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MODELING OF THE FRICTION PROCESS IN A FRICTIONAL PAIR OF VEHICLE DISC BRAKE USING PHASE SPACE

Key-words

Modeling of friction, phase space, Lyapunov exponent.

Summary

The paper presents the possibility of applying selected topological tools in the process of the modeling, diagnosing, and monitoring of a friction pair. The paper includes a presentation of the parameters describing the working conditions of a frictional pair in the phase space (PS), which permits the elimination of the time component from the data in the form of a time series, which enables the analysis of non-linear periodic behaviours of the phase point. The trace of the trajectory of the phase point presents the character of the process concerned. This information is unavailable in the traditional analysis preceded by a process of the preparation of experimental data (statistical processes). Interpretation of the oscillation of the system is possible by using a graphic presentation of the results in the phase space. Such a manner of analysing the parameters of the operation of a frictional pair enables the prediction of the condition of the system in a short period of time.

Introduction

The main function of modeling a physical phenomenon is the prognostic function. The second one, but not less important to science development, is an explanation of the physics of examined phenomena. Friction phenomena include many basic physical, chemical, and mechanical mechanisms [6], [7]. Most of them are not recognised as an aspect of the friction phenomena and are not presented in a formal model. An analysis of the literature of friction phenomena modeling points to the lack of a model that would describe this phenomenon in its entirety. This situation influences the basic function of the modeling and understanding its character, making correct interpretation of the operating conditions parameters of the frictional pair difficult. Analysis of the friction process shows its dynamic, non-linear character. The observations of the operating condition parameters of the real frictional pair show the non-regularity and unpredictable behaviour of the parameters value in an assumption of long-term observation. The observed dynamic of the friction phenomena demonstrates a system that undergoes change in a time and can evolve.

1. Implemented modeling method

In order to analyse a complicated dynamic of the friction process, we have to use the tools that serve to analyse non-linear systems, which undoubtedly is the friction phenomena [8].

Phase space

The dynamic systems can be graphically and numerically analysed in phase space (Fig. 1).

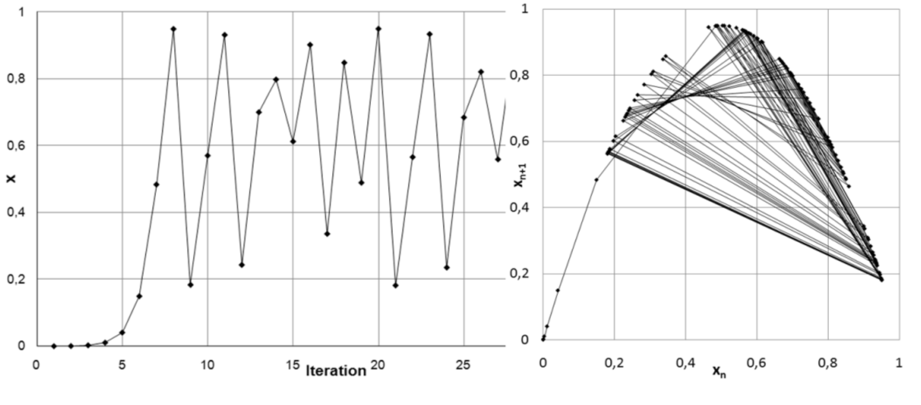

Fig. 1. Classical diagram and phase portrait for logistic mapping, a=3.8

It is a space in which the coordinates are all values needed to unequivocally describe the trajectories of the examined system [1], [2]. The temporary dynamic state of the system is a point in a phase space, and the trajectory is a history of its movement in some period of time. Phase space permits a convenient description of the evolution of the dynamic system as an evolution line in a space of the parameter values. This manner of presenting the data allows the elimination of a time component from a time delay data. This space shows all possible states of the dynamic system. It can be two-, three- or multidimensional. The possibility of showing the trajectories of the phase point in a phase space concerns every system regardless to its characteristic. However, it is a basic tool for the observation and analysis of the trajectories of the phase point.

Lyapunov exponent

The Lyapunov exponent is a measure of the rate at which nearby trajectories in phase space diverge. Chaotic orbits have at least one positive Lyapunov exponent. For periodic orbits, all Lyapunov exponents are negative. The Lyapunov exponent is zero near a bifurcation. In general, there are as many exponents as there are dynamical equations. Only the most positive exponent is calculated here. It is given in units of bits per data sample. Thus, a value of +1 means that the separation of nearby orbits doubles on the average in the time interval between data samples [9].

