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NUMERICAL STRESS ANALYSIS OF HARMONIC DRIVE ELEMENTS BY BEM

Key words

flexspline, harmonic drive, stress analysis, boundary element method.

Summary

The paper presents an analysis of the influence of design features of the flexspline in a harmonic drive, such as the relative radial deformation w_o/m, the relative coating thickness g/d_f, and the design features of the basic rack tooth profile, on stress values in the bottom lands of a toothed rim. In the analysis of the state of stress, the boundary element method (BEM) was applied.

Introduction

An increase of working loads transferred by harmonic drives causes an increase of forces acting on the teeth of their toothed rims. An important issue concerning the strength calculation of harmonic drives is the evaluation of the influence of stress values in the bottom lands of the toothed rim on the strength of the flexspline. As the results of empirical tests show, the bottom lands of the toothed rim are the places of reduced strength [1]. A common phenomenon preceding the destruction of a flexspline is its cracking in the place of a local increase of stress values in the toothed rim [1].

In order to make the strength calculations and the choice of design features of a harmonic drive more specific, it is justified to use precise methods of determining stresses, which will facilitate an analysis of the influence of geometric features of toothed wheels and the basic rack tooth profile parameters on the strength. Currently, in order to calculate the stress values in the tooth base, one of the two numerical methods is used: the finite element method (FEM) [2], [3], [4], or the boundary element method (BEM). The latter, in comparison to FEM, requires a considerably smaller number of computing nodes located within the tooth profile and a respectively smaller number of linear equations necessary to solve the task.

In the analysis of the state of stress presented in the paper, the boundary element method was used to determine the stress values in the bottom lands of the toothed rim of a flexspline in a harmonic drive co-operating with a mechanical cam wave generator. In numerical calculations, the software developed at the Faculty of Transport of the Silesian University of Technology was used [5]. The way in which the flexspline was loaded and supported as well as the numerical BEM model employed in the calculations are presented in paper [6], [7].

In this project, an analysis has been carried out regarding the effect of the adopted types of mechanical wave generators and the design features of the flexspline, including the relative radial deformation wo/m and the relative coating thickness g/df, on the distribution of stresses in the bottom land. In addition, an analysis has been made of the influence of the selected design features of the basic rack tooth profile on the stress values in the bottom lands of the teeth in the toothed rim of a flexspline.

1. Calculation results

The results presented concern the influence on the stresses of the selected design features of the flexspline: the relative radial deformation $w_o/m \in (0.9; 1; 1.1)$, and the relative thickness, $g/d_f \in (0.009; 0.012; 0.015)$, where: w_0 - is the maximum radial deformation of the wheel, m – the module, g – the wall thickness under the toothing, and d_f – the inner diameter of the wheel.

The influence of the following relative values was also analysed: the height of the head of basic rack tooth profile h_{ao} and the curve radius of the head of basic rack tooth profile assuming the following values: $\rho_{ao} \in (0.2; 0.25; 0.3; 0.38)$. Toothing of the flexspline was made assuming the following parameters for the basic rack tooth profile:

– Profile angle $\alpha_{on} = 20^o$,
– Relative curve radius of the head of the basic rack tooth profile $\rho_{ao} = 0.2–0.38$, and
– Relative height of the head of the basic rack tooth profile $h_{ao} = 1.25; 1.35$.

The influence of the adopted parameters on the stress values in the bottom lands of the teeth was investigated for cases when the gear worked without external load and with nominal load. Figure 1 shows an exemplary deformation of the half of flexspline under the influence of the applied load coming from the wave generator and the forces between the teeth. As presented in Fig. 1, the buckling of the flexspline in the respective area indicates that the numerical model developed is correct.

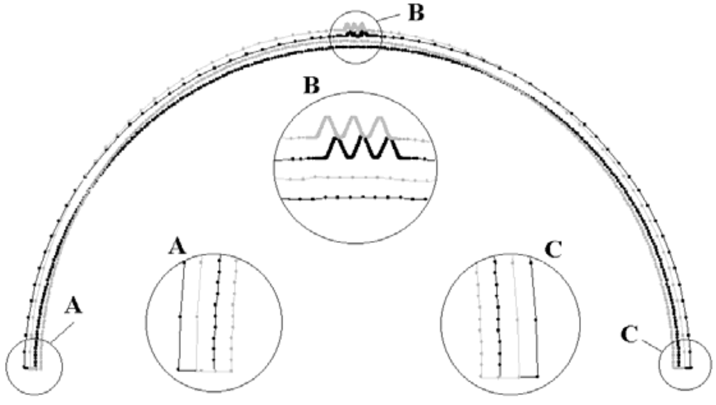


Fig. 1. Deformation of flexspline

An example of a bottom land profile for ρ_{ao} = 0.38, with a marked numeration of the BEM lattice nodes, is presented in Fig. 2. In the figures, the computation point A refers to the maximum value of stress in the proximity of the bottom land of the teeth; whereas, point B refers to the value of stress in the tooth base.

