SYMPTOM LIMIT VALUE: A STATISTICAL APPROACH

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Summary

Quantitative technical condition assessment may employ a scale provided by symptom baseline and limit values. Prognosis is then based on fitting a suitable function to recorded symptom time history. Such approach assumes the deterministic symptom concept. Due to the influence of factors other than object technical condition, however, symptom often has to be regarded as a random variable. With such approach it is necessary to consider the probability of limit value excess and hence of a false alert. This pertains to the object operation policy. An example is provided by vibration-based symptoms relevant to a steam turbine fluid-flow system. On the basis of experimental data it is shown that this probability can be unacceptably high well before the limit value is attained.

Keywords: technical diagnostics, diagnostic symptom, limit value, prognosis.

WARTOŚĆ GRANICZNA SYMPTOMU W UJĘCIU STATYSTYCZNYM

Streszczenie

Ilościowa ocena stanu technicznego może być oparta na bazowej i granicznej wartości symptomu. Prognozowanie jest wówczas realizowane przez dopasowanie odpowiedniej funkcji do zarejestrowanego przebiegu czasowego symptomu. Zakłada to deterministyczny charakter symptomu. Ze względu na wpływ czynników innych niż stan techniczny symptom musi jednak często być traktowany jako zmienna losowa. Należy wówczas rozważyć prawdopodobieństwo przekroczenia wartości granicznej, a tym samym fałszywego alarmu. Jest to związane z polityką eksploatacji obiektu. Podany przykład dotyczy drganiowych symptomów stanu układu przepływowego turbiny parowej. Na podstawie danych z rzeczywistego obiektu wykazano, że prawdopodobieństwo to może być niedopuszczalnie wysokie na długo przed osiągnięciem wartości granicznej.

Słowa kluczowe: diagnostyka techniczna, symptom diagnostyczny, wartość graniczna, prognoza.

1. INTRODUCTION

Three principal areas of interest in condition monitoring are fault detection, diagnosis and prognosis [1]. Fault detection can be alternatively referred to as qualitative diagnosis, its aim being fault identification and localization. Similarly, diagnosis can be more precisely termed quantitative diagnosis, as it is aimed at determining damage extent [2].

Quantitative technical condition estimation of any object must, by necessity, involve a reference scale. For a given symptom S such scale may be provided by its baseline and limit values (S_0 and S_l, respectively). Interpretation of the baseline value is straightforward, as it refers to a new object with no malfunctions and generalized damage D equal to zero ($D = \theta/\theta_b$, where θ denotes time and θ_b is the time to breakdown). Limit value is the indication of technical condition deterioration to a point where 'some action should be taken'; in other words, $S = S_l$ indicates an 'accelerated wear problem' [3]. It must not be confused with the maximum admissible value, which pertains to operational safety considerations and should, in principle, be determined by the object manufacturer.

Symptom limit value concept may be based on symptom reliability [3-5]. If we employ the Neyman-Pearson rule, known from the statistical decision theory, we obtain

$$R(S_l) = A/G \quad , \tag{1}$$

where A is the acceptable probability of performing an unnecessary repair (i.e. of the object able for normal operation) and G is object availability. $R(S)$ is the symptom reliability, given by

$$R(S) = P(S > S_e) = \int_{S_e}^{\infty} p(S^*)dS^* \,, \tag{2}$$

where $p(S)$ is the symptom probability density function. With sufficient database $p(S)$ can be estimated, so that, for given values of A and G, an estimation of S_l can be obtained. Note that A and G are related to the plant operation philosophy and, in a way, represent acceptable risk level. This is an important issue that shall be recalled later.

Symptom probability density function is estimated from experimental data, so limit value determination inevitably involves some measure of uncertainty. In the following, however, we shall assume that the available database is large enough for this uncertainty to be neglected.

Symptom limit value is particularly important for prognosis. Forecasting technical condition development usually involves some form of symptom time history (trend) analysis and fitting a curve to experimental data [1, 2]. In this manner, it is possible to estimate, at a given moment θ, the 'time to alert' $\Delta\theta$:

$$\Delta\theta = \theta_l - \theta; \;\; S(\theta_l) = S_l \; , \qquad (3)$$

so that a subsequent repair can be timed and its extent adjusted. This is important especially for large and critical machines, such as power generating units, aircraft engines etc.

