# MATHEMATICAL MODELING OF NONSTATIONARY PROCESSES IN ENGINE AGGREGATES OF MUD PUMPS

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Summary

The mathematical model of dynamic processes in the engine aggregate of the mud pump with the operating friction clutch is offered. Inconstancy of reduced moment of inertia of crank mechanism of the pump and also interrelation of electromagnetic effects in asynchronous engine with non-linear oscillations of mechanical system are taken into account. The integration of system of the differential equations of movement of pump is realized by numerical methods.

Keywords: mud pumps, engine aggregates, nonstationary processes, mathematical modeling.

## 1. THE ANALYSIS OF RESEARCHESAND PROBLEM DEFINITION

The pumping aggregate of a drilling rig is one of primary elements of circulating system. Effectiveness of lifting of drilled solids depends on reliability of its functioning. Uninterrupted feed of flush fluid into borehole is important requirement for system functioning, because the temporary stopping of feed can cause heavy failure - sticking of drilling string as a result of a settling of slime and heavy fractions of a mud. It causes necessity of a raise of a technical level of mud pumps and their driving systems.

The drive of mud pump is realized by explosion engine or the electric engine through operating friction clutch, belting and tooth gear. With the purpose of definition of loads and prediction of fastness and durability of elements of drive appears a problem of execution of comprehensive analysis of nonstationary operating modes of a pumping aggregate. The theory of mud pumps is stated in transactions [2, 3, 7].

General methodology of modeling of electroconductive systems is enough completely stated in transactions [1, 6], examples of mathematical modeling of electromechanical systems which include asynchronous engine, reducing gear and operating mechanism – in transactions [5, 8]. In monograph [4] is offered method of calculation of nonstationary processes in engine aggregates of mud pumps, which one is grounded on combined integration of differential equations of movement, which are composed taking into account inconstancy of reduced moment of inertia of crank mechanisms of the pump, and equations of electromagnetic processes in asynchronous engine.

In this paper is offered mathematical model of dynamic processes in a pumping aggregate with any amount of pistons and with the operating friction clutch, which is composed taking into account inconstancy of reduced moment of inertia of crank mechanisms of the pump and continuity of interrelation of mechanical and electromagnetic oscillating appearances.

## 2. THE DIFFERENTIAL EQUATIONS OF MOVEMENT OF PUMPING AGGREGATE

The mechanical system of pumping aggregate, which consists of asynchronous engine, air clutch, belting, reducing gear and piston pump, is schematically represented on fig. 1.

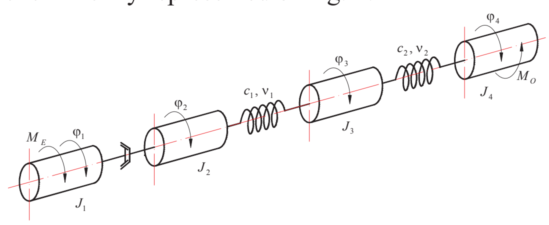


Fig. 1. The computation scheme of mechanical system of pumping aggregate

There are such designations on the scheme: $J_1$ – the reduced moment of inertia of rotor of the electric engine with driving part of air clutch; $J_2$ – the reduced moment of inertia of driven part of air clutch with shaft and with driving pulley of belting; $J_3$ – the reduced moment of inertia of transmission shaft with pinion and with driven pulley of belting; $J_4$ – the reduced moment of inertia of crank mechanism of the pump; $c_1$ – the reduced rigidity of V-belts; $\nu_1$ – the reduced damping coefficient of V-belts; $c_2$ – the reduced rigidity of reducing gear; $\nu_2$ – the reduced damping coefficient of reducing gear; $M_E$ – the reduced electromagnetic moment of engine; $M_O$ – the moment of forces of resistance to movement which acts on mainshaft of pump; $\varphi_1$, $\varphi_2$, $\varphi_3$, $\varphi_4$ – reduced angular coordinates. Inertial and elastic-dissipative characteristics we result in to the mainshaft of the pump.

