FAILURE ORIENTED DIAGNOSTIC MODELS IN CONDITION MONITORING

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Summary
Main aim of paper is to present problems of use of physical probabilistic models in task of decreasing of uncertainty of prognosis of raise and evolution of failure. Presented considerations are supported by example of fatigue analysis of tooth breaking in gearbox.

Keywords: diagnostic, physical probabilistic models, fatigue failure of tooth in gearbox.

USZKODZENIOWO ZORIENTOWANE MODELE DIAGNOSTYCZNE
W MONITOROWANIU STANU OBIEKTÓW TECHNICZNYCH

Streszczenie
Głównym celem pracy jest przybliżenie problematyki zastosowania fizykalnych modeli probabilistycznych w zadaniu zmniejszenia niepewności prognozy odnośnie powstawania i ewolucji uszkodzenia. Prezentowane rozważania oparte są na przykładzie zmęczeniowego wyłamania zęba w przekładni zębatej.

Słowa kluczowe: diagnostyka, modele diagnostyczne, zmęczeniowe wyłamanie zęba.

1. INTRODUCTION

The extension of equipment lifetime is the main purpose of maintenance strategy. Correct maintenance strategy can extend mean time to the failure, reduce the frequency of service interruptions and allow to avoid the undesirable consequences of catastrophic damages.

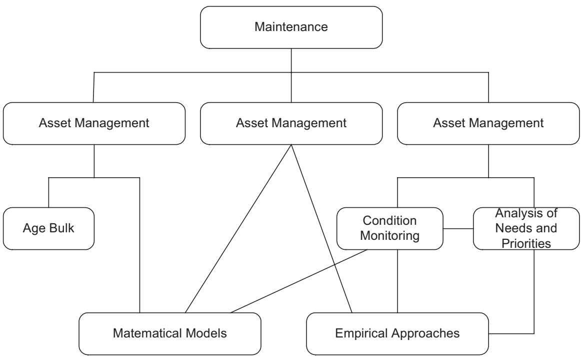

Fig. 1. Schedule of maintenance approaches

In other words, maintenance has relevant impact on components and system reliability. Therefore many companies replaced their scheduled maintenance with more flexible procedures based on a study of information obtained thought condition monitoring and predictive maintenance strategy. Figure 1 presents a specification of the various maintenance activities.

Maintenance models range from very simple to quite sophisticated. The simple rigid maintenance schedule defines activities that are carried out at fixed time intervals. The maintenance intervals are evaluated on the long-term experiences. For a complete evaluation one would have to know how the monitoring will extend the lifetime of component. To find this out the deterioration process of components have to be modeled.

2 MODELING OF CHANGES IN PROBABILITY DISTRIBUTIONS

In models that have been analyzed up to now probability of defect occurrence with assumption of invariability of examined distributions in operation time was determined. In reality, as a result of occurrence of wear and tear processes and the associated changes of the conditions of mating of elements and kinematic pairs, the evolution of probability distribution can be observed, both in quantitative terms (change of the parameters of probability density function) as well as in qualitative terms (change of the function describing the distribution). In addition, the degradation processes which accompany performance of the functional task can cause similar variation of probability distributions describing the load-carrying capacity. In this case it can expected that location of the separating line and the probability of defect occurrence will be dependent on the operating time. Recently numerous methods of examination and analysis of time-dependent reliability functions have been developed [1, 2, 3]. Examples of such changes are presented in Figure 2.

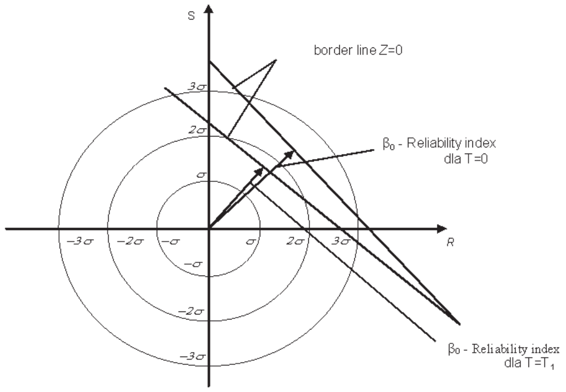

Fig. 2. Illustration of impact of wear and tear on the value of the reliability index

