

DYNAMIC BEHAVIOR OF A ROTOR SUPPORTED BY ANGULAR BALL BEARINGS

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Summary

The wide range and large scale usage of rolling bearings indicates their necessity and vital contribution to the performance of modern industries. In this paper, a non linear model, predicting mechanical behavior of the loaded angular ball bearing, have been developed. The dynamic behavior of a rotor supported by two angular ball bearings is analyzed. The finite element method is used and the rotor is discretized on beam elements. A mathematical modal taking account different sources of non linearity: the Hertzian contact force and the action of all balls on the bearings inner races, is developed. The Newmark algorithm coupled with Newton Raphson iterative method is used to solve the non linear differential equation iteratively.

Keywords: angular ball bearing, rotor, dynamic analysis, bearing interaction.

1. INTRODUCTION

Rotating contact bearings act as rotary joints between two or more links of a mechanism, with a minimum friction. In rotating machines, rotating contact bearings are a source of internal excitation. They transmit vibrations generated by gears and shafts to the housing.

Stribeck R. [1] and Palmgren A. [2] have developed an analytic model of a rolling bearing loaded in the radial and/or axial direction. These representations are based on two degrees of freedom model. In this model, the authors don't take account of the variation of the loaded contact angle. Jones A. B. [3] has developed five degrees of freedom model (three translations and two rotations of the races), the 6th d.o.f. is the bearing revolution around his axis. He has introduced then the inertia effect (centrifugal force and gyroscopic moment). Simple formulas presented by While M. F. [4] consist on modeling the roller bearing by axial and radial stiffnesses in the ball bearing and cylindrical roller bearing. His study is based on a numeric radial model and the coupling between radial and axial directions is inexistent. Gupta P. K. [5] proposes an analytical model of the rolling bearing (ball and roller) dynamic behavior. He determines the interaction between the rolling elements. He takes note of the cage presence and the lubricant. This work shows the importance of lubrication on the instability of the cage movement. The study proposed by Wardle F. P. [6] shows the relation between the rolling element number and the order of the waviness (geometry imperfection). The author notes the vibration frequencies resulting from the non-linearity relation between displacement and force. These results are validated experimentally. In 1990 years, an interesting and complete study is proposed by Lim T. C. and Singh R. [7]. They suggested an analytic approach based on the determination of a stiffness matrix associated to five

degrees of freedom of the inner race (three translations and two rotations) in its relative movement with respect to the outer race.

The proposed matrix includes the beam flexion and housing coupling for the two types of rolling bearings (ball and roller). A three-dimensional model is proposed by Yhland E. [8]. This model introduces the geometry imperfections and calculates the stiffness matrix. An algebraic non linear eruptional system joining the forces and moments on displacement vector and geometric parameters is developed by Houpert L [9]. No rigidity matrixes are formulated. Datta J. and Farhang K. [10] propose a non linear dynamic model in witch they introduce the masses of each bearing element (cage, inner and outer races and rolling elements). This study permits the prediction of the rolling bearing dynamic behavior in different operating conditions.

Three degrees of freedom model is suggested by Akturk N.¹¹ in which he introduces a geometric imperfection. Lahmar F.¹² has used the formulation developed by Lim and Singh to resolve a non linear dynamic problem of an helical gear system, but any bearing defect formulation in this case is introduced. Jang G. and Jeang S. W. [13, 14] have resolved the dynamic equilibrium of a rotor supported by two angular ball bearing having five degrees of freedom. In next time, they introduce the centrifugal and gyroscopic effects. But the rotor flexibility is ignored.

In our study, we propose in a first time a ball bearing model using the non linear contact between the bearing trail and the rolling elements. The model presented is inspired of the Lim and Singh development (Lim T. C. and Singh R. [7]). The deflection between the rolling elements and the race trail is determined, we can deduct then the forces exerted by balls on the inner race. We are interested in a second time, to resolve the dynamic rotor system equation of motion, the Newton-Raphson

method coupled with Newmark algorithm are used successively. The initial structure degrees of freedom is computed from a static analysis taking account the static effort exerted by bearings on the structure.

2. BALL BEARING MODELLING

The main fundamental components of a ball bearing are the inner race, the cage, the outer race and the rolling elements. The important geometrical characteristics are presented in figure 1. We note the ball diameter D_b , the pitch diameter D_m , the outer and inner raceway groove diameters D_o and D_i , the rolling elements number Z , and the unloaded contact angle α_0 .

