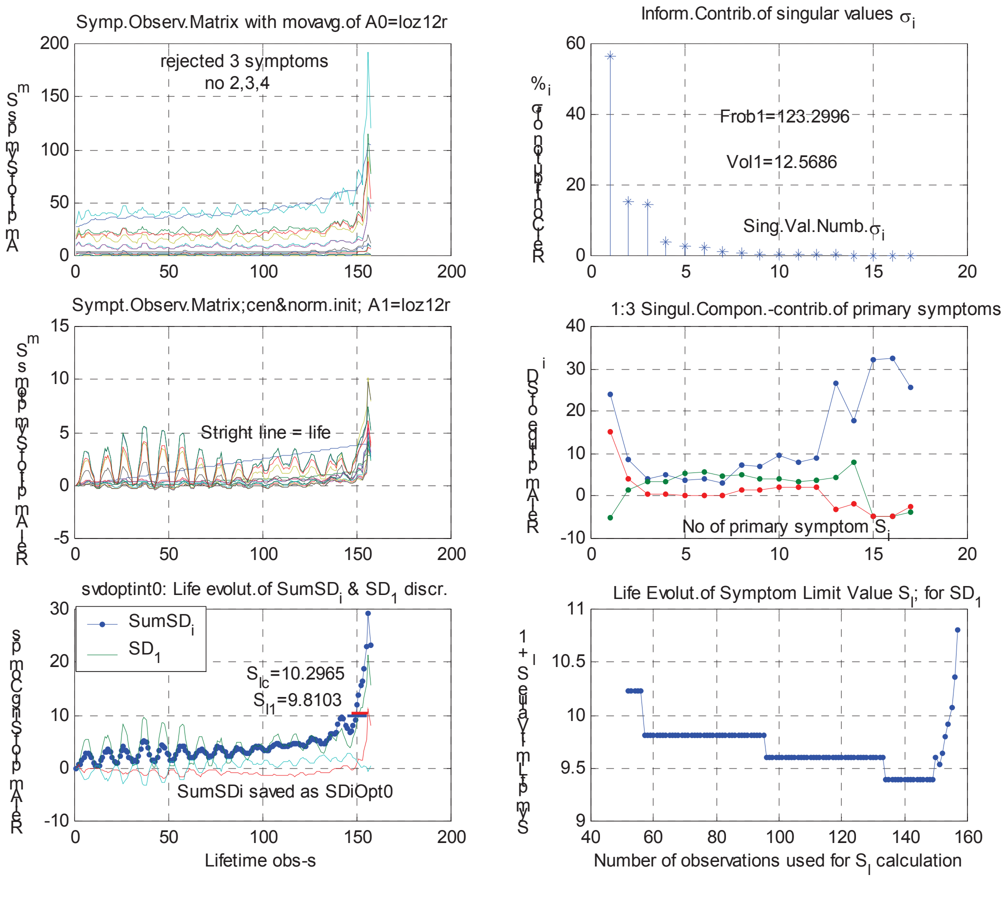

Fig. 5. Optimized SOM of ball bearing at the testing stand with slowly pulsating load

[3]This time author is obliged to Professor F. Tomaszewski for providing the data.