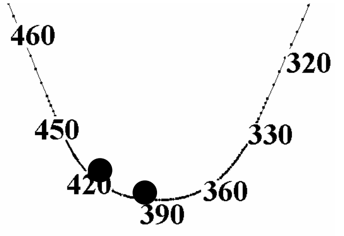


Fig. 2. Example of a bottom land of the teeth with a numbered BEM lattice nodes for ρ_{ao} = 0.38

Figure 3 presents the distribution of stresses in the bottom land, depending on the adopted types of mechanical wave generators. With respect to the state of stress, it is advantageous to use cam or disc generators, which cause the stress values in the bottom land to be similar, which is respectively lower than while using a roll generator.

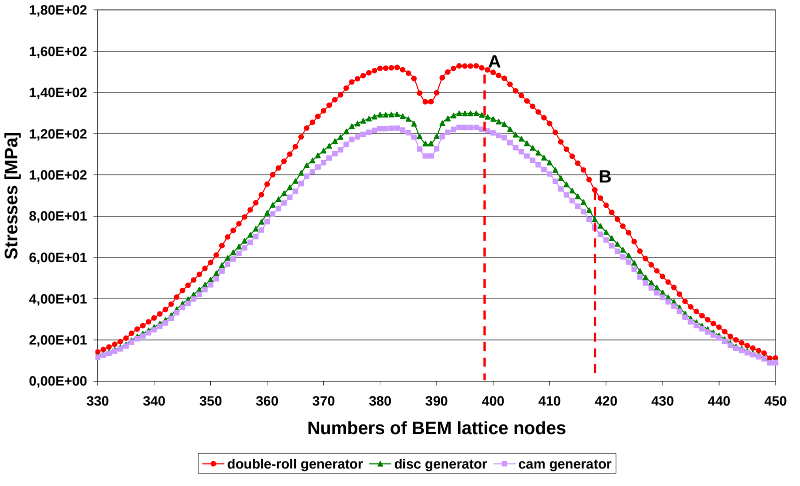


Fig. 3. Stress distribution in the bottom land of a tooth, depending on the wave generator, with: $g/d_f = 0.012$; $w_0/m = 1$; $M_{nom} = 0$

The rest of the calculation results, presented in Figs. 4 to 6, refer to the interaction of the flexspline with the cam wave generator. The influences of the design features of the flexspline, relative radial deformation w_o/m, and the relative coating thickness g/d_f, are presented in Figs. 4 and 5, respectively.

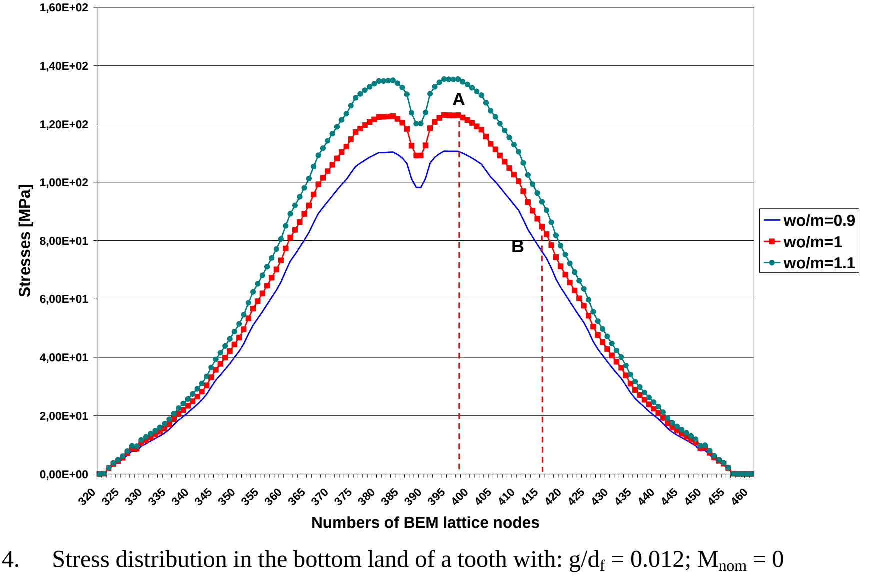


Fig. 4. Stress distribution in the bottom land of a tooth with: $g/d_f = 0.012$; $M_{nom} = 0$

