DAMAGE DETECTION WITH USE OF ADAPTIVE MODAL FILTER

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Summary

In 1992 J. C. Shelley presented the adaptive modal filter. It was used in active vibrations reduction systems. It is however possible to apply the adaptive modal filter to damage detection according to the modal based diagnostics rules. Proposed method is very simple and bases on tracking of changes of adaptive modal filter coefficients. There are two variants of adaptive modal filter presented in the paper. For both of them the simulation and experimental verification was performed.

Keywords: modal filter, adaptive modal filter, damage detection.

WYKRYWANIA USZKODZEŃ Z ZASTOSOWANIEM ADAPTACYJNEGO FILTRU MODALNEGO

Streszczenie

W 1992 roku J. C. Shelley zaprezentował adaptacyjne filtr modalny. Był on stosowany w układach aktywnej redukcji drgań. Istnieje jednak możliwość zastosowania adaptacyjnego filtru modalnego do wykrywania uszkodzeń zgodnie z zasadami diagnostyki opartej na modelu. Proponowane podejście jest bardzo proste i opiera się na śledzeniu zmian współczynników adaptacyjnego filtru modalnego. W pracy pokazano dwa warianty adaptacyjnego filtru modalnego. Dla obu przypadków przeprowadzono symulacyjną i eksperymentalną weryfikację proponowanego podejścia.

Słowa kluczowe: filtr modalny, adaptacyjny filtr modalny, wykrywanie uszkodzeń.

1. INTRODUCTION

Nondestructive methods of damage detection applied to structural health monitoring can be divided into three groups according to the principle, that they use: methods based on the analysis of the mechanical quantities changes, methods based on the analysis of the electrical, electro-mechanical and electro-magnetic quantities changes and so called other methods. First of these groups can be further divided onto methods which use the measurements of static quantities (deflection, stress etc.) and dynamic quantities in low and high frequency range. The author is interested in the low frequency dynamic methods, that is the methods where vibration measurements are performed in the range up to 1 kHz. The main idea of these methods is so called model based diagnostics, that is comparison of selected model parameters identified for the reference state of the object with the same parameters obtained for the object in the current stage. The difference in the compared value can indicate damage [1]. The most often used model in this approach is the modal model. The modal model is relatively easy to identify, and by means of operational modal analysis, it may be identified only from response data; it is, therefore, very useful in diagnostics. Nevertheless, application of the model-based diagnostics within damage detection has several limitations and faults. First of all there is a serious problem with distinction of the parameters' change resulting from damage and being the consequence of environmental changes e.g. temperature or humidity. From the large group of methods based on the modal model one of the most efficient seems to be the one which uses modal filtration [6, 7]. The method has the following advantages: low sensitivity to the environmental conditions, full modal analysis has to be performer only at the beginning, possibility of automation of the diagnostic procedures, low computational cost. It solves then the basic problems with application of the modal model to the model based diagnostics. Application of the modal filter to damage detection was proposed for the firs time by Gawronski and Sawicki in 2000 [4]. As a damage indicator they used the modal norm, calculated for each measuring sensor location and for each mode from the frequency band of interest. For calculation of these norms the reciprocal modal vector matrix is required, that is the modal filter parameters. Next entire set of obtained modal norms is compared with analogical set stored for the system in reference state. The method allows for damage detection and localization. Disadvantage of the method consists mainly in large number of calculations which are required to be done (modal norms are calculated for each mode and each measuring location). It also suffers from lack of unequivocal damage index, which would allow to detect damage properly by inexperienced personnel.

Another manner of application of the modal filter to damage detection can be found in El-Ouafi Bahlous 2007 [5]. The suggested approach requires vibration data of the system in the undamaged and current stage along with FE model parameterized by means of specified damage parameters. With use of modal filtration of the system response in current stage the residuum function is calculated. The residua turn to be normally distributed with mean value equal zero for undamaged system data and mean value different than zero for the damaged case. To verify the statistical quantities of the residua the generalized log-likelihood ratio test was proposed. This test allows for damage detection. Next the procedure for damage localization and identification is started. It bases on multiple sensitivity and rejection tests (the number of tests equals the number of parameters). Also in this method the required computational power is very high. The biggest disadvantage of this technique is necessity of usage of finite element (FE) model. Additionally the FE model has to be updated with respect to the large number of modes.