In general random nature of additional load resulting from environmental impact is assumed, however the basic load associated with realization of functional tasks remains a determined value. Still, in the models which take into account relation between the load as well as manufacturing and assembly errors and also for the impact of wear and tear, also this type of load will have random nature. For example, the analysis of the process of emergence of tooth defects in toothed gears points to the fact that the growth of disturbance of contact conditions is accompanied by the phenomenon of amplitude-and-frequency modulation of a signal's parameters [3]. This means that the diagnostically-essential information in such a case is contained in a restricted frequency band where the structure of distribution of power among respective components depends on the type and size of modulation. Assuming the random nature of the input signal $x(t)$ (normal distribution), the probability density of the envelope takes the form which is compatible with Rayleigh distribution [2]:

$$f(A,\sigma) = \frac{A}{\sigma^2}\exp\left[\frac{A^2}{2\sigma^2}\right] \quad \text{dla } A > 0 \qquad (1)$$

where:
$A(t)$ - signal envelope,
σ^2 - variance of the analyzed narrowband process.

The form of probability density, as described by relationship (1) does not cover the disturbance associated with the occurrence of additional non-linear adjoints, what is connected with the necessity of analysis of third and fourth order moments (2) (Fig. 3).

Thus to extract the information on the type and size of disturbance one must first select the frequency band and apply relevant signal demodulation techniques. The difficulties which occur when using the more complex method of evaluating defect probability result in more interest shown in the attempts of updating the values of strength and load distribution parameters in time of observation and measurements. Assuming that the distribution parameter is a random variable, which defines the a priori form of the probability density function, the a posteriori distribution upon conducting an experiment (observation) can be determined. It is often assumed that the obtained distribution is conditional, which enables Bayes theorem to be applied. Another example of a method of updating the value of parameters is to determine the a posteriori distribution with the use of credibility function.

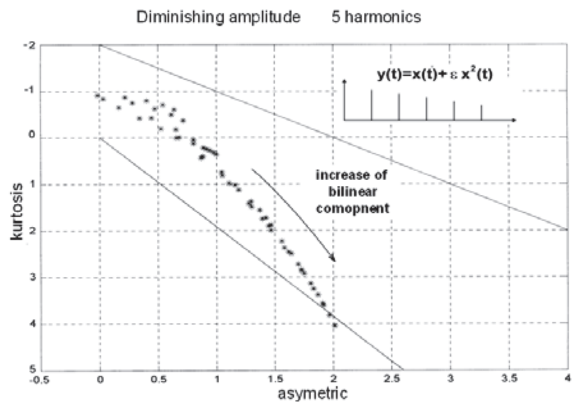

Fig. 3. Example of changes of higher moment values caused by non-linear disturbance

In any approach the main issue is extraction of the relevant diagnostic information. The results of the research confirm that as a result of wear and tear the quality of mating of individual elements and kinematic pairs changes, leading to evolution of load probability distribution. The observation concerns both, the changes of the values of density function's parameters as well as the class of the distribution itself. Similarly, the occurring degradation processes can cause changes in distribution of probability of permitted load capacity. Research [3] confirms the possibility of occurrence of such disturbance.

3. EXAMINATION OF RELIABILITY IN THE FIELD OF STANDARDIZED VARIABLES

The basis for conducting such analyses is such a knowledge of basic parameters and architecture of the system so that it is possible to define the sources and the types of uncertainty. This enables us to contemplate the models of designed objects while accounting for future behavior, state of knowledge of operational processes in the conditions of load which varies in time. In such a case the basic tool is the assessment of reliability while using probabilistic models. To present these problems closer, let us put the formula which defines the possibility of defect occurrence in the following way [4]:

$$Z = f(X_1 \dots X_n) = R - S \qquad (2)$$

where:

$f\left(X_1 \dots X_n\right)$ - the function of random variables which defines the difference between the loads and the load-carrying capacity. Generally it depends on numerous structural, technological, assembly, operational and environmental factors,

R - load-carrying capacity, e.g. strength,

S - load, e.g. stress.