The equation of movement of elements of aggregate we compose under the scheme of the equation of Lagrange of the second sort

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)-\frac{\partial T}{\partial q_j}+\frac{\partial \Pi}{\partial q_j}+\frac{\partial \Phi}{\partial \dot{q}_j}=Q_j \quad \left(j=1,4\right), \quad (1)$$

де where $T$ and $\Pi$ – kinetic and potential energies of mechanical system; $\Phi$ – dissipation function of the Rayleigh; $Q_j$ – the generalized force; $q_j$ – the generalized coordinate; $t$ – time.

Accepting for generalized coordinates such magnitudes

$$q_1=\varphi_1; \quad q_2=\varphi_2; \quad q_3=\varphi_3; \quad q_4=\varphi_4, \quad (2)$$

kinetic energy of system we give in such form

$$T=\frac{J_1\omega_1^{\ 2}}{2}+\frac{J_2\omega_2^{\ 2}}{2}+\frac{J_3\omega_3^{\ 2}}{2}+\frac{J_4\omega_4^{\ 2}}{2}, \quad (3)$$

where $\omega_1$, $\omega_2$, $\omega_3$, $\omega_4$ – reduced to mainshaft the angular velocity of rotor of the electric motor, angular velocity of the half-coupling, which is coupled to driving pulley of belting, angular velocity of the high-speed shaft of the reducer, which is connected to driven pulley of belting, and real angular velocity of mainshaft of the pump,

$$\frac{d\varphi_1}{dt}=\omega_1, \frac{d\varphi_2}{dt}=\omega_2, \frac{d\varphi_3}{dt}=\omega_3, \frac{d\varphi_4}{dt}=\omega_4. \quad (4)$$

Potential energy and dissipation function of the Rayleigh, taking into account (2) and (4), we give in such view

$$\Pi=\frac{c_1\left(\varphi_2-\varphi_3\right)^2}{2}+\frac{c_2\left(\varphi_3-\varphi_4\right)^2}{2};$$

$$\Phi=\frac{\nu_1\left(\omega_2-\omega_3\right)^2}{2}+\frac{\nu_2\left(\omega_3-\omega_4\right)^2}{2}. \quad (5)$$

For operating mode, when there is slippage in air clutch, finding derivatives from (3) and (5) and substituting them in the equations of Lagrange of the second sort (1), we gain the equations of movement in such form

$$J_1\frac{d\omega_1}{dt}=M_{Ез}-M_T;$$

$$J_2\frac{d\omega_2}{dt}=M_T-c_1\left(\varphi_2-\varphi_3\right)-\nu_1\left(\omega_2-\omega_3\right);$$

$$J_3\frac{d\omega_3}{dt}=c_1\left(\varphi_2-\varphi_3\right)+\nu_1\left(\omega_2-\omega_3\right)-$$
$$-c_2\left(\varphi_3-\varphi_4\right)-\nu_2\left(\omega_3-\omega_4\right);$$

$$J_4\frac{d\omega_4}{dt}=-\frac{1}{2}\frac{\partial J_4}{\partial \varphi_4}\omega_4^{\ 2}+$$
$$+c_2\left(\varphi_3-\varphi_4\right)+\nu_2\left(\omega_3-\omega_4\right)-M_O, \quad (6)$$

where $M_T$ – reduced to mainshaft the friction torque in air clutch, which varies according such law:

$$M_T=\alpha_t\cdot t, \quad \text{as} \quad t<t_r;$$

$$M_T=\alpha_t\cdot t_r, \quad \text{as} \quad t\geq t_r,$$

here $t_r$ – a time of filling up of clutch; $\alpha_t$ – coefficient determined by dependence

$$\alpha_t=\frac{M_{T0}}{t_r},$$

where $M_{TO}$ – maximal torque of friction in clutch.

The reduced moment of engine is receivable by formula

$$M_E=M_{E0}\cdot u, \quad (7)$$

where $M_{EO}$ – real electromagnetic torque of engine; $u$ – drive ratio.