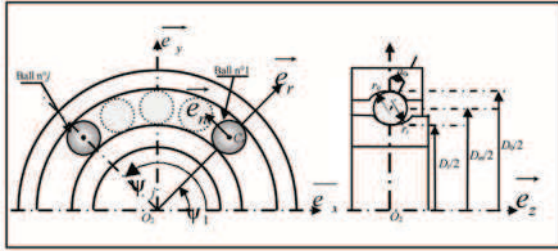


Fig. 1. Ball bearing geometric characteristics

Two co-ordinate systems, shown in Fig. 1, are used. The first is the overall outer race co-ordinate system \mathfrak{R}_1 , where $(x, y, z, \theta_x, \theta_y)$ corresponds to the degrees of freedom of the inner race centre. The second is a local cage co-ordinate system \mathfrak{R}_2 , having the origin at an initial rolling element centre C_r . The outer race centre O_1 is assumed to be a fixed. The degree of freedom θ_z is null corresponding to the bearing axis rotation. A positive nominal contact angle α_0 implies that the angular contact bearing should be loaded in the positive \vec{e}_z direction.

An external load is applied on the inner race, (outer race fixed in her lodging), generates an inner race centre translation $u_2^{\mathfrak{R}_1}(O_2)$ and angular displacement $\Omega_2^{\mathfrak{R}_1}(O_2)$ in the global frame. They are written as:

$$u_2^{\mathfrak{R}_1}(O_2) = \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_{R_1} \quad \Omega_2^{\mathfrak{R}_1}(O_2) = \begin{Bmatrix} \theta_x \\ \theta_y \\ 0 \end{Bmatrix}_{R_1} \quad (1)$$

An elastic deformation of the j th rolling element occurs. It is defined as the total interaction following the normal direction, we can write:

$$\Delta_j = d(\psi_j) - d_0 = \sqrt{\Delta_{rj}^{*2} + \Delta_{zj}^{*2}} - d_0 - w_j \quad (2)$$

where $d(\psi_j)$ and d_0 are, respectively, the loaded and the unloaded relative distance between the inner and the outer raceway groove curvature centers O_{1j} and O_{2j} , and Δ_{rj}^* , Δ_{zj}^* are the radial and axial elastic deformations.

When centrifugal forces are neglected, the loaded contact angles between rolling element – inner race and rolling element – outer race are the same. The loaded contact angle α_j is given:

$$\tan \alpha_j = \frac{\Delta_{zj}^*}{\Delta_{rj}^*} \quad (3)$$

A negative elastic deformation indicates no contact between ball and the two races. For a positive elastic deformation, the ball races contact can be computed from the classical Hertz point – contact theory. The forces exerted by rolling elements on the inner race are computed:

$$\vec{F}_{pal} = \sum \vec{F}_{pal} = \begin{cases} \sum_{j=1}^Z -Kp\Delta_j^{1.5} \cdot \begin{Bmatrix} \cos\alpha_j \cos\psi_j \\ \cos\alpha_j \sin\psi_j \\ \sin\alpha_j \end{Bmatrix} \\ \sum_{j=1}^Z -Kp\Delta_j^{1.5} \cdot \begin{Bmatrix} R_i \sin\alpha_j \sin\psi_j - \alpha_j \cos\alpha_j \sin\psi_j \\ -R_i \sin\alpha_j \cos\psi_j - \alpha_j \cos\alpha_j \cos\psi_j \end{Bmatrix}_{\mathfrak{R}_1} \end{cases} \quad (4)$$

3 FINITE ELEMENT DISCRETIZATION

The study consists on analyzing structural vibrations generated by a rotor coupled by ball bearings. Bearings outer races are fixed in the rigid support (logging) and the inner races are fixed rigidly on the rotating shafts. The finite element method is used: shafts are discretized using beam finite elements with 2 nodes and 6 degrees of freedom per node. The beam section is constant in \vec{e}_z direction. These elements take into account the effects of torsion, bending and tension-compression. The generalised displacement of the j th node is given by:

$$\{q\}^T = \{u_j, v_j, w_j, \varphi_j, \psi_j, \theta_j\} \quad (5)$$

where (u_j, ψ_j) and (v_j, φ_j) are respectively the beam bending in the (\vec{e}_x, \vec{e}_z) plane and in the (\vec{e}_y, \vec{e}_z) plane, w_j and θ_j are respectively the degrees of freedom associated with the axial and torsional deformations.

