VIBRATION BASED DAMAGE DETECTION USING LAPLACE WAVELET

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Summary

In work the Laplace wavelet definition is given and it utilization to the damage diagnostics. The Laplace wavelet is a complex, analytic damped exponential wavelet and it is desirable wavelet basis to analyze signals of impulse response. A correlation filtering approach is introduced using the Laplace wavelet to identifying the modal parameters from vibration signals.

Considered in work damage were: small crack in steel beam and change (the decrease) the tension force in the prestressed element. Based on simulated (cracked beam) and measured (prestressed beam) signals the natural frequencies and modal damping ratios have been determined.

Keywords: damage detection, Laplace wavelet, natural frequency, modal damping.

WYKORZYSTANIE FALKI LAPLACE'A W DIAGNOSTYCE USZKODZEŃ

Streszczenie

W pracy podano definicję falki Laplace'a i sposób jej wykorzystania do celów diagnostyki uszkodzeń. Falka Laplace'a jest to pewna analityczna, zespolona i tłumiona funkcja wykładnicza co ułatwia analizę sygnałów odpowiedzi impulsowych. Dla tak zdefiniowanej falki zastosowano filtrowanie korelacyjne co pozwala na wyznaczenie parametrów modalnych analizowanego układu, na podstawie analizy jego odpowiedzi impulsowej.

Analizowanymi uszkodzeniami były: „małe" pęknięcie w belce prostoliniowej i zmiana siły sprężającej w elemencie wstępnie sprężonym.

Słowa kluczowe: diagnostyka, falka Laplace'a, częstości własne, tłumienie modalne.

1. INTRODUCTION

Damage in structural element caused changing in its natural frequencies [3, 6, 7]. The simplest way to detect natural frequency is using the Fourier transform of impulse response of damaged element. The Fourier transform uses the basis of infinitely long sinusoids, and it is thus not ideal for non-stationary signals. In work the Laplace wavelet definition is given and it utilization to the damage diagnostics. The Laplace wavelet is a complex, analytic damped exponential wavelet and it is desirable wavelet basis to analyze signals of impulse response. A correlation filtering approach is introduced using the Laplace wavelet to identifying the modal parameters from vibration signals.

Considered in work damage were: small crack in steel beam and change (the decrease) the tension force in the prestressed element.

In case of crack, these changes in natural frequencies can be used for detection [2, 6, 8] and in some cases identification of crack [4, 5].

2. ANALYSED SYSTEMS

Based on natural frequency and Fourier transform it is difficult to find a little damage in systems. The changes in physical properties due to a little damage and associated with it changes in natural frequencies can be smaller than error in signal processing (i.e. FFT).

To overcome this problem the utilization the Laplace wavelet in two different systems is showed.

<u>Cracked beam model.</u> In work the simply supported with crack showed in fig.1 was analysed.

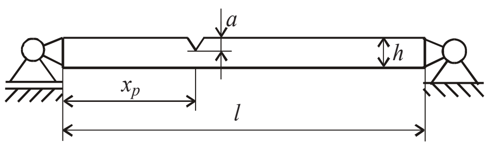

Fig. 1. Analysed beam with crack

Proposed method signal processing allow to detect a crack in an early stage. At work the crack of 5 % beam height is analysed. In simulation the crack is substituted by rotational spring, which flexibility is calculated by using Castigliano theorem and laws of the fracture mechanics.

<u>Prestress losses.</u> In work the decrease in tension force N in the prestressed element was analysed.

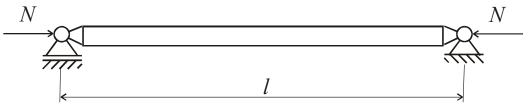

Fig. 2. Analysed prestressed beam

Detection of prestress losses is very important especially in vibration based diagnosis or monitoring.

Prestress is applied mostly to concrete beam because it has very little tension strength and due to prestress concrete beams can be bend [1]. Loss in prestress causes to occur tensile stress in concrete what can lead to crack formation in element. Unfortunately both failure occur in different directions in changes of natural frequency.

In fig. 3 the curve for beam with both damages but having constant value of natural frequency is showed. It means for each point of this curve beam have two different damage but have natural frequency equal to natural frequency of undamaged beam.