If the slippage in operative air clutch is absent, elements with moments of inertia $J_1$ and $J_2$ realize combined movement. The equation of movement of system is receivable by substitution of expressions (3) and (5) in relation (1) provided that $\varphi_1$=$\varphi_2$,

$$\omega_1=\omega_2; \quad \varphi_1=\varphi_1\left(t_1\right)+\varphi_2\left(t\right)-\varphi_2\left(t_1\right);$$

$$\left(J_1+J_2\right)\frac{d\omega_2}{dt}=M_E-c_1\left(\varphi_2-\varphi_3\right)-\nu_1\left(\omega_2-\omega_3\right);$$

$$J_3\frac{d\omega_3}{dt}=c_1\left(\varphi_2-\varphi_3\right)+\nu_1\left(\omega_2-\omega_3\right)-$$
$$-c_2\left(\varphi_3-\varphi_4\right)-\nu_2\left(\omega_3-\omega_4\right);$$

$$J_4\frac{d\omega_4}{dt}=-\frac{1}{2}\frac{\partial J_4}{\partial \varphi_4}\omega_4^{\ 2}+$$
$$+c_2\left(\varphi_3-\varphi_4\right)+\nu_2\left(\omega_3-\omega_4\right)-M_O. \quad (8)$$

Conversion from mode of movement with slippage to mode without slippage happens, if such demands are fulfilled

$$\omega_2=\omega_1 \ \text{ i } \ M_E-J_1\frac{d\omega_1}{dt}\leq M_T. \quad (9)$$

If during system's movement without slippage in operative clutch in some instant the condition, which is expressed by second relation (9), is violated, then mode of movement again begins, that is accompanied by slippage.

During numeric integration of differential equations (4), (6) and (8) it is necessary to define at every step the derivative of the function $J_4$ regarding coordinate $\varphi_4$ and also the electromagnetic moment of engine $M_E$.

The reduced moment of inertia of pump's mechanism, which includes $n$ crank mechanisms, we give in such form [5]:

$$J_{3B}\left(\varphi\right)=\sum_{i=1}^{n}\left[J_{S1}+m_1a_1^{\ 2}+m_2u_i\frac{l_1^{\ 2}\left(\cos\varphi_i\right)^2}{h_i}+\right.$$
$$\left.+J_{S2}\frac{l_1^{\ 2}\left(\cos\varphi_i\right)^2}{h_i}+m_3\left(-l_1\left(\sin\varphi_i+\frac{l_1\sin 2\varphi_i}{2\sqrt{h_i}}\right)\right)^2\right], \quad (11)$$

where

$$h_i = l_2{}^2 - l_1{}^2 (\sin \varphi_i)^2 ,$$

$$u_i = \frac{h_i}{(\cos\varphi_i)^2} + a_2{}^2 - 2a_2 \frac{h_i}{l_2} +$$

$$+ 2a_2 \frac{l_1 \cdot (\sin \varphi_i)^2 \cdot \sqrt{h_i}}{\cos\varphi_i \cdot l_2} ;$$

where $\varphi_i$ ($i$ = 1, 2, …, $n$) – angular coordinate of the driving element of crank mechanism – the crank; $m_1$, $m_2$, $m_3$ – masses, accordingly, of crank, of crank mechanism and of piston; $J_{S1}$ i $J_{S2}$ – central moments of inertia of crank and of connecting rod; $l_1$ and $l_2$ – linear dimensions of elements; $a_1$ – distance from center of mass of crank to its rotation axis; $a_2$ – distance from center of mass of connecting rod to its axis of joint with crank.

Turn's angles of driving elements of pump $\varphi_i$ ($i$ = 1, 2, ..., $n$) are linked with turn's angle of mainshaft of pump $\varphi$ by such relations:

– for a simplex pump $\varphi_1 = \varphi$ ;

– for a 2-cylinder single direct-action pump $\varphi_1 = \varphi, \quad \varphi_2 = \varphi + \pi$ ;

– for a 2-cylinder bidirectional pump $\varphi_1 = \varphi, \quad \varphi_2 = \varphi + \pi / 2$ ;

– for a 3-cylinder single direct-action pump $\varphi_1 = \varphi, \quad \varphi_2 = \varphi + 2\pi / 3, \quad \varphi_3 = \varphi + 4\pi / 4$ .

Derivative $dJ_{3B}/d\varphi$ is receivable as sum of derivatives $p_i$ from expression (11), and it depends on amount $n$ of crank mechanisms of the pump and shift of turn's angles of driving elements of these mechanisms

$$\frac{dJ_{3B}}{d\varphi} = \sum_{i=1}^{n} p_i \; .$$

The moment of forces of useful resistance of mainshaft is receivable according to [5] by formula

$$M_O = \sum_{i=1}^{n} M_{Oi} \; , \tag{12}$$

where $M_{Oi}$ – moment of forces of useful resistance, which is created by fluid pressure on $i$-th piston,

$$M_{Oi} = P_i l_1 \Theta_i \; , \tag{13}$$

where $P_i$ – force of piston pressure, $\Theta_i$ – trigonometric function of turn's angle of crank.