3.1. Static Analysis

In order to compute the initial structure displacements, a static study is treated. The static equilibrium system is written as:

$$[K_{tot}]\{X_0\} = \{F_0\} + \sum_{i=1}^{N_{balls}} \{F_{pal}(X_0)\} \quad (6)$$

where:

$[K_{tot}]$ global stiffness matrix deduced from beam elements matrices,

$\{F_0\}$ static external force,

$\sum_{i=1}^{N_{balls}} \{F_{pal}(X_0)\}$ action of balls on the inner races

and $\{X_0\}$ is the system degrees of freedom.

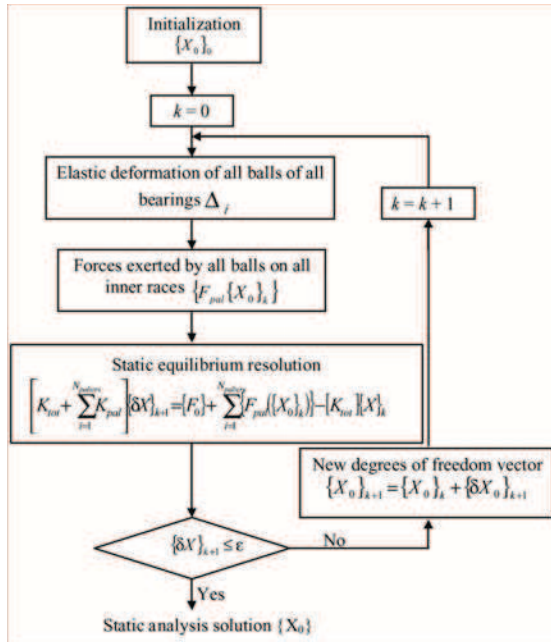


Fig. 2. Static analysis steps

To resolve the equation of motion, we have used the iterative Newton Raphson method, the adopted method for non linear problems, which resolve the system equilibrium (fig. 2).

A shaft supported by two ball bearings is studied. The finite element method is used. An external radial load $F_r = 6000$ N is applied on the middle of the shaft. The shaft has a length L_{palier} , an external diameter $d_{ext} = 50$ mm and internal diameter d_{int} (fig. 3).

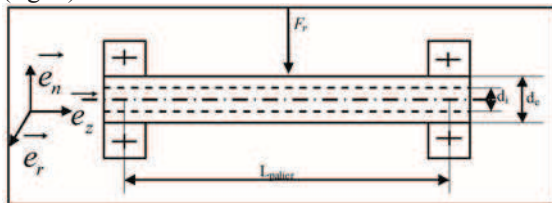


Fig. 3. Shaft modeling

The load distribution on the right ball bearing is presented in the next figures for different configurations.

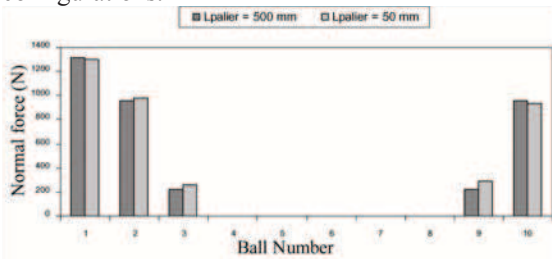


Fig. 4. Shaft length influence ($d_{int} = 0$ mm)

For an internal diameter equal to zero and a shaft length L_{palier} equal to 50 mm and 500 mm, figure 4 presents the load distributions in the right bearing. The distance variation between the load distribution: the shaft bending influence is traduced by a highest value of

the maximal load which changes from 1300 to 1320 N.

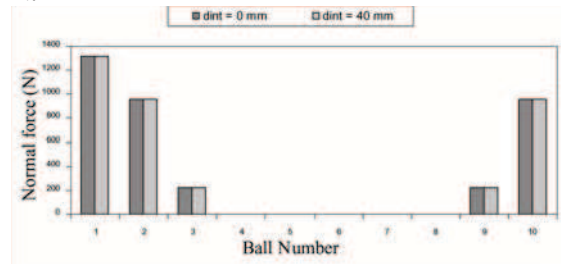


Fig. 5. Shaft diameter influence ($L_{palier} = 50$ mm)

Fig. 5 presents the load distribution on the right ball bearing for an internal shaft diameter equal to 0 and 40 mm. No difference is observed. In fact for a small distance L_{palier} , any bending is observed.

When we change the shaft length $L_{palier} = 500$ mm, figure 6 show a large increase in the maximal load distribution in the right bearing from 1300 N to 1588 N for a hallow shaft. The number of loaded balls increases from 5 to 8. We can interpret that the bearing stiffness is changed when we change the shaft stiffness matrix and naturally when we change the shaft characteristics and the deflection will be more important for a shaft having an internal diameter equal to $d_{int} = 40$ mm.