For any real object, especially a large and complex one, we have to keep in mind that a measured symptom value depends not only on object technical condition. In fact,

$$S = S[\mathbf{X}(\theta), \mathbf{R}(\theta), \mathbf{Z}(\theta)] \; , \qquad (4)$$

where **X**, **R** and **Z** denote condition parameters, control and interference vectors, respectively. In principle, influence of control parameters can be normalized [6], although procedures may be tedious and their applicability is often limited to a given machine type or even a particular example. In most cases it is reasonable to assume that **R** and **Z** components have no monotonic trends and can be treated as random variables with parameters that do not change with time. This implies that, for any symptom, its measured value is also a random variable and, in principle, should be dealt with in a statistical rather than a deterministic manner. This immediately brings about a question whether the above approach to symptom limit value excess, based on a deterministic symptom life curve $S(\theta)$, is appropriate.

2. BASIC CONSIDERATIONS

Let us, for simplicity, assume that we are dealing with only one symptom and only one condition parameter. In such case, Eq.(4) takes the form of

$$S = S[X(\theta), R_1(\theta), \ldots, R_m(\theta), Z_1(\theta), \ldots, Z_n(\theta)]. \qquad (5)$$

In many important cases X can be identified with the above-mentioned generalized damage D, so that $X(\theta) = \theta/\theta_b$. If the conditions

$$\bigwedge_{i\in\langle 1,m\rangle} \frac{\partial S}{\partial X} >> \frac{\partial S}{\partial R_i} \quad , \qquad (6)$$

$$\bigwedge_{i\in\langle 1,n\rangle} \frac{\partial S}{\partial X} >> \frac{\partial S}{\partial Z_i} \qquad (7)$$

are fulfilled, we may assume that recorded symptom time history is dominated by technical condition evolution and influence of other factors may be neglected. This justifies a deterministic approach. If this is not the case, we may infer that a measured symptom value is, to a large extent, influenced by control and/or interference.

In practice, deterministic approach is often acceptable if technical condition deterioration is fast, i.e. when we are dealing with a rapidly developing fault. Example is given in Fig.1a. It is easily seen that, in each life cycle,[1] there is an almost linear increase (in this case caused by increasing rotor bow) and fluctuations are comparatively small. The opposite case is illustrated by another example, given in Fig.1b. Prior to rotor replacement, large fluctuations can be observed, superimposed on a continuous (approximately exponential) curve, related to 'normal' lifetime consumption – a natural damage or 'soft fault' [8, 9]. A deterministic approach in such case may prove inadequate.

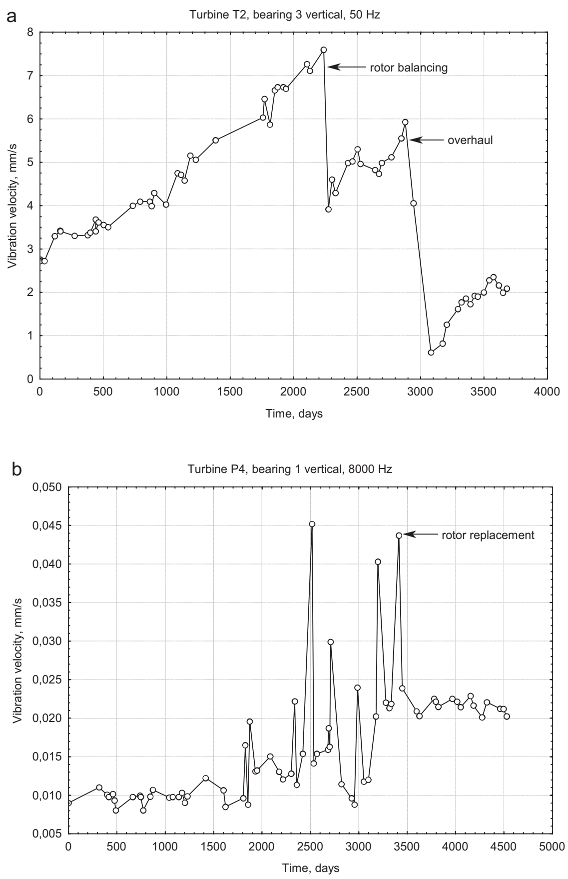

Fig. 1. Examples of vibration time histories, obtained for power steam turbines; a – 13CK230 unit, rear intermediate-pressure turbine bearing, vertical direction, 50 Hz band; b – K-200 unit, front high-pressure turbine bearing, vertical direction, 8 kHz band. See main text for details.