For a bidirectional pump $P_i$ is receivable by formula

$$P_i = -pF_n , \text{ if } v_i > 0; \quad P_i = 0 , \text{ if } v_i = 0;$$

$$P_i = p(F_n - F_ш), \text{ if } v_i < 0,$$

where $F_n$ and $F_ш$ – cross-sectional areas of piston and rod, $p$ – fluid pressure on piston; $v_i$ – piston's speed

$$v_i = -\omega_2 l_1 \Theta_i \; . \tag{14}$$

Trigonometric function of turn's angle of crank $\Theta_i$, which figures in associations (13), (14), has view

$$\Theta_i = \sin\varphi_i + \frac{\sin\varphi_i \cos\varphi_i}{\sqrt{\left(\frac{l_2}{l_1}\right)^2 - (\sin\varphi_i)^2}} \; .$$

Real electromagnetic torque of engine $M_{E0}$ is receivable by equations of electromagnetic processes in asynchronous engine.

## 3. EQUATIONS OF ELECTROMAGNETIC PROCESSES IN ASYNCHRONOUS ENGINE

The differential equations of electromagnetic transients in an induction motor, tacking into account filling of magnetic conductor, are represented in such form [6]:

$$\frac{di_S}{dt} = A_S (u + \Omega_S \Psi_S - R_S i_S) + B_S (\Omega_R \Psi_R - R_R i_R) ;$$

$$\frac{di_R}{dt} = A_R (\Omega_R \Psi_R - R_R i_R) + B_R (u_S + \Omega_S \Psi_S - R_S i_S) , \tag{15}$$

where $i_S$, $i_R$ i $u_S$ – matrixes-columns of currents and voltages; $A_S$, $B_S$, $A_R$, $B_R$ – matrixes of links; $\Omega_S$, $\Omega_R$ – matrixes of rotation frequencies; $\Psi_S$, $\Psi_R$ – matrixes-columns of magnetic linkages. $R_s$, $R_r$ – pure resistances of windings.

The index $s$ specifies an inhering of magnitude to a stator winding, and $r$ - to a rotor winding.

Matrixes-columns $i_s$, $i_r$ and $u_s$ are determined by means of dependences

$$i_j (j = S,\ R) = \mathrm{col}\,(i_{jx}, i_{jy}); \quad u_S = \mathrm{col}\,(U_m, 0) ,$$

where $i_{jx}$, $i_{jy}$ – projections of currents to axes of coordinates $x$, $y$; $U_m$ – a peak voltage of a network.

Square matrixes $A_S$, $B_S$, $A_R$, $B_R$ are determined by dependences

$$A_S = \alpha_S (1 - \alpha_S G); \quad B_S = -\alpha_S \alpha_R G;$$

$$A_R = \alpha_R (1 - \alpha_R G); \quad B_R = B_S ,$$

where:

$$G = \frac{1}{i_m{}^2} \begin{bmatrix} Ri_x{}^2 + Ti_y{}^2 & (R - T) i_x i_y \\ (R - T) i_x i_y & Ti_x{}^2 + Ri_y{}^2 \end{bmatrix},$$

$$R = \frac{1}{\rho + \alpha_S + \alpha_R}; \quad T = \frac{1}{\tau + \alpha_S + \alpha_R} .$$

Here $i_m$, $i_x$, $i_y$ – a current of magnetization and its components along *x,y-axes*; $\tau$, $\rho$ – the magnitudes determined on a curve of magnetization, representing dependence of working magnetic linkage $\Psi_m$ on a current of magnetization; $\alpha_s$, $\alpha_r$ – magnitudes, inverse to inductances of diffusing of stator windings and a rotor.