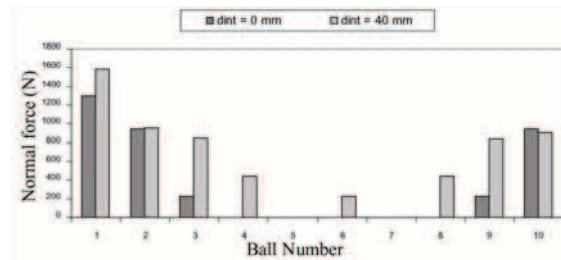


Fig. 6. Shaft diameter influence ($L_{palier} = 500$ mm)

The obtained results show the importance of a coupled model: the bearing stiffness matrix is functioning of the shaft geometric characteristics and the bearing site.

3.2. Modal Analysis

A modal analysis is treated; the natural frequencies and corresponding rotor mode shapes of are computed.

For the time invariant case, the eigenvalue problem of the gear system is:

$$[K_\phi] \{\phi_i\} = \Omega_i^2 [M_\phi] \{\phi_i\} \quad (7)$$

where Ω_i are the natural frequencies, $[K_\phi]$ is the modal stiffness matrix, $[M_\phi]$ is the modal mass matrix normalized to the identity, and ϕ_i the eigenvector matrix.

The eigenfrequencies associated to the rotor are recapitulated on table 1.

Table 1. Rotor Natural Frequencies

Mode number	Frequency (Hz)
1	19,87
2	19,87
3	54,18
4	54,18
5	575,22

3.3. Dynamic Response

We are interested now to the system dynamic behaviour. The system equation of motion, taking account of the beam and bearings presence is written:

$$\begin{aligned}
 & [M_{tot}]\{\ddot{X}\} + [C]\{\dot{X}\} + [K_{tot}(t)]\{X\} \\
 & = \{F_0\} + \sum_{i=1}^{N_{pal}} \{F_{pal}(t, X)\}
 \end{aligned}
 \tag{8}$$

Where

$[M_{tot}]$, $[C]$ global mass and damping matrices,

$[K_{tot}(t)]$ global stiffness matrix,

$\{F_0\}$ static external force,

$\sum_{i=1}^{N_{pal}} \{F_{pal}(t, X)\}$ forces exerted by ball bearings on the inner race given by equation (4).

In order to resolve equation (8), Newmark method coupled with the iterative Newton Raphson method, which resolve the system equilibrium at each step, are used. The system equation of motion is projected and resolved on a chosen modal basis, and then the temporal responses are therefore obtained by modal recombination. (figure 7)

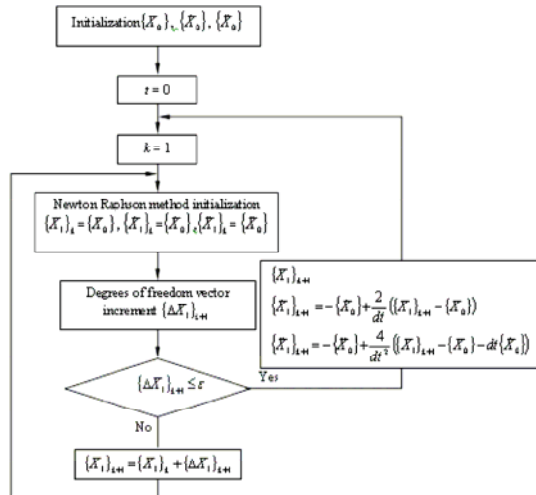


Fig. 7. Dynamic Analysis process

A disk is placed on the shaft middle. The rotor geometric characteristics are given by table 2. The rotating frequency is $f_i = 50 \text{ Hz} = 3000 \text{ tr/mn}$.

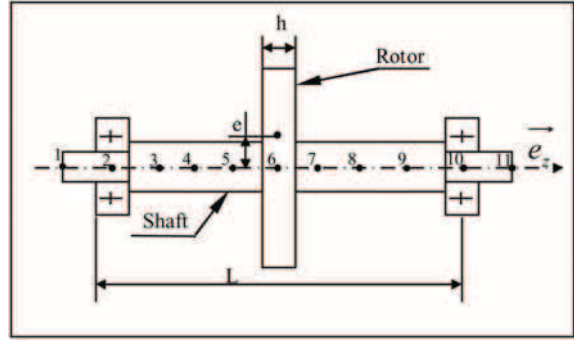


Fig. 8. Rotor supported by two angular ball bearings

Table 2. Rotor Geometric Characteristics

Geometric characteristics	Dimension
Shaft length L	500 mm
Shaft Diameter d_{ext}	50 mm
Rotor Mass M_D	10 Kg
Disc thickness h	50 mm
Rotor Mass Unbalance m	0,1 Kg