Hurst Exponent

The Hurst exponent can be estimated such that:

$$H_q = H(q), \tag{1}$$

for a time series

$$g(t) \text{ where } (t = 1,2,\ldots), \tag{2}$$

may be defined by the scaling properties of its structure functions $\mathbf{S}_q(\tau)$

$$S_q \left\langle \left| g(t+\tau) - g(t) \right|^q \right\rangle_T \sim \tau^{qH(q)} \tag{3}$$

where $\mathbf{q} > \mathbf{0}$, τ is the time lag and averaging is over the time window $\mathbf{T} >> \tau$ usually the largest time scale of the system.

In such a case, the value of **X** on average moves away from its initial position by an amount proportional to the square root of time, and we say the Hurst exponent is 0.5. Exponents greater than 0.5 indicate persistence (past trends persist into the future), whereas exponents less than 0.5 indicate antipersistence (past trends tend to reverse in the future). Thus, if we have data with a relatively flat power spectrum, we might integrate it and see if the exponent is close to 0.5, which would imply that it is random and uncorrelated. For real data, the plot of displacement versus time seldom falls along a straight line, in which case the Hurst exponent depends upon the time scale [9].

Topological entropy

The topological entropy of a dynamic system is a nonnegative real number that measures the complexity of the system.
Given any **ε** > **0** and ***n*** **≥ 1**, two points of **X** are **ε** -close with respect to this metric if their first **r** iterates are **ε** -close. This metric allows one to distinguish in a neighbourhood of an orbit the points that move away from each other during the iteration from the points that travel together. A subset **E** of **X** is said to be **(*n*, ε**)-separated if each pair of distinct points of **E** is at least **ε** apart in the metric d_n. Denote by **N(*n*, ε**) the maximum cardinality of an **(*n*, ε**)-separated set. The topological entropy of the map **f** is defined by

$$h(f) = \lim_{\varepsilon \to 0} \left(\lim_{n \to \infty} E \frac{\sup 1}{n} \log N(n, \varepsilon \right) E \tag{4}$$

Topological dynamic systems of positive entropy are often considered topologically chaotic. Positive entropy always implies Li-Yorke chaos defined as the existence of an uncountable scrambled set.

There exist also a possibility to predict the next value of dynamic system by using the maximum-entropy method. This method is good for extracting sharp, discrete lines from an otherwise noisy data record [2].

2. Experimental methods

2.1. Materials

All tests carried out in this research were performed for the cooperating brake pad-brake disc couple friction. The material of the brake pad is a standard one used in an automotive industry. The brake pad is a product of Lucas, series GDB, type 101. Brake disc were performed from gray cast iron ZL 250, with graphite in a form of flakes-table 2, [5], [7].

Table 1. Characteristic of friction couple

kind of specimen	element specification	supplier, type	material
brake pad	element available on the market	Lucas, GDB 101	-
brake disc	designer and performer especially for tests	-	ZL 250, graphite in a form of flakes

2.2. Test stand

The investigation was performed at a stand for carrying out model examinations of disc brakes. The stand was used within the scope of the European grant COST “Superior Friction and Wear Control in Engines and Transmissions” within the project “Friction processes in automotive disc brakes in the presence of hard abrasive particles” [3], [4].

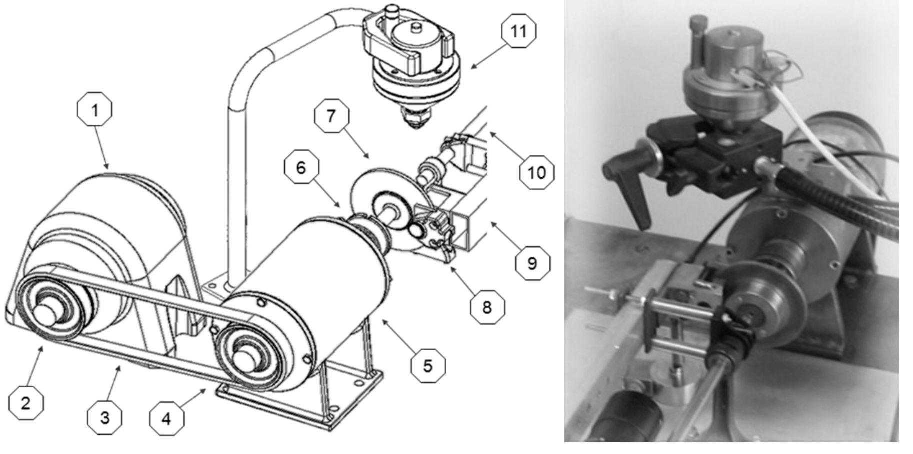

Fig. 2. General schema of the test stand: 1-electric motor, 2-drive wheel, 3-belt, 4-driven wheel, 5-housing of spindle, 6-spindle, 7-brake disk, 9-calliper support, 10-calliper guidance system, 11-feeder of hard abrasive particles [4]