Matrixes of rotation frequencies:

$$\Omega_S = \begin{bmatrix} 0 & \omega_0 \\ -\omega_0 & 0 \end{bmatrix}; \quad \Omega_R = \begin{bmatrix} 0 & \omega_0 - \omega_R \\ \omega_R - \omega_0 & 0 \end{bmatrix},$$

where $\omega_0$ and $\omega_r$ – synchronous angular velocity of the motor and angular velocity of the rotor, expressed in electric radians per second. Magnitudes $\omega_0$ and $\omega_R$ have values

$$\omega_0 = 314\ ;\ \omega_R = \omega_1 \cdot u \cdot p_0\ ,$$

for second step:

$$\omega_R = \omega_2 \cdot u \cdot p_0\ ,$$

where $u$ – drive ratio; $p_0$ – amount of pairs magnetic-poles.

Matrixes-columns of full magnetic linkages of stator windings and rotor look like:

$$\Psi_S = \frac{1}{\alpha_S} i_S + \frac{1}{\tau} i; \quad \Psi_R = \frac{1}{\alpha_R} i_R + \frac{1}{\tau} i,$$

where

$$i = \mathrm{col}\left(i_x, i_y\right).$$

Magnitudes

$$i_x = i_{Sx} + i_{Rx}\ ; \quad i_y = i_{Sy} + i_{Ry}\ ; \quad i_m = \sqrt{i_x^{\,2} + i_y^{\,2}}\ .$$

Magnitudes $\tau$ and $\rho$ are determined by dependences

$$\tau = \frac{i_m}{\psi_m}; \qquad \rho = \frac{di_m}{d\psi_m}\ .$$

The electromagnetic moment of the motor is discovered by formula

$$M_E = M_{E0} \cdot u = \frac{3}{2} \cdot u \cdot p_0 \frac{1}{\tau}\left(i_{Rx} i_{Sy} - i_{Ry} i_{Sx}\right).$$

If movement with slippage – $\omega_R=\omega_1 \cdot u$; if movement without slippage – $\omega_R= \omega_2 \cdot u$.

## 4. EXAMPLE OF CALCULATION

Let's consider the engine aggregate of mud pump У8-6М equipped by asynchronous engine АКЗ-15-41-8Б. As the reduced moment of inertia of moving parts of the pump is great, for launching the aggregate apply operative air clutch. At first operative clutch unlocks for pumping aggregate's activating and idle engine firing is carried out. The further acceleration of system occurs by gradual turning on the clutch. The moment of forces of useful resistance is receivable by formula (12). Engine's characteristic: amplitude of voltage $U_m$=4,9 kV; active resistances of phases of stator and rotor $r_S$=0,38 Ω, $r_R$=0,318 Ω; parasitic inductances $L_S$=1,048·10$^{-2}$ H, $L_R$=1,112·10$^{-2}$ H; operative inductance $L_m$=0,505 H; Amount of pairs magnetic-poles $p_0$=4; rotor's moment of inertia $J_1$=55 kg·m$^2$. Mud pump У8-6М is piston-like, 2-cylinder bidirectional pump and consists of driving and hydraulic parts, which are mounted on one frame. Angle between driving elements pump's crank mechanisms is 90° ($\varphi_1=\varphi$, $\varphi_2=\varphi+\pi/2$). Masses of elements: $m_1$=1000 kg, $m_2$=800 kg, $m_3$=420 kg; geometrical dimensions of elements: $l_1$ =0,2 m, $a_1$=0,13 m, $l_2$=0,85 m, $a_2$=0,25 m; central moments of inertia of crank and rod, accordingly, $J_{S1}$=42 kg·m$^2$, $J_{S2}$= 137 kg·m$^2$.

Initial conditions of integration of differential equations (4), (6), (15) for phase of acceleration of the pump by turning on operative clutch are such:

$$\omega_1(0) = 78{,}5\ ;\ \ \omega_2(0) = 0\ ;\ \ \omega_3(0) = 0\ ;\ \ \omega_4(0) = 0\ ;$$

$$\varphi_1(0) = 0\ ;\ \ \varphi_2(0) = 0\ ;\ \ \varphi_3(0) = 0\ ;\ \ \varphi_4(0) = 0\ ;$$

$$i_{Sx}(t) = 0{,}071\ ;\ i_{Sy}(t) = -30{,}27\ ;$$

$$i_{Rx}(t) = 6{,}777 \cdot 10^{-6}\ ;\ i_{Ry}(t) = 5{,}441 \cdot 10^{-6}\ .$$

Given here values of currents and angular velocity of rotor are gained after calculation of idle engine firing before steady mode.