Another way of using modal filtering to structural health monitoring was presented by Deraemaeker and Preumont in 2006 [6] Frequency response function of an object filtered with a modal filter has only one peak corresponding to the natural frequency to which the filter is tuned. When a local change occurs in the object – in stiffness or in mass (this mainly happens when damage in the object arises), the filter stops working and on the output characteristic other peaks start to appear, corresponding to other, not perfectly filtered natural frequencies. On the other hand, global change of entire stiffness or mass matrix (due to changes in ambient temperature or humidity) does not corrupt the filter and the filtered characteristic has still one peak but slightly moved in the frequency domain. The method apart from the earlier mentioned advantages, which results from its low sensitivity to environmental conditions has very low computational cost, and can operate in autonomous regime. Only the final data interpretation could be left to the personnel. This interpretation is anyhow not difficult and it does not require much experience. Another advantage of the method results from the fact that it can operate on the output only data.

Method described above was in 2008 extended to damage localization by K. Mendrok [7]. The idea for extension of the method by adding damage localization, bases on the fact, that damage, in most of the cases, disturbs the mode shapes only locally. That is why many methods of damage localization use mode shapes as an input data. It is then possible to divide an object into areas measured with use of several sensors and build separate modal filters for data coming from these sensors only. In areas without damage, the shape of modes does not change and modal filter keeps working – no additional peaks on the filter output. When group of sensors placed near the damage is considered, mode shape is disturb locally due to damage and modal filter does not filters perfectly characteristics measured by these sensors.

There is however another possibility of modal filter application for damage detection. In 1992 J. C. Shelley [8] presented the adaptive modal filter. The basic idea of this technique consisted in on-line tracking of system modal parameters changes and correction of modal filter parameters to make it work for the changed system. It was used in active vibrations reduction systems. It is however possible to apply the adaptive modal filter to damage detection according to the modal based diagnostics rules. The idea is to detect changes in modal filter parameters, which are directly connected with the system structural changes (local changes in mass or stiffness matrices of the system model)..

2. THEORETICAL BACKGROUND

Depending if the excitation force is known or not two variants of adaptive procedure for reciprocal modal vectors updating was proposed [8].

Variant no. 1 with known excitation operates in time domain. What is also important in this version it can work on-line. Entire adaptive procedure is given for single input, real normal mode system, but the technique can be easy extended to the multiple input, complex mode case. The method requires measurements of the inputs and outputs of the systems, and an estimate of the pole for which the reciprocal modal vector will be calculated.

The discrete time modal filter is adaptive modal filter is updated as in equation (1):

$$\psi_{k+1} = \psi_k - 2\mu \cdot e_k \cdot x_k \qquad (1)$$

where: ψ_k – is the estimate of the reciprocal modal vector for sample *k,*
ψ_k – is the updated estimate of the reciprocal modal vector,
x_k – is the vector of system response measurements for sample *k,*
e_k – is the error in the estimate of the modal coordinate for sample *k,*
μ – is the adaptive gain.

The estimate of the modal coordinate is formed by modal filtering the response data:

$$\hat{\eta}_k = \psi_k^T x_k \qquad (2)$$

where: $\hat{\eta}_k$ – is the modal coordinate estimated by the modal filter at sample period k.

The error e_k is the difference between the estimated and actual modal coordinate:

$$e_k = \eta_k - \hat{\eta}_k \qquad (3)$$

The true modal coordinate η_k is not known. An estimate of the modal coordinate, based on the input force history, may be generated by driving a second order system by the measured force *f*. For a real normal mode system, the continues time reference system is:

$$\begin{Bmatrix} \dot{\eta} \\ \ddot{\eta} \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -\Omega^2 & 2\sigma \end{bmatrix} \begin{Bmatrix} \eta \\ \dot{\eta} \end{Bmatrix} + \begin{bmatrix} 0 \\ 2\omega_d \end{bmatrix} f \quad (4)$$

where: Ω – is the undamped natural frequency,
ω_d – is the damped natural frequency,
σ – is the real part of the system pole.

The corresponding discrete time system is:

$$\begin{Bmatrix} \eta_k \\ \dot{\eta}_k \end{Bmatrix} = [A] \begin{Bmatrix} \eta_{k-1} \\ \dot{\eta}_{k-1} \end{Bmatrix} + [B] f_{k-1} \quad (5)$$

where: A and B are the corresponding discretized versions matrices of the system matrices of the equation (4).