[1] Life cycles are determined by repairs and overhauls; for more details, see e.g. [5, 7].

As already mentioned, R_i and Z_i components can usually be treated as random variables with constant parameters. What we in fact observe, however, is not these components by themselves, but rather the object reaction to their changes. This obviously leads to a question how does the object sensitivity to R_i and Z_i, given by partial derivatives in Eqs. (6) and (7), change with time. This can be estimated only indirectly. Analysis of experimental data for power steam turbines shows [10] that, for vibration-based symptoms, standard deviation determined within a time 'window' changes rapidly as this window moves along the time axis. We may therefore infer that, with D increasing, standard deviation of a symptom, treated here as a random variable, will also increase. Thus, for a fixed S_l, probability of recording a symptom value $S > S_l$ will increase as $\theta \rightarrow \theta_b$, not only as a result of increasing symptom expected value $\hat{S}$.

There is still no model suitable to account for such processes in a quantitative manner. It seems, however, justified to perform a simulation based on data obtained for real objects. Results turn out to be of importance from the point of view of plant operational policy.

3. OBJECT AND MEASUREMENT DATA

Results dealt with in the following were obtained with a K-200 power steam turbine, operated as a base-load unit in a utility power plant. This turbine had logged about 150,000 hours of operation before investigations started (this particular moment corresponds in the following to $\theta = 0$), which for this turbine type means a considerable lifetime consumption degree.[2] After about nine years the high-pressure rotor was replaced during a scheduled overhaul (Fig.1b refers to this particular turbine).

Data for analysis were obtained from 23% constant-percentage bandwidth (CPB) absolute vibration velocity spectra, recorded on turbine bearings. Amplitudes in individual frequency bands determined from the vibroacoustic model were treated as individual symptoms. More details can be found in references [12, 13]. It seems necessary to recall here that for steam turbines (and in fact for all rotating machines that produce broadband vibration spectra) two frequency ranges can be distinguished. The so-called harmonic or low range contains components generated directly as a result of the rotating motion, while the blade or high range contains those resulting from interaction between fluid-flow system elements and steam flow. The latter range, which is of particular interest for this study, is in this turbine approximately between 500 and 9000 Hz. It should be noted here that vibration-based symptoms from this range are typically much more sensitive to factors other that technical condition evolution, their typical behavior being similar to that shown in Fig. 1b.

Due to comparatively long period covered by observation it was possible to estimate limit values for the above-mentioned symptoms. Details of relevant procedures are beyond the scope of this study and can be found in references [7, 14].

In the following, two symptoms are analyzed in detail, namely vibration velocity amplitudes in the 6300 Hz and 8000 Hz bands, recorded at the front high-pressure turbine bearing in vertical direction. For brevity these symptoms are hereinafter referred to as S_1 and S_2, respectively. These frequency bands contain components generated by the high-pressure rotor, which was replaced during the overhaul. Both these symptoms exhibited a marked increase tendency prior to the replacement, which indicates damage acceleration. For obvious reasons, data obtained after the overhaul have not been taken into account.

4. RESULTS AND DISCUSSION

Fig.2 shows raw trends of S_1 and S_2 with exponential fitting; it has been shown, on the basis of experimental data analysis and some model considerations, that this type of $S(\theta)$ function is appropriate for this frequency range [7]. In both graphs limit value is indicated (S_{1l} = 0.349 mm/s, S_{2l} = 0.121 mm/s). This obviously allows for a prognosis: had it not been for the overhaul, exponential fit of the S_1 would have attained its limit value in about 5,400 days and of S_2 in about 9,500 days, starting from $\theta = 0$. As it can be seen in Fig.2a, S_1 actually exceeded its limit value three times before the overhaul.