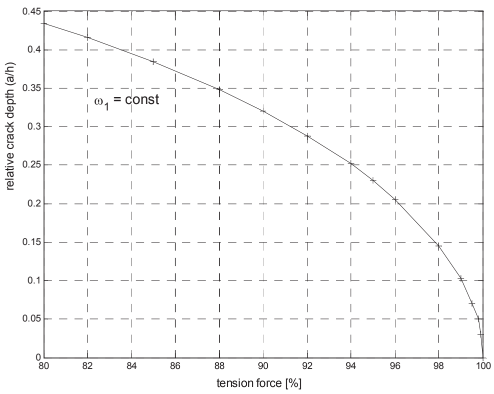

Fig. 3. Damaged beam with no change in natural frequency

In fig. 4 the impulse response function is showed for undamaged and damaged (cracked) beam.

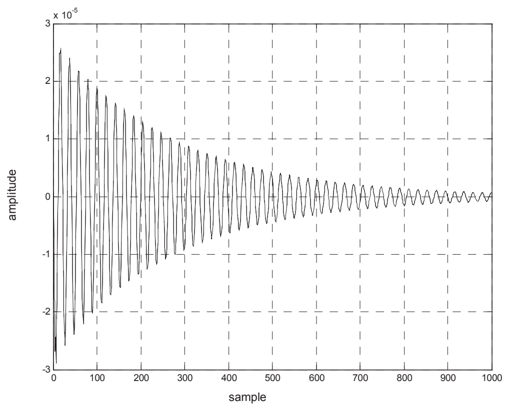

Fig. 4. Impulse response function

For FFT in both cases one can obtain the same natural frequency. Because of this the Laplace wavelet is introduced in next point of this work.

3. LAPLACE WAVELET

The Laplace wavelet is a complex, analytic damped exponential wavelet and it is desirable wavelet basis to analyse signals of impulse response. Using the Laplace wavelet one can identify not only natural frequency but also modal viscous damping ratio.

The Laplace wavelet is defined as:

$$\psi(f,\zeta,\tau,t) = Ae^{-\frac{\zeta}{\sqrt{1-\zeta^2}}\cdot 2\cdot\pi\cdot f(t-\tau)} \cdot e^{-j2\cdot\pi\cdot f(t-\tau)} \qquad (1)$$

for $t \in \langle \tau, \tau + D \rangle$ and $\psi(f,\zeta,\tau,t) = 0$ for the others t, where $j = \sqrt{-1}$, f - frequency $f \in \Re^+$, ζ - viscous damping ratio $\zeta \in (0,1)$, and time index τ. The range D ensures the wavelet is completely supported and has nonzero finite length equal D.

Example of Laplace wavelet in fig. 5 is shown.

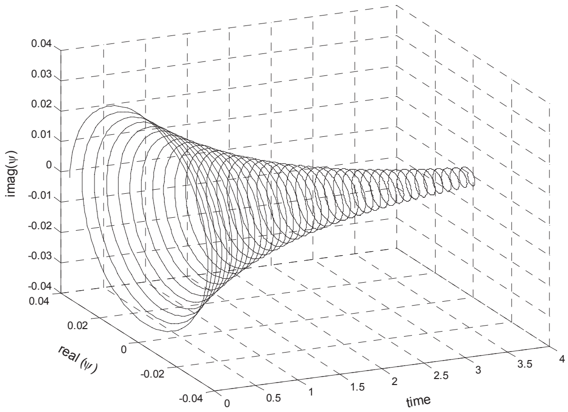

Fig. 5. Laplace wavelet for f = 10Hz, ζ = 0.01, D = 4s

4. CORRELATION FILTERING

Correlation can be measured by using an inner (dot) product operation written as:

$$\langle \psi_\gamma(t), x(t) \rangle = \|\psi_\gamma(t)\|_2 \cdot \|x(t)\|_2 \cdot \cos(\theta) = \int_{-\infty}^{+\infty} \psi_\gamma(t) \cdot x(t)\, dt \qquad (2)$$

where: $\gamma = \{f, \zeta, \tau\}$.

A correlation function $c_\gamma \in \Re$ is defined by:

$$c_\gamma = \frac{|\langle \psi_\gamma(t), x(t) \rangle|}{\|\psi_\gamma(t)\|_2 \cdot \|x(t)\|_2} \qquad (3)$$

Peaks of c_γ relate the wavelets with the strongest correlation to the signal:

$$\chi = \max(c_\gamma) \qquad (4)$$

The frequency f and damping ζ associated with the χ indicate the modal parameters of the system.