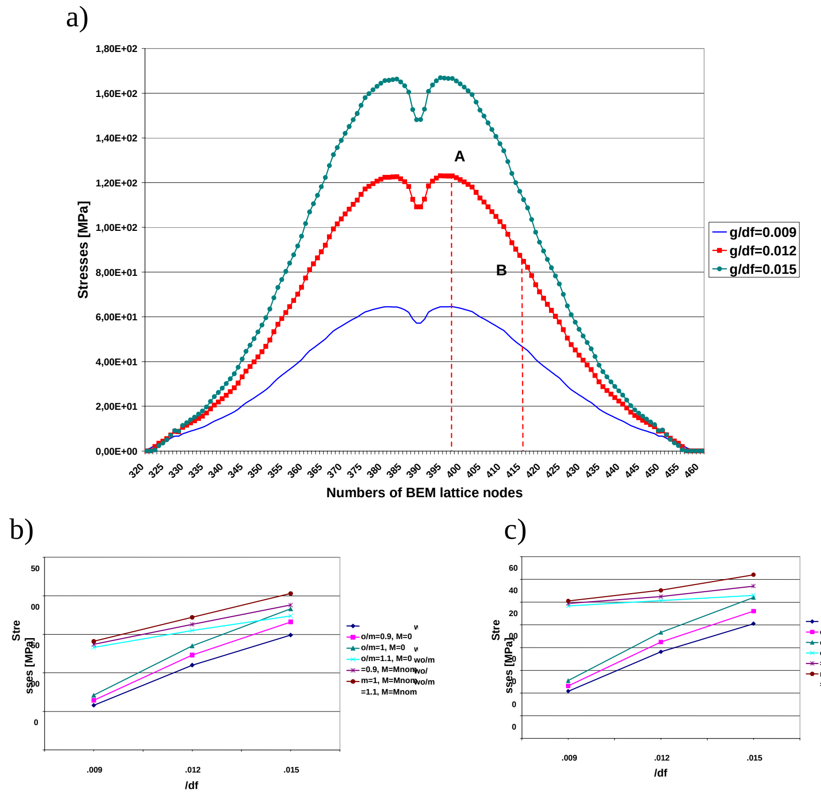
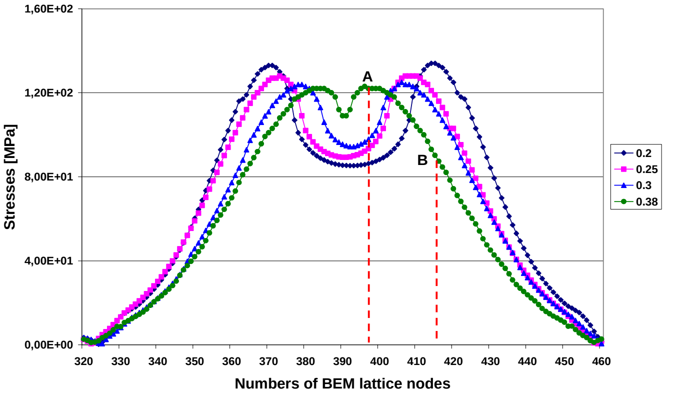


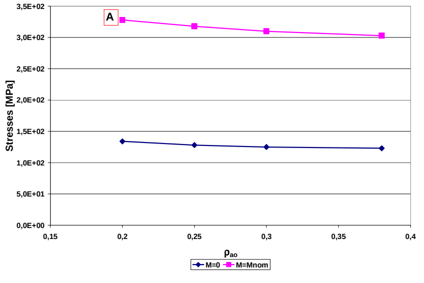
Fig. 5. The influence of the relative coating thickness g/d_f on a) the distribution of stresses in the bottom land of a tooth with $w_o/m = 1$; $M_{nom} = 0$, b) the value of stresses in the bottom land (point A), and c) the value of stresses in the tooth base (point B)