Assuming that both, load S and load-carrying capacity R are random variables, we can determine their functions of probability distribution density $f_r(x)$, $f_s(x)$.

The method of determining the function is schematically presented in figure 4 where the striped area denotes the possibility of occurrence of *z<0*, which is interpreted as the probability of failure occurrence.

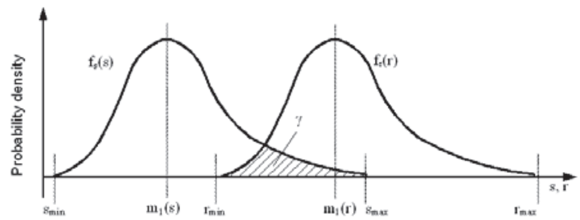

Fig. 4. Exemplary functions of density of load and load-bearing capacity probability

Let us note that the probability depends on the following parameters:

- Relative location of both curves – the expected values and standard deviations of stress and resistance are the parameters which can represent the location;
- In the function of probability density, for non-Gaussian distributions, one should account for the impact of skewness and flatness.

Minimization of probability of defect occurrence is one of the elements of the procedure of reducing the risk level. Determination of the probability of defect p_f can be presented in various ways, for example [1]:

$$P_f = \int_{-\infty}^{s_{\max}} \int_{r_{\min}}^{+\infty} f_{R,S}(r,s)\,\mathrm{d}r\,\mathrm{d}s \qquad (3)$$

where:

r, s – the relevant variables.

Making an assumption that f_S and f_R are independent and taking into account the technical restrictions related to minimum loads and maximum load-carrying capacity, relationship (3) shall be noted as follows:

$$P_f = \int_{s_{\min}}^{s_{\max}} \int_{r_{\min}}^{r_{\max}} f_R(r) f_S(s)\,\mathrm{d}r\,\mathrm{d}s \qquad (4)$$

Thus an event corresponding to a defect will occur when

$$Z = (R,S) \Rightarrow P_f = P\left(Z\left(R,S\right)\right) \le 0 \qquad (5)$$

On the assumption that the variables describing load capacity and load have normal distributions, let us now consider the possibility of determining a curve separating the area threatened with the defect from the remaining areas:

$$Z = 0 \qquad (6)$$

Let us note that thanks to introduction of standardized variables (2):

$$R' = \frac{R - \mu_R}{\sigma_R} \qquad \mathrm{S}' = \frac{S - \mu_S}{\sigma_S} \qquad (7)$$

the equation of the boundary line Z=0, defined by the formula (2), while assuming that the contemplated variables have normal distribution, shall take the following form:

$$Z = \sigma_R R' - \sigma_S \mathrm{S}' + \mu_R - \mu_S \qquad (8)$$

In analyzed case of standardized variables plane, it will be obtained:

$$p = \beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \qquad (9)$$

where:

β - reliability index in n-determined spaces.

Hence

$$P = \phi\left(-\frac{\mu}{\sigma}\right) \qquad (10)$$

Thus, we have obtained a measure which indicates, in the (S', R') system, that as the boundary line moves away from the beginning of the coordinate system, hence the probability of a failure decreases.

Referring now to the relationship which defines the distribution function of the standardized normal distribution:

$$\Phi(x) = \frac{1}{2\pi} \int \exp\left[-\frac{1}{2}t\right] \mathrm{d}t \qquad (11)$$

as well as the probability densities for such distribution (5), the relation defining the probability of a defect (10), as defined by β reliability indicator, shall finally take the following form:

$$P\left(Z \le 0\right) = P = 1 - \Phi(\beta) \qquad (12)$$

which in turn enables using the values of the distribution function Φ(x), found in a table for x >0, or the standardized programs for calculating the probability of normal distribution. Tables containing the values of the distribution function are found in majority of handbooks related to probability calculus and mathematical statistics.