The reference system of equation (5) is initiated with zero initial conditions and driven by the measured force f. The error in the reference modal coordinate, due to the incorrect initial conditions, will be inconsequential after the initial condition error decays to the noise floor of the response measurements. If the reference modal coordinate is sufficiently accurate, and an appropriate adaptive gain μ, is chosen, the adaptive modal filter defined by equations (1) through (5) will converge to the modal filter vector which extracts the response of only the mode of interests from the vector of physical response measurements.

The idea of using this approach to damage detection bases on the fact that the update of a system is required only when some changes in the system occur. In this variant of adaptive modal filter the best damage indicator would be the error function e_k. Its value will be different then zero whenever the adaptive procedure will have to work which is always caused by the system structural changes.

The second variant of adaptive modal filter developed by J. C. Shelley [8] does no require the knowledge of operational forces. For this variant it is necessary to assume that the robust modal filter for the structure in a suitable baseline configuration exists. The baseline configuration will be designated as the open loop system, and the system requiring an update of the modal filter will be called closed loop system.

Given a modal filter for an open loop system, an approximation to the closed loop modal filter can be made using the response spectra of the open loop modal filter output. The technique is based on using singular value decomposition (SVD) of the response autospectral matrix to approximate the modal vectors of the system. Obviously, this approach assumes broadband noise as an input and sufficiently smooth response spectra. Likewise, it cannot be implemented in realtime, but can be implemented online. The approach is very similar to the pseudoinverse technique for enhancing single-degree of freedom responses in the complex mode indicator function method of parameter estimation.

The vector of sensor outputs, $x(\omega)$, are transformed to the open loop space, $\eta_o((\omega))$, by premultiplying $x(\omega)$ by the modal filter matrix, ψ_o for the open loop system.

$$\eta_0(\omega) = \psi_0^T x(\omega) \quad (6)$$

One can form the autospectral matrix of the open loop coordinates and decompose it via SVD at each frequency:

$$\eta_0(\omega)\eta_0(\omega)^T = U(\omega)\Sigma(\omega)V(\omega)^T \quad (7)$$

where, $U(\omega) = \lfloor u_1(\omega), u_2(\omega), \cdots, u_{N_0}(\omega) \rfloor$ is the matrix of orthonormal vectors at each frequency that span the column space, $\Sigma(\omega)$ is the diagonal matrix of singular values (sorted in decreasing magnitude), and $V(\omega)$ is the orthonormal matrix that spans the row space of the autopower matrix. N_0 is the number of modes active in the frequency range being considered (i.e. the number of columns in ψ_o)..

By performing SVD at each frequency and plotting out the singular values versus frequency, the dominant motion in the data should show up as peaks in this plot. The dominant shape at a given frequency corresponding to these peaks is the singular vector in the first column of U. For the repeated root case or pole-crossing case, it may be acceptable to choose both the first and the second singular vectors at a given frequency.

The objective is to assemble a matrix of singular vectors corresponding to the dominant motion in the data. Since the data is transformed to the modal space of the open loop system, there must be one singular vector for each mode in the system. Assuming then that this matrix of singular vectors is a reasonable approximation to the closed loop modes represented in open loop space, then a transformation from the open loop modal coordinates to the closed loop modal coordinates is the inverse of this matrix. Thus, the approximation to the mode shapes is represented by:

$$U_{C0} = \lfloor u_1(\omega_1), u_1(\omega_2), \cdots, u_1(\omega_{N_0}) \rfloor \quad (8)$$

and the transformation to the approximate modal space of the modified system is accomplished via equation:

$$\eta_C(\omega) = U_{C0}^{-1}\eta_0(\omega) \quad (9)$$

This approach will break down if U_{C0} is singular or when the response operating shapes do not adequately approximate mode shapes.

In this variant of adaptive modal filter good damage indicator would be the difference between transformation matrix U_{C0} obtained for the system in reference state and the same matrix but calculated for the system in current state. If some damage in the system appears, the adaptive procedure will have to work and the difference will be non-zero.

3. SIMULATION VERIFICATION

Firstly, the author decided to test the procedure on the data from numerical simulation. The 4 DOF model of 4 masses connected in series was used for that purpose. Physical parameters established for the model are gathered together in table 1.