The influences on stress of relative values of the curve radius of the tool head ρ_{ao}, assuming the following optimal design features of the flexspline are the relative radial deformation, $w_o/m = 1$, and the relative wheel thickness, $g/d_f = 0012$, are presented in Fig. 6.

a)



b)



c)

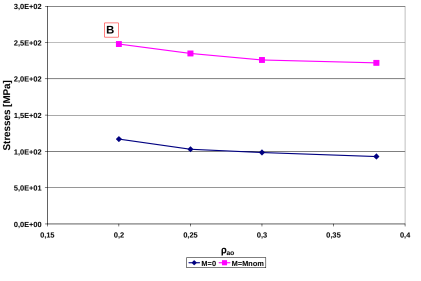
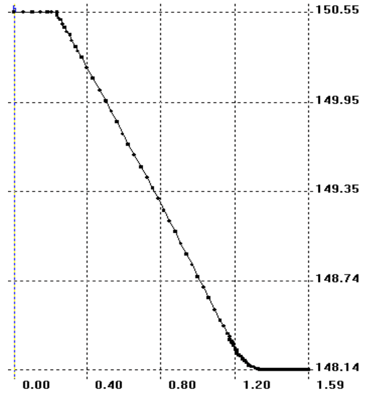


Fig. 6. The influence of the curve radius of the tool head ρ_{ao} on a) the distribution of stresses in the bottom land of a tooth, with: g/d_f = 0,012; w_0/m = 1; M_{nom}= 0, b) the value of stresses in the bottom land (point A), and c) value of stresses in the tooth base (point B)

a)



b)

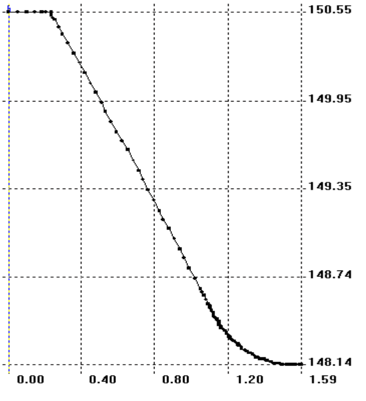


Fig. 7. The lateral contour lines of the tooth for a) ρ_{ao} = 0.1; b) ρ_{ao} = 0.38

Conclusions

This paper presents strength calculations for the teeth in toothed rims of a flexspline of a double harmonic drive by means of BEM. In the numerical analysis conducted, the influence was investigated of the design features of a flexspline and of the basic rack tooth profile on the value of stresses in the bottom lands of the rim teeth. The results of numerical calculations correspond in terms of their quality to the results presented in the literature [8], [9]. On the basis of the analysis of the results the following conclusions can be formulated:
1. With respect to the state of stress in the bottom lands of the teeth, it is beneficial to use cam or disc wave generators (Fig. 3).
2. The bottom lands of toothed rims are the places exposed to increasing stresses. The values of stress in the tooth base are respectively smaller than those in the proximity of the bottom land, which corresponds to the results of the empirical research [1].
3. The increase in the relative thickness of the coating (Fig. 5) and the growth of the relative radial deformation (Fig. 4) cause an increase of stress in the bottom lands of the rim teeth.
4. A decreased value of the curve radius of the head of the basic rack tooth profile results in a change in the width of the tooth bottom land, a shortening of the transition curve in the tooth base, and a reduction of the tooth thickness at its base (Fig. 7), which in turn leads to increased values of stress. Yet, the influence of the relative curve radius of the head of the basic rack tooth profile on the stress value is insignificant (Fig. 6).

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Recenzent:
Wiesław OSTAPSKI

Numeryczna analiza stanu naprężenia elementów przekładni falowej z wykorzystaniem MEB

Słowa kluczowe

Koło podatne, przekładnia falowa, analiza naprężenia, metoda elementów brzegowych.

Streszczenie

W opracowaniu przeprowadzono analizę wpływu przyjętych rodzajów mechanicznych generatorów fali oraz cech konstrukcyjnych koła podatnego: względnej deformacji promieniowej w_o/m i względnej grubości powłoki g/d_f na rozkłady naprężeń w dnie wrębu. Ponadto wykonano analizę wpływu wybranych cech konstrukcyjnych zarysu odniesienia na wartości naprężeń w dnach wrębów międzyzębnych wieńca zębatego. W przeprowadzonej analizie stanu naprężenia wykorzystano metodę elementów brzegowych (MEB).