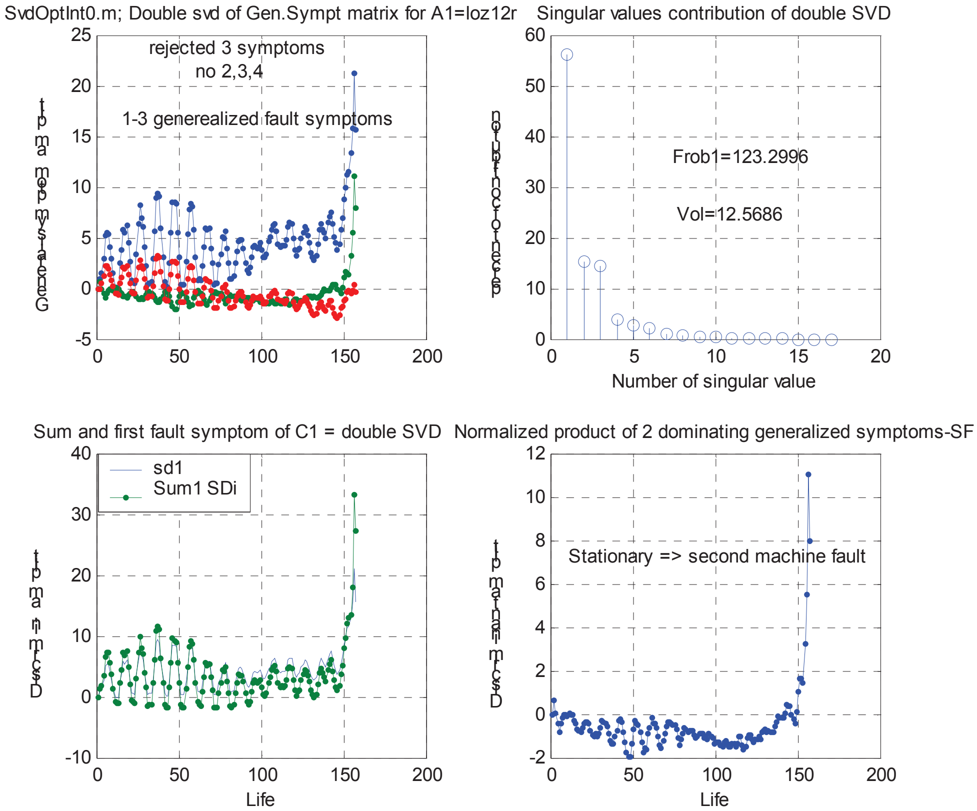

Fig. 6. Ball bearing data from the Fig5 after second SVD of generalized fault matrix SD