Table. 1. Simulation model parameters

Mass [kg]	$m_1 = 5$; $m_2 = 1$; $m_3 = 1$; $m_4 = 1$;
Stiffness coefficient [N / m]	$k_{01} = 800$; $k_{12} = 150$; $k_{23} = 150$; $k_{34} = 150$;

The following notation was used: the stiffness between mass i and j – k_{ij}. The proportional damping was applied. In order to determine the analytical model of the established system, its equation of motion was formed in the following matrix form:

$$\{f\} = [M]\cdot\{\ddot{x}\} + [C]\cdot\{\dot{x}\} + [K]\cdot\{x\} \qquad (10)$$

For the equation (10), the eigenvalue problem was solved assuming zero initial conditions for displacements and velocities. As a result, 4 conjugated pairs of the system eigenvalues were obtained. On their basis, the modal parameters were calculated: natural frequencies, modal damping coefficients and modal vectors. Next the model was excited in the mass no. 3 with band noise signal of frequency range 0 – 64 rad/sec. The responses in all masses was then calculated and with use of Hv estimate the frequency response functions where computed. In the next step, the reciprocal modal vectors were calculated, i.e. the modal filter was formulated. Now both variants of the adaptive modal filter was tested separately.

Because the first variant operates in the time domain, the model was build in Matlab/Simulink environment. The simulation was performed as follows: in 50 second the stiffness coefficient k_{23} was reduced of 20 % (simulation of damage) and the adaptive procedure for modal filter update was launched. In the figure 1 entire verification procedure is presented. In the figures 2 and 3 the results of modal filtration without the adaptive procedure are presented both in time and frequency domain. In the figure 4 and 5 the result of adaptive procedure operation is shown also in time and frequency domain. In the figure 4 additionally the error function e_k time history is plotted.

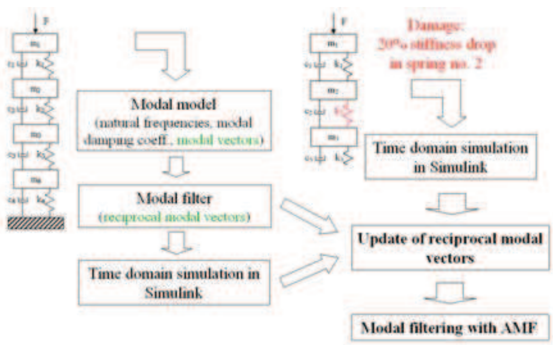


Fig. 1. Procedure for simulation verification of the first variant of the method

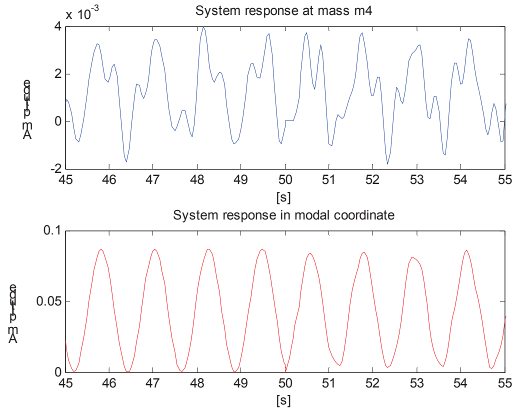


Fig. 2. Results of modal filtration without adaptive procedure in time domain – visible drop of filtration accuracy after 50 s.

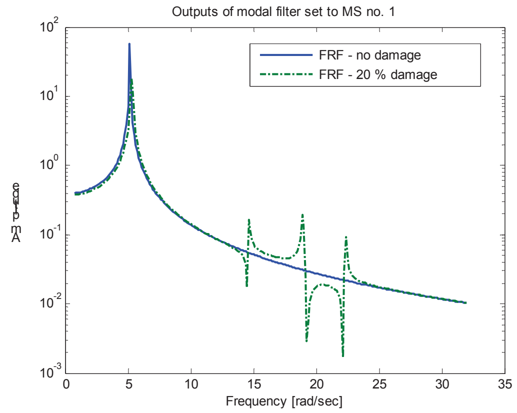


Fig. 3. Results of modal filtration without adaptive procedure in frequency domain