In fig. 6 an example signal correlation is showed.

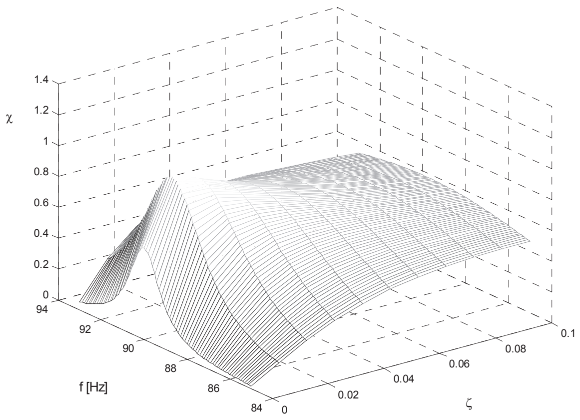

Fig. 6. Example χ function

5. DAMAGE IDENTIFICATION

Laplace wavelet can be used for identification not only first but also others natural frequencies. In this case the first, second and third natural frequencies and modal viscous damping ratio are identified.

Crack in beam. In tab. 1 and 2 the changes in first and second natural frequency and modal damping ratio are given.

Tab. 1

relative crack depth *a*/*h*	first natural frequency	damping ratio
0	48.94	0.011
3	48.93	0.011
5	48.91	0.011
10	48.83	0.011
15	48.71	0.011

Tab. 2

relative crack depth *a*/*h*	second natural frequency	damping ratio
0	195.79	0.011
3	195.78	0.011
5	195.76	0.011
10	195.68	0.011
15	195.34	0.011

There is no change in damping ratio because all date comes from computer simulation.

The changes in natural frequency in fig. 7 are showed.

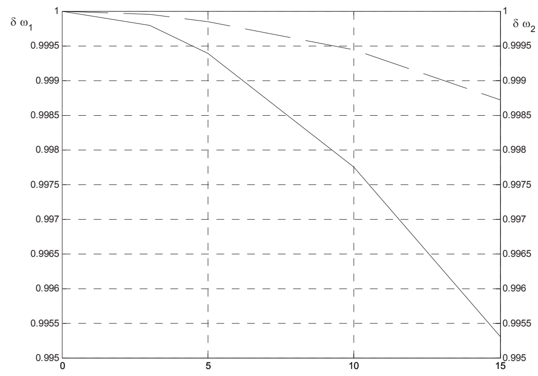

Fig. 7. Changes in fist (—) and second (- -) natural frequency of a cracked beam

Prestress losses. Author would like to thank dr inż. Ronan Barczewski for vibration data measured on laboratory specimens. In tab. 3 the changes third natural frequency and modal damping ratio are given.

Tab. 3

tension force [N]	third natural frequency	damping ratio
0	350.60	0,0051
20	350.88	0,0053
40	351.27	0,0055
60	354.77	0,0058
80	356.91	0,0074
100	357.93	0,0087

The changes in natural frequency and damping ratio in fig. 8 are showed.

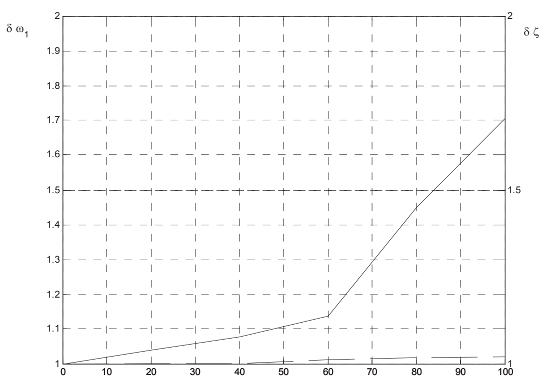

Fig. 8. Changes in fist natural frequency (- -) and damping ratio (—)

In fig. 8 changes of the both analysed value are given with the same scale. As one can see the damping ratio are more sensitive to prestress changing.

6. SUMMARY

In work the Laplace wavelet was introduced and utilized to find a little changes in natural frequencies of damaged element. Of course in real object so little changes in frequencies can occur not only due fault but due change temperature, humidity and so on.

Using the Laplace wavelet one can also find the modal viscous damping ratio. Analysed problems shows that changes in damping ratio are more sensitive to prestress changing.

This work was done as a part of research project
N 504 042 32/3443

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