The stand was equipped with major functional systems, such as, a brake disc drive permitting continuous adjustment of rotational speed, a brake system enabling the application of any braking force, and a feeder of hard abrasive particles enabling adjustment of the time and quantity of the particles being fed. The stand had been equipped with systems enabling the taking measurements of the following parameters: rotational speed of the brake disc, braking force, wear, and temperature. The following were adapted for the stand: a hydraulic disc brake calliper, a hydraulic piston pump, a set of weights, a feeding head with electromagnetic drive, a driver of the type PLC SR-12MTDC equipped with transistor outputs, a measuring system Spider 8 with Catman 3.0 software, a set of sensors.

The measuring system is a measuring set by Hottinger Baldwin Messtechnik Gmbh consisting of a central unit called Spider 8, with accuracy class 0.2% and measuring elements. The measuring elements applied are sensors by the HBM Company of accuracy class 0.2%. The measuring system is handled using the Catman 3.2 software.

2.3. Tests conditions

The investigations were performed on a stand for carrying out model examinations of disc brakes according to Table 2. All tests were performed at the same environmental conditions: $T=16^{\circ}C$, p=990 hPa.

Tests no. III, IV, V consist of the following hard abrasive particles fading character:

- t = 0-60 s operating without hard abrasive particles,
- t = 60 s start delivering of hard abrasive particles,
- t = 60–120 s operating with hard abrasive particles.

Table 2. Characteristic of tests

Cooperation parameter\ test name	no. I	no. II	no. III		no. IV		no. V	
			part: a	part: b	part: a	part: b	part: a	part: b
Relative friction Velocity [m/s]	1.5	1.5	3.0		3.0		3.0	
Brake pad pressure [MPa]	0.1	0.1	0.1		0.2		0.5	
operation condition	without hard abrasive particles	with hard abrasive particles	without hard abrasive particles	with hard abrasive particles	without hard abrasive particles	with hard abrasive particles	without hard abrasive particles	with hard abrasive particles

3. Results

Figure 3, 4 presents the course of friction force fluctuation during the cooperation of the elements in the presence of hard abrasive particles (t = 60 s - the moment of dispensing the hard particles). In order to lucidly present the data represented in Graphs 3 and 4 in the phase space, two time intervals were selected. Figure 5 presents the course of the friction force for Test no. IV. The first one (Fig. 5a) is for cooperation without the presence of hard abrasive particles (range t = 20 s - 40 s), the other one (Fig. 5b) is for the cooperation in the presence of hard abrasive particles (range t = 80 s – 100 s). Similarly, Fig. 6 presents the course of the friction force for Test no. V. Figure 6a presents the course for cooperation without the presence of hard abrasive particles (range t = 20 s - 40 s), with the second one (Fig. 6b) for the cooperation in the presence of hard abrasive particles (range t = 80 s - 100 s).

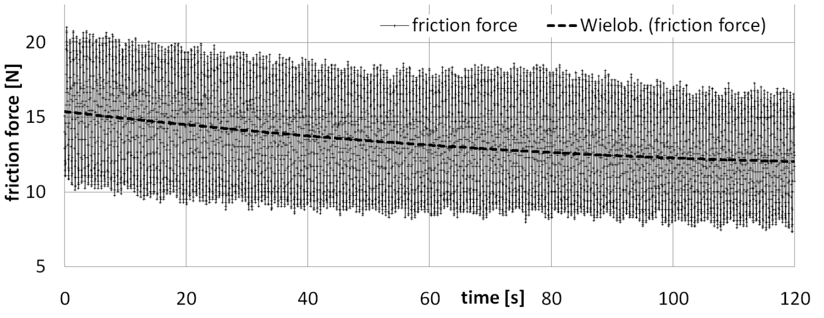

Fig. 3. Friction force vs. time trace for Test no. IV, t = 60 s - the initial moment of dispensing the hard abrasive particle