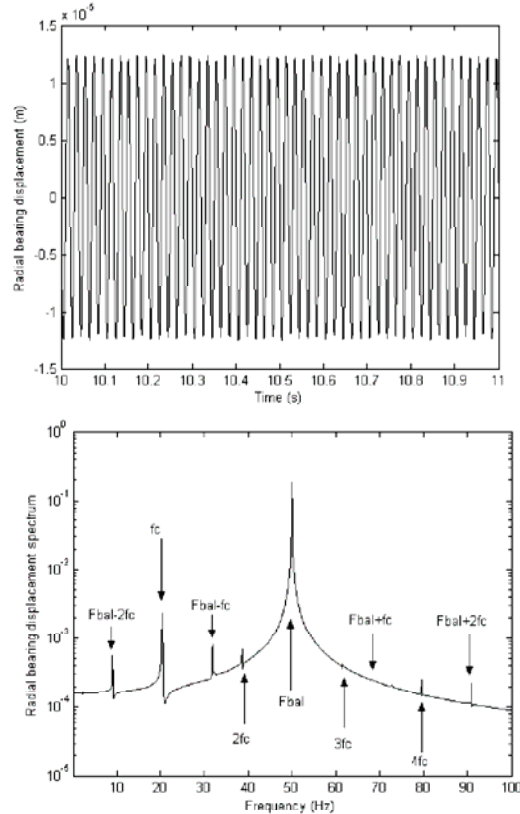


Fig. 9. Temporal and Spectral Signature of the left inner race center displacement

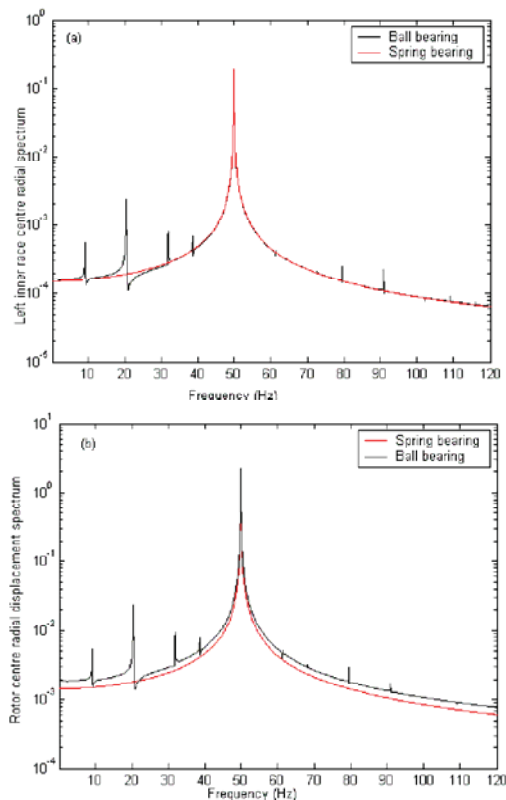


Fig. 10. Left inner race center (a) and disc center (b) displacements spectrum for a rotor supported by angular ball bearings and spring bearings

Figure 9 show the left inner race centre spectrum. We note the presence of the unbalance frequency $F_{bal} = 50$ Hz, its origin is the time variation of the unbalance load, and the presence of the cage frequency $f_c = 20$ Hz, where the origin is the bearing rotation and then the variation of the load exerted by all rolling elements on the rotor.

We note also a modulation of the unbalance frequency due to the cage frequency having a value $F_{bal} \pm if_c = 30$ Hz and 70 Hz for $i = 1$, $F_{bal} \pm if_c = 10$ Hz and 90 Hz for $i = 2$.

Figure 10 show the left inner race centre (a) and rotor centre displacements (b) spectrum. We note the presence of unbalance frequency in the two cases and for the two modelizations (ball bearing and spring bearing). The vibratory level is more important for the rotor centre displacement (point of application of the unbalance) than the bearing displacement.

4 CONCLUSION

In this study, a rotor is studied taking account the ball bearing incidence. In a first time a static study is treated. We note the importance of a coupled model; the bearing stiffness matrix is functioning of system geometric characteristics and bearings positions.

In a second time, a modal analysis is realised. The system natural frequencies are computed.

Finally, a dynamic study is presented. The displacements spectrums show the presence of two

characteristics frequencies: the unbalance frequency which the origin is the unbalance load variation, the cage frequency which the origin is the bearing rotation. We note also a modulation of the unbalance frequency due to the bearing frequency.

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