It seems reasonable to assume, at least as a first approximation, a normal distribution of S_1 and S_2; we shall recall this issue later in Section 5. Within the framework of a statistical approach, we may assume that exponential fit represents the time history of symptom expected value $\hat{S}$. We thus obtain:

$$\hat{S}_1(\theta) = 0.011 \times \exp(0.0006 \times \theta) \; , \qquad (8)$$

$$\hat{S}_2(\theta) = 0.0077 \times \exp(0.0003 \times \theta) \; , \qquad (9)$$

where symptom value is given in mm/s and θ in days. As already mentioned, experimental evidence shows that standard deviation should be expected to increase with θ. We may apply a 'moving window' procedure similar to that described in [10]: standard deviation σ is estimated within a window that includes ten consecutive measurements. Such approach obviously implies the assumption that technical condition deterioration during the period covered by this window can be neglected. Due to the 'accelerated wear' condition, we may also assume

[2] K-200 turbines had been designed in early 1950s, with a very conservative (by today's standards) service life estimation of about 100,000 hours. In practice service life of thick-walled elements (casings) and rotors has been about 200,000 to 250,000 hours; see e.g. [11].

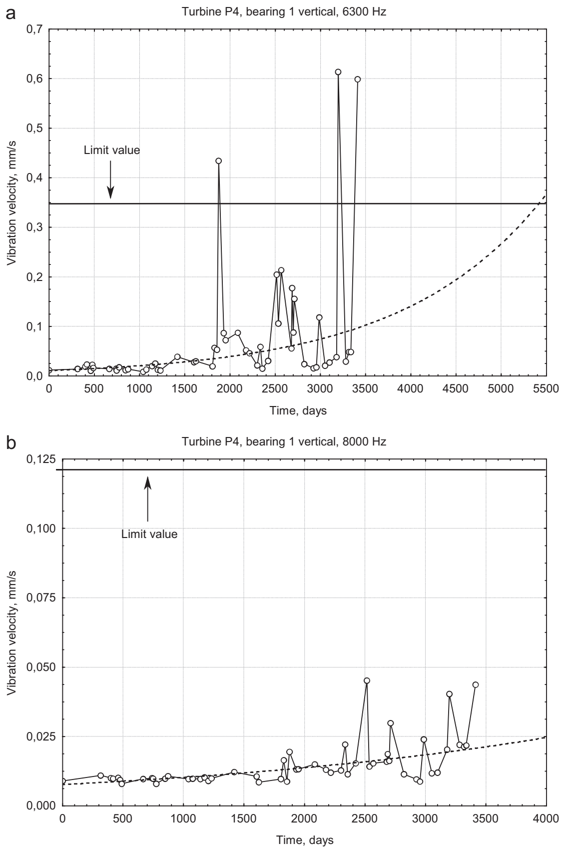

Fig. 2. Time histories of the S_1 (a) and S_2 (b) symptoms; broken line represents exponential fit and horizontal line marks the symptom limit value. Intersection of these lines has not been shown in (b) for clarity (θ_l is about 9,500 days).