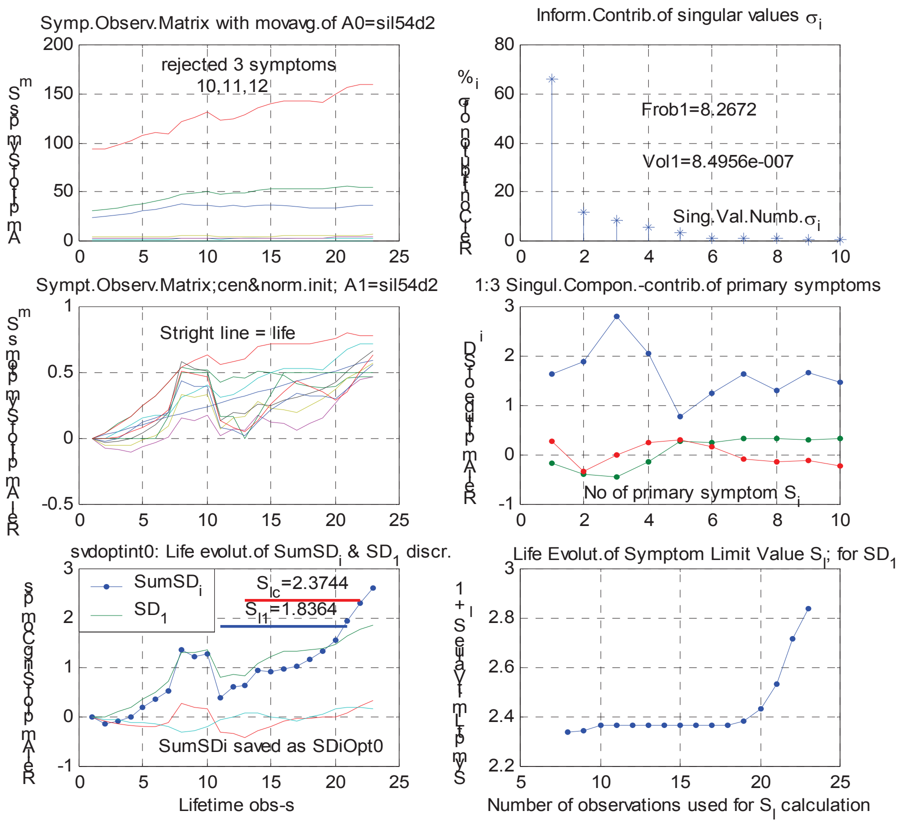

Fig. 7. Optimized SOM of railroad diesel engine diagnostically processed by SVD

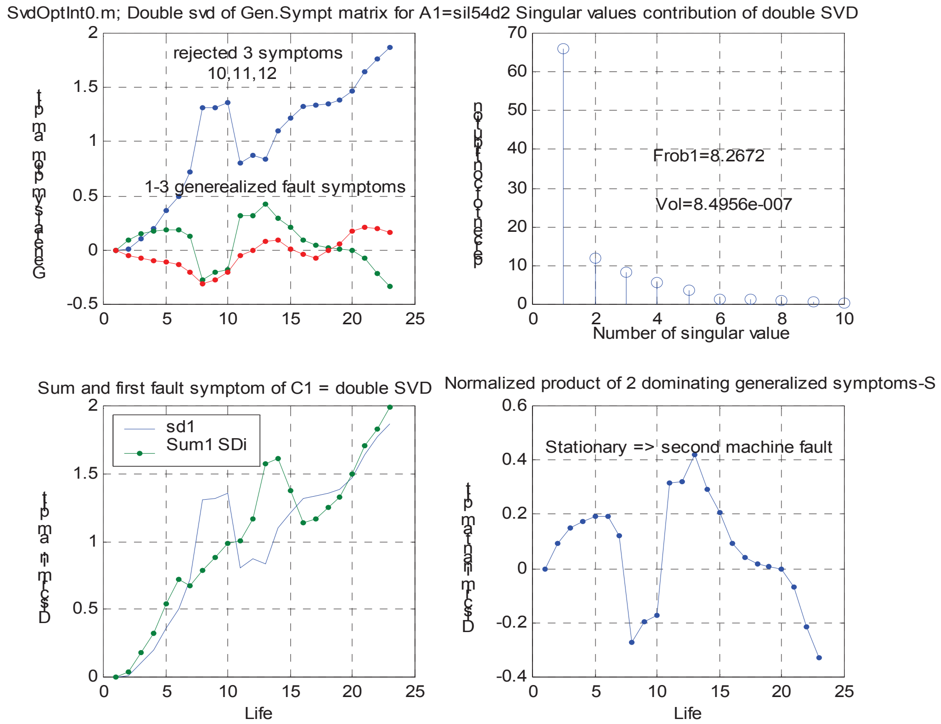

Fig. 8. Second SVD of railroad diesel generalized fault matrix, and respective SF measure life coarse

5. CONCLUSIONS

The premise for writing of this paper was unwritten assumption that singular value decomposition used in condition monitoring for fault information extraction may have some errors, in dependence of rank and the dimension of SOM of monitored object. This was amplified by the fact that correlation coefficient matrix of generalized fault symptom matrix SD has large off-diagonal elements. So, the second SVD of SD matrix of three diagnostic cases has been performed. The two of them with rather long observation history, (*60 and 160 rows of SOM as system observation*) and the last very short with 24 rows of SOM. In all three cases the results were the same, no increase in accuracy of calculation, the same singular vectors and singular values, independently of the matrix row dimension. This may be the proof of **validity of SVD use** as the method of fault information extraction, but it is not the proof of the goal of this paper.

Additionally, along this consideration some new diagnostic measure of second fault existence - SF has been introduced. In two first cases SF confirmed the existence of second fault presence, as really it was the case. This seems to be one of the concrete results of this paper

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