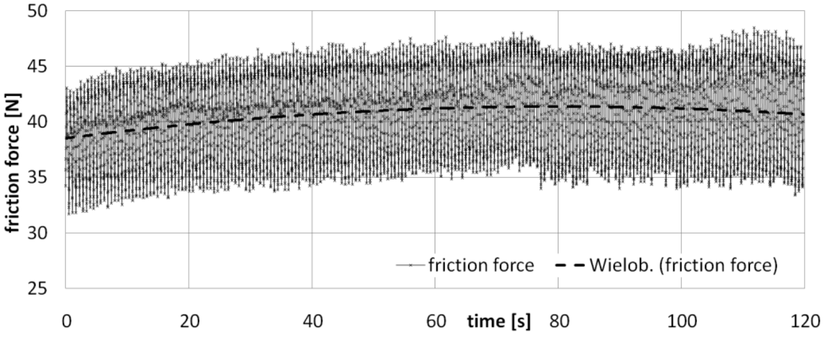

Fig. 4. Friction force vs. time trace for Test no. V, t=60s-the initial moment of dispensing the hard abrasive particle

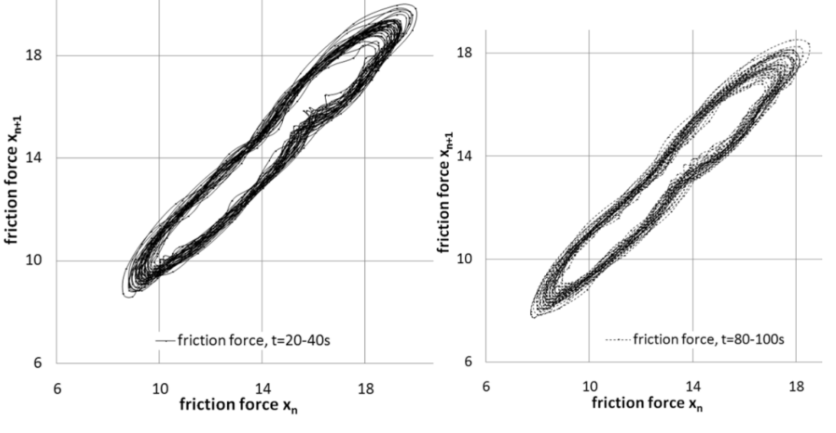

Fig. 5. The curve of friction force course for subsequent orbits of the phase point for Test no. IV: cooperation without participation of solid particles (t=20s-40s)-a, in the presence of hard particles (t=80s-100s)-b

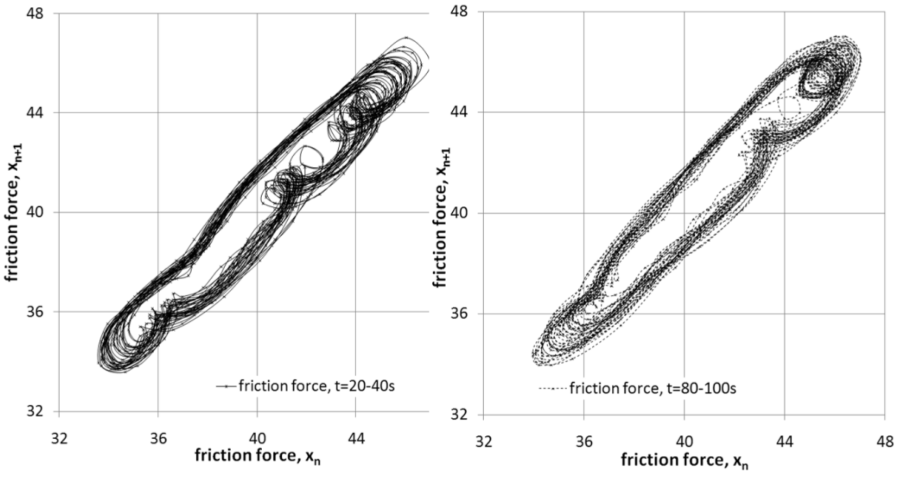

Fig. 6. The curve of friction force course for subsequent orbits of the phase point for Test no. IV: cooperation without participation of solid particles (t=20s-40s)-a, in the presence of hard particles (t=80s-100s)-b

For the presented tests (Table 2), an estimation of some exponents has been performed. The results are presented in Table 3.