This means that regarding the methods of calculating the probability of a defect, which were mentioned in the first part of the paper, we can use iteration procedures of increasing forecast precision by updating probability density function for permitted load and load-capacity. The specific method depends on the scope of the adopted diagnosis as well as the type of the observed defect. In general one can note that the biggest possibilities of correct solving the problem formulated this way are connected with the fact of reaching of such a diagnostic information for which diagnostic model can be built, which in its essence refers to the process of defect origin and development.

In task outlined this way it becomes important to select the threshold values which will define the selection of ranges and variability classes, respectively for the parameters of distribution and the probability density function.

Summarizing attention can be drawn to the fact that the main reasons of uncertainty of evaluations and analyses are:

- the procedures determining the values of parameters, e.g. small size of samples can have influence on the width of confidence intervals, extrapolation of data from one facility to another, disturbances in observation of data;
- modeling procedures which are focused on determining the essential variables and usually take into account their mutual detailed relations in an unsatisfactory manner;
- the nature of the phenomenon; the status of knowledge that does not always enable to take into account all the important factors, especially the course of defect-generation processes.

Higher credibility of forecasts can be obtained by including the changes of the diagnostic parameter resulting from wear and tear processes. As a result, when forecasting the probability of an emergency situation it becomes necessary to introduce the information about these material changes.

Authors traced such a procedure while analyzing the phenomenon of fatigue-related tooth fracture in a toothed gear. An assumption was made of variability of parameters of load distribution without change of its form and in addition the disturbance of their changes, resulting from defect initiation and propagation, was taken into account. For this purpose information was used from an experiment in which changes of stress at the foot of a tooth in the course of accelerated fatigue tests of a toothed gear (Fig. 5) was traced.

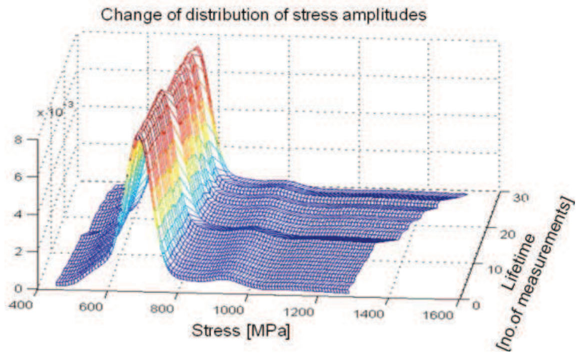

Fig. 5. Change of distribution of stress amplitudes at the foot of a tooth during the experiment

The values of these parameters were standardized in such a way so that they were adequate to the changes of undisturbed distribution. The reliability indicator was the parameter that was observed.

It turned out that if changes of material properties, resulting from wear and tear fatigue-related tooth fracture, were not took into account, forecasting of the emergency state of a toothed gear is burdened with too big error (Fig. 6).

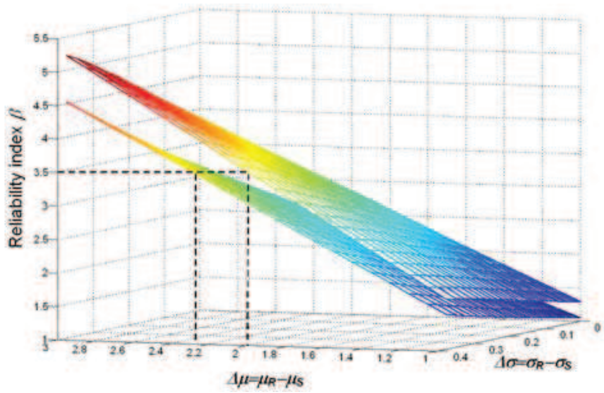

Fig. 6. Change of reliability indicator while not including and while including the wear and tear processes.

4. CONCLUSIONS

Presentation of a relatively simple example was aimed at demonstrating the use of physical probabilistic models in the tasks of reducing the uncertainty of forecasting related to emergence and evolution of defects. The direct application of the presented approach may call for applying, at least, the transformation of probability distribution, procedures of system, linearization and examination of the degree of variables correlation. Still, the main proceedings will preserve their nature.

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