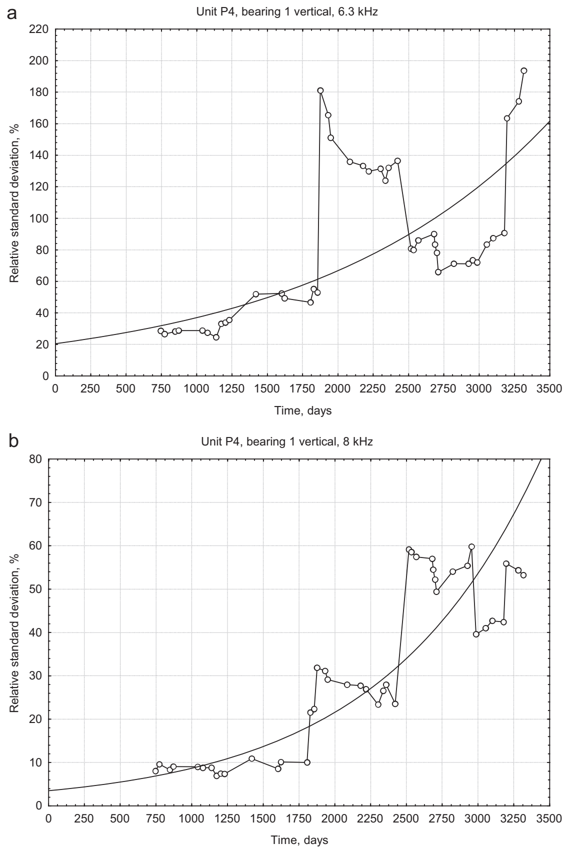

Fig. 3. Exponential fits of σ_{r1} (a) and σ_{r2} (b) time histories.

the exponential fit for σ. Corresponding results for symptoms S_1 and S_2 are shown in Fig. 3. It is easily seen that fitting is far from perfect, due to large 'jumps' of the symptom value that cause stepwise changes of σ. This is particularly evident for σ_1. Exponential increase can nevertheless be seen. For relative standard deviation σ_r ($\sigma_r = \sigma / \hat{S}$) we obtain:

$$\sigma_{r1}(\theta) = 20.51 \times \exp(0.0006 \times \theta) \ , \qquad (10)$$

$$\sigma_{r2}(\theta) = 3.50 \times \exp(0.0009 \times \theta) \ , \qquad (11)$$

where σ_r is given in percent and θ in days.

Functions given by Eqs. (8 ÷ 11) and estimated values of S_{1l} and S_{2l} allow for numerical simulations of limit value excess probability $P = P(S > S_l)$ as a function of time for both S_1 and S_2. Results are shown in Fig. 4. It should be noted here that, in order to reduce the influence of randomness, baseline values S_{01} and S_{02} have been determined by averaging first five measurements rather than simply taking the first measured value.

Simulation results shown in Fig. 4 show that for $\theta < 2500$ days and $\theta < 3000$ days for S_1 and S_2, respectively, limit value excess probability is very low ($P_{1,2} < 0.00001$). At the end of the period covered by observation, i.e. for $\theta \approx 3300$ days, P is still quite low ($P_1 = 0.159$ and $P_2 = 0.017$ for S_1 and S_2, respectively). It can be easily seen, however, that for $\theta = 5000$ days, i.e. about 400 days before estimated θ_l, P_1 is only slightly lower than 0.5. From the point of view of unit operation this means that more than a year before symptom limit value is attained the probability of a false alert is almost 0.5. For S_2 initial values of $\hat{S}$ and σ are substantially lower, but due to higher exponential factor the increase is faster. For $\theta = 5000$ days, i.e. over twelve years before estimated θ_l, P_2 is already over 0.4.

We may note here that $S(\theta)$ fitting and S_l estimation correspond to a 'conventional', or deterministic approach to technical condition development prognosis. Such approach involves some method of symptom limit value determination and hence, as already mentioned, implies a certain plant operation policy. The above considerations show, however, that such policy should also determine the acceptable level of false alert probability. Analysis of data pertaining to real objects clearly shows that this probability is certainly not negligible well before the symptom limit value is attained. With an on-line condition monitoring system, wherein measurements can be taken at arbitrary time intervals, an averaging procedure may be a reasonable alternative: in this way, symptom time history is 'smoothed' and the impact of its statistical nature is reduced. Such systems are, however, usually very costly and their parameters may be limited by the industrial plant environment