Table 3. The numerical results of tests

test no.	I, t=20-100	II, t=20-100	III a, t=10-40s	III b, t=80-110s	IV a, t=20-40s	IV b, t=80-100s	V a, t=20-40s	V b, t=80-100s
Hurst exponent	0.194	0.318	0.179	0.431	0.438	0.445	0.296	0.279
Largest Lyapunov exponent	0.478 ±0.014	0.634 ±0.014	0.449 ±0.030	0.605 ±0.021	0.287 ±0.050	0.293 ±0.046	0.250 ±0.057	0.331 ±0.055
Entropy (approx.)	0.372	0.481	0.388	0.304	0.334	0.296	0.257	0.311
Standard deviation	0.389	0.865	0.330	1.395	3.284	3.088	3.607	3.925

Figures 5 and 6 present the course of the subsequent orbits of the phase point for cooperation while dispensing hard particles (Fig. b), as well as without dispensing (Fig. a). The trajectories obtained of the phase point present some non-linear oscillations. A preliminary analysis of the curves presented in the phase space points to a different character of the orbits for the individual forms of cooperation between the friction elements. This observation suggests the possibility of modeling the operating conditions of a frictional pair based on long-term graphic analysis of the operating parameters.

4. Discussion

For all ranges presented in Table II, estimation of the Lyapunov exponent has been performed. The dominant value of the Lyapunov exponent is positive for each test (Table 3). Moreover, within separate test, significant increase of the Lyapunov exponent for operating in the presence of hard abrasive particles has been observed. This relation can be used for the modeling and diagnostics of the working conditions of the friction couple.

The increase of the standard deviation of the friction force for operating in the presence of hard abrasive particles, for a great majority of the cases, was observed. Only for Test no. IV does this value decrease, which means that proposed criterion in the aspect of the hard abrasive particle detection in friction couple is false.

The value of the Hurst exponent in a range between 0 and 0.5 shows that examined systems do not come under the probabilistic laws. For each characteristic range in tests, the topological entropy was calculated. For an individual range in a certain test, the value of entropy is different, but there is no simple way for correlating of the entropy with the physical condition of friction couple.

5. Conclusion

Presentation of parameters describing the working conditions of a frictional pair in the phase space permits the elimination of the time component from the data in the form of a time series, which enables analysis of non-linear periodic behaviours of the phase point. The trace of the trajectory of the phase point presents the character of the process concerned. This information is unavailable in the traditional analysis, preceded by the process of the preparation of experimental data (statistical processes).

A positive value of the Lyapunov exponent has been obtained, which leads to the following observations:

- It proves that this system is not subject to the laws of probabilistic distribution.

- It implements the occurrence of mathematical chaos in the system being considered.
- It enables prediction of the condition of the system in a short period of time.
- It enables the modeling of the friction process as a chaotic system using symbolic dynamics.

Moreover, the value of the Lyapunov exponent significantly shows the change of the operating conditions of the friction couple (operating with and without participation of solid particles).

On this level of the investigation, the topological entropy does not distinguish and faultless parameter for describing the friction couple because of its sensitivity.

The authors draw the plans for his next investigations in this aspect.

It is very important to say that such a examination (in a PS) can only be a supplement for the examination of the friction couple and cannot exist without the earlier classical description.

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Modelowanie procesu tarcia w hamulcach tarczowych pojazdów za pomocą przestrzeni fazowej

Słowa kluczowe

Modelowanie procesu tarcia, przestrzeń fazowa, wykładnik Lyapunov'a.

Streszczenie

Artykuł przedstawia możliwość zastosowania wybranych narzędzi topologii matematycznej w procesie modelowania, diagnostyki oraz monitoringu pary ciernej. Przedstawienie parametrów opisujących stan pracy pary trącej w przestrzeni fazowej prowadzi do eliminacji składowej czasowej z rozpatrywanego szeregu czasowego danych wejściowych, co pozwala na analizę nieliniowych, okresowych zachowań punktu fazowego. Droga trajektorii punktu fazowego przedstawia charakter procesu tarcia. Informacja ta jest niedostępna w tradycyjnej analizie danych, poprzedzonej wstępną obróbką danych (procesy statystyczne). Interpretacja oscylacji generowanych przez system jest możliwa za pomocą graficznej prezentacji wyników w przestrzeni fazowej oraz parametrów opisujących zachowanie trajektorii punktu fazowego.