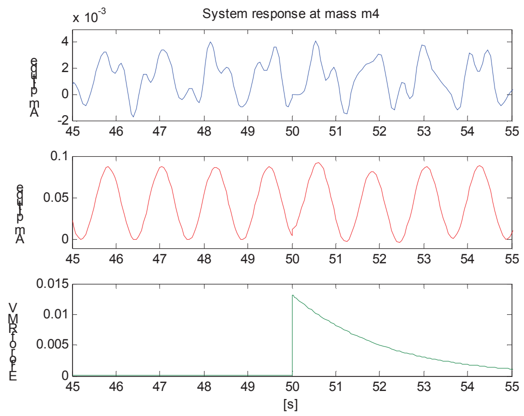


Fig. 4. Results of modal filtration with adaptive procedure in time domain

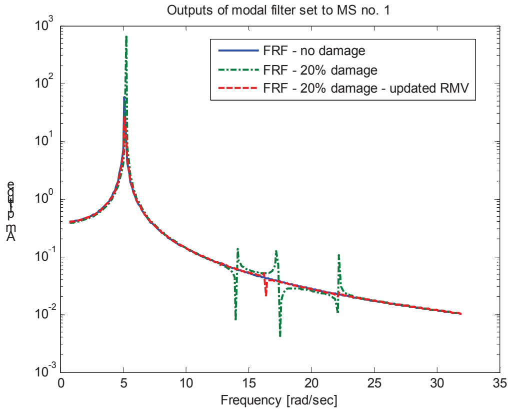


Fig. 5. Results of modal filtration with adaptive procedure in frequency domain

This stage of the first variant of adaptive modal filter verification showed that the procedure works very efficient. Algorithm needed about 5 second to adopt reciprocal modal vectors to the new system. Level of accuracy of modal filtration after updating was satisfying. It can be seen on the frequency domain plots, which were taken in logarithmic scale. The plot of error function presented in the lower part of figure 4 confirms that it can be treated as a good damage indicator. The value of the function grew rapidly when the damage was introduced.

In similar manner the second variant of adaptive modal filter was verified. The same numerical model was used but damage introduced in the system was reduced to the 95 % of original k_{23} stiffness coefficient value. In the figure 7 the plot of first singular value obtained by SVD performed on modal coordinate at each frequency is presented. Results of reciprocal modal vectors update is placed in the figure 8.

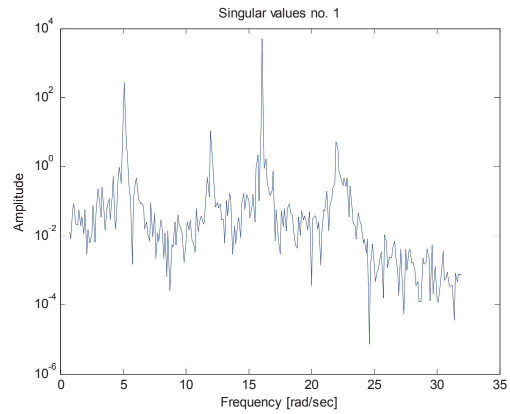


Fig. 7. Plot of first singular value obtained by SVD performed on modal coordinate at each frequency for the damaged model.

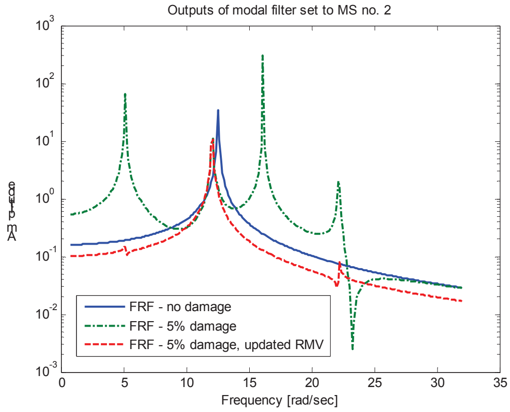


Fig. 8. Results of modal filtration with adaptive procedure in frequency domain

In the figure 9 plot of the difference between transformation matrix U_{C0r} obtained for the system in reference state and the same matrix but calculated for the system with damage U_{C0d}.