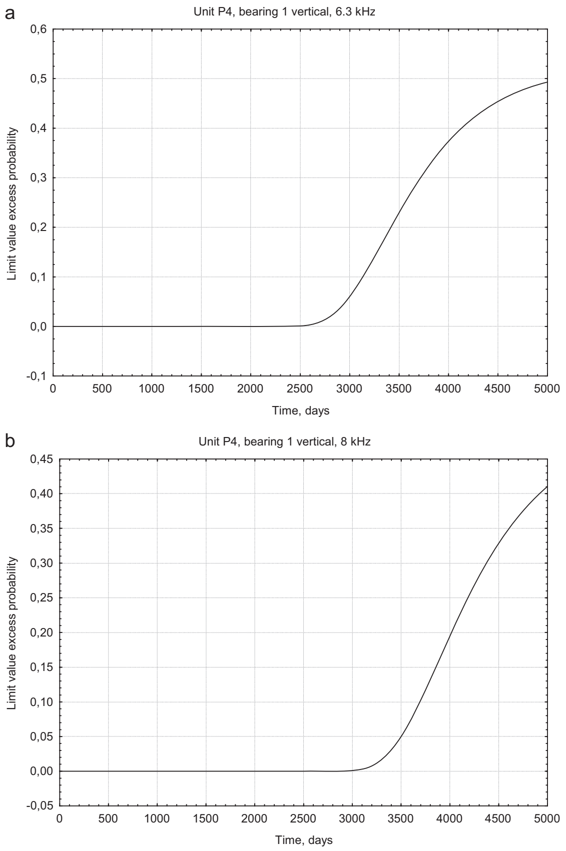

Fig. 4. Symptom limit value excess probability $P(S > S_l)$, calculated for S_1 (a) and S_2 (b).

requirements.[3] On the other hand, with an off-line system, wherein measurements are usually performed at certain time intervals, such averaging is not practical and statistical nature of symptoms has to be accounted for.

5. FUTURE PROSPECTS

The example presented in Section 4 immediately raises at least two questions that have to be addressed.

The first one is related to the method of determining statistical parameters involved. It seems reasonable to assume that $\hat{S}(\theta)$ is adequately represented by fitting a curve to experimental results, providing that the function is properly selected and fitting quality satisfactory. Exponential fitting seems justified for objects that approach θ_b, due to the destructive feedback [1]. It has also been shown experimentally that such fitting yields good results for vibration-based symptoms pertaining to the blade frequency range [2]. Fitting a curve to the $\sigma(\theta)$ experimental time histories is, however, more problematic, which is clearly seen in Fig. 3. One possible reason, and probably the most important one, is the very method of determining σ. Time window should be kept short, in order to fulfill the condition of negligible technical condition change. The shorter the window, however, the worse is the accuracy of estimation. This contradiction is particularly severe if intervals between individual measurements are long, and this is exactly the case in this particular cause. The situation would certainly have been much better if data from a purpose-designed diagnostic experiment had been available. Such experiment should probably be performed with an object characterized by much shorter θ_l. This opens the field for possible future research.

The other question is whether normal distribution of S is justified. It is well known that many physical phenomena can be described by this type of distribution [14]. In this particular case this implies that, at a given moment θ, with a corresponding expected value $\hat{S}(\theta)$ and an arbitrary ΔS, it is equally probable to record symptom value of $S = \hat{S}(\theta) + \Delta S$ and of $S = \hat{S}(\theta) - \Delta S$. In fact this does not seem to be the case. Figs. 1b and 2 clearly indicate that 'upward' jumps are encountered more often than 'downward' ones; in other words, influence of control and interference is likely to result in a measured symptom value increase rather than decrease. A right-hand skewed distribution would thus be more appropriate. Weibull distribution, with the probability density function given by

$$p(S) = \frac{c}{b}\left(\frac{S-a}{b}\right)^{c-1} \exp\left[-\left(\frac{S-a}{b}\right)^{c}\right] \qquad (12)$$

(a – threshold, b – scale, c – shape factors, $S \geq a$) might be more suitable. It seems justified to assume $a = 0$, as no lower limit for the symptom value can be determined. Both b and c must in such case be estimated within a narrow time window, which exacerbates the above-mentioned accuracy problem (with the normal distribution only σ is determined in this manner). In the author's opinion, assumption of a right-hand skewed distribution is very unlikely to change qualitative conclusions presented in Section 4 and render the symptom statistical nature unimportant in technical condition development prognosis. In fact, $P(S > S_l)$ curves might in such case even be steeper than those shown in Fig. 4. Again, a purpose-designed diagnostic experiment might be decisive.

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[3] On-line vibration measurement systems installed on steam turbines usually have the upper frequency limit of a few hundred Hz, due to sensors used. The entire blade frequency range is thus cut off.

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