As a result of combined numeric integration of differential equations of movement of mechanical system (4), (6) or (4), (8) and the equations, which present electromagnetic appearances in asynchronous engine (15), we gain temporal dependences of magnitudes $\omega_1$, $\omega_2$, $\omega_3$, $\omega_4$, of electromagnetic moment $M_E$, and also of torque $M_{1c}$ and $M_{2c}$ in elastic elements of aggregate, which are defined by dependences:

$$M_{1c} = c_1\left(\varphi_2 - \varphi_3\right) + \nu_1\left(\omega_2 - \omega_3\right);$$

$$M_{2c} = c_2\left(\varphi_3 - \varphi_4\right) + \nu_2\left(\omega_3 - \omega_4\right).$$

Graphs, which are represented on fig. 2, illustrate temporal changing of angular velocity of rotor of the electric motor (*a*) and driving pulley of belting (*b*). As we can see, the rotor of the drive goes into the steady mode during acceleration of the pump, which is loaded by useful resistance, during incomplete three seconds. The time, during which angular velocities $\omega_1$ and $\omega_2$ are equalizing, essentially depends on the law of changing of friction torque in clutch $M_T$. Insignificantly oscillations of angular velocities of elements of mechanical system are caused by cyclic functioning of pumping aggregate. With increasing loading on pistons the amplitude of these oscillations essentially increases.

The curve, which is represented on fig. 3. (a), illustrates temporal changing of the electromagnetic moment of engine АКЗ-15-41-8Б during beginning of turning on air clutch before intromission of system in steady mode.

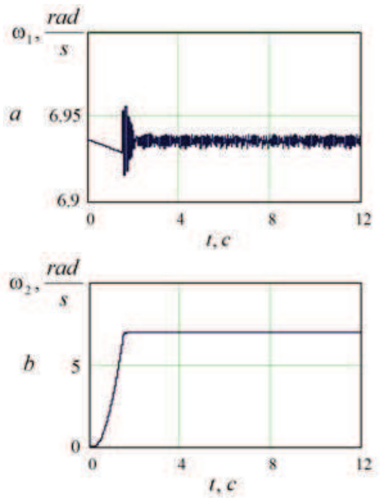


Fig. 2. Calculated dependences of angular velocity of rotor of the electric motor (a) and angular velocity of the half-coupling, which is coupled to driving pulley of belting (b)

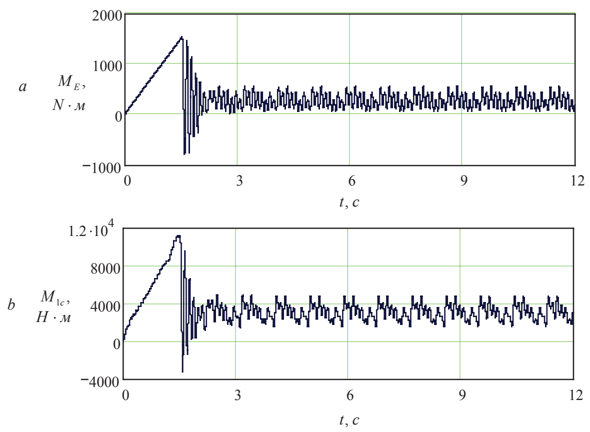


Fig. 3. Graphs of changing of electromagnetic moment of engine (a) and moment in belting (b) during gradual activation air clutch.

Curve, which is represented on fig. 3. (b), illustrates temporal dependences of torques in elastic elements of mechanism, that is in belting and tooth gear. Intensive oscillations of torques are caused by cyclic changing of load on piston and also great inertia of its elements.

## SUMMARY

1. Created mathematical model enables to ensure necessary exactitude of strength analysis and prediction of resource of elements of pumping aggregate. By rational sampling of operating mode of drive system is possible considerably to decrease dynamic loads of its elements and to increase at the expense of it reliability of the engine aggregate.
2. The gained results allow to define rational performance of air clutch, namely, a friction torque in clutch $M_T$ and fill-up time of clutch by air, which ensure enough acceleration of the aggregate under condition of limitation of loads of elements of the pump and its drive.

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