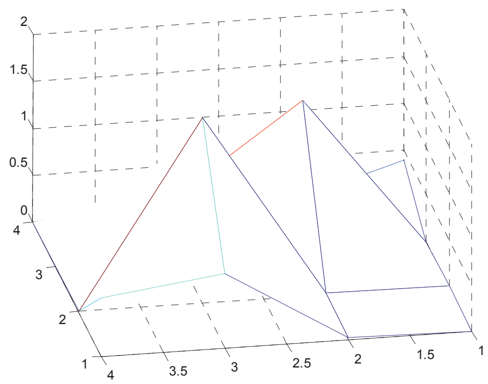


Fig. 9. Plot of proposed damage indicator

The characteristics presented in the figure 8 proves that also this variant of adaptive modal filter works properly. Proposed damage indicator showed that the damage occurred in the system.

4. EXPERIMENTAL VERIFICATION

The laboratory stand used for experimental validation of the proposed damage detection procedure consists of steel frame excited with an electrodynamic shaker. Vibrations were measured by accelerometers placed on the frame. The photo of the test setup without sensors and measuring equipment is presented in figure 10, the network of measuring points is presented in figure 11. The frame has been tested for different damage size. Damage in the frame has been introduced by nicking the upper bar at the measuring point top:7.

There were 4 modal tests carried out for different damage sizes:

- TEST 1 for an undamaged structure,
- TEST 2 for damage at point 7 with a 5 mm-deep gash (12 %),

- TEST 3 for damage at point 7 with a 14 mm-deep gash (35 %),
- TEST 4 for damage at point 7 with a 20 mm-deep gash (50 %)

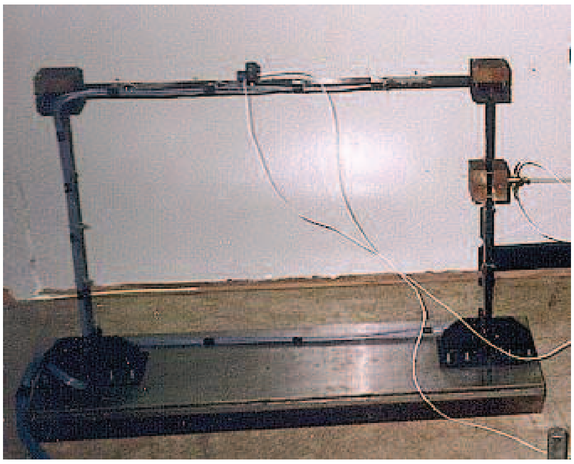


Fig. 10. Laboratory stand

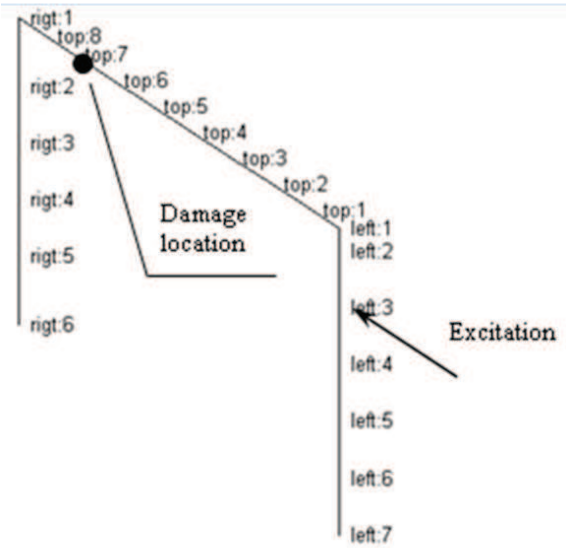


Fig. 11. Measuring point net

Because there were no time data available from this experiment, only the second variant of the method could been verified. The results of the verification are presented in the following figures: no. 12 presents the plot of first singular value obtained by SVD performed on modal coordinate at each frequency, no. 13 – results of reciprocal modal vectors update.

Similarly as it was done for the simulation verification, the difference between transformation matrix U_{C0r} obtained for the system in reference state and the same matrix but calculated for the system with consecutive level of damage U_{C0d} was calculated. The plots of resulting matrices are presented in the figure 14. Additionally the damage index was calculated from this matrices as a Frobenius norm.

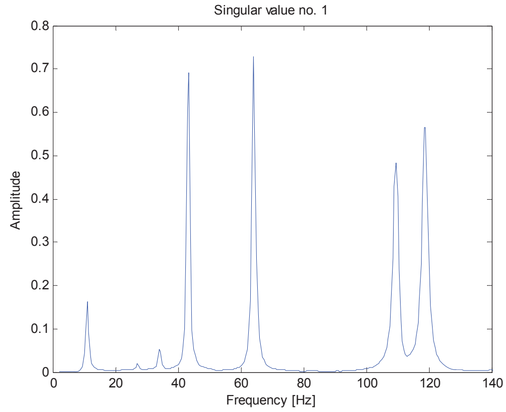


Fig. 12. Plot of first singular value obtained by SVD performed on modal coordinate at each frequency for the damaged model.

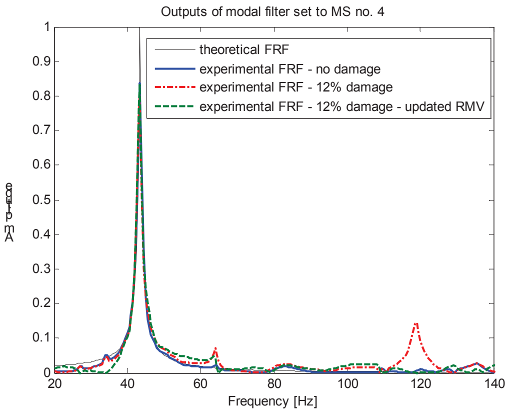
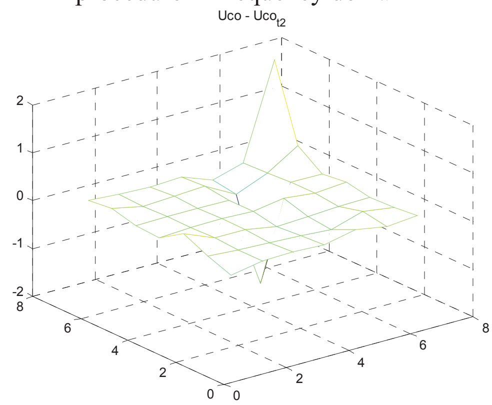
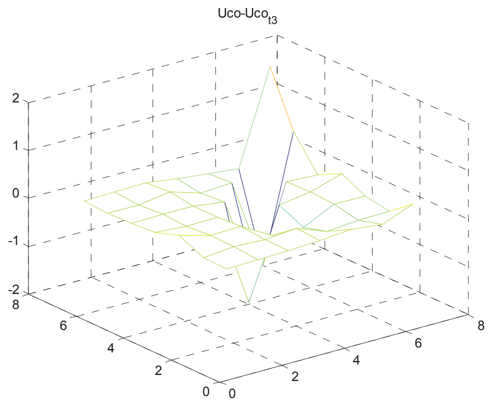


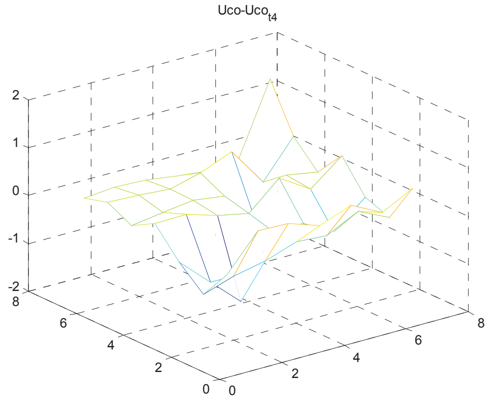
Fig. 13. Results of modal filtration with adaptive procedure in frequency domain



Test T2 - DI = 2.8262



Test T3 - DI = 3.3550



Test T4 - DI = 3.9342

Fig. 14. The differences between transformation matrices U_{C0r} and U_{C0d} for the consecutive tests

Also the experimental verification confirmed good applicability of adaptive modal filter for damage detection.

5. CONCLUSIONS

Both presented types of AMF proved to be efficient for simulation and experimental data. It means that they were able to track the system changes and update the modal filtration results for varying system parameters. Non of the methods however does not allow for 100 % filtration that is full natural modes separation.

In variant with known excitation force very annoying is to determine the value of the gain *m*, which should be selected for each of the measuring direction separately. There are no rules or hints how to find it proper value.

It is possible to use both variants of AMF for damage detection. The error function e_k proposed as a damage indicator for the firs variant of the AMF is much easier in interpretation.

It is necessary, as the future work, to find the way for convenient presentation of the transformation matrix *UC0* changes.

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