USE OF TASK–ORIENTED DYNAMIC RESAMPLING IN REDUCTION OF SIGNAL NON–STATIONARITY

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Summary

The article presents method of reducing the non–stationarity of a signal which relies on task–oriented dynamic signal resampling. The characteristic feature of this method is lack of preliminary assumptions regarding the phenomenon causing the eliminated non–stationarity. Thanks to this the method enables not only elimination of the non–stationarity caused by linear change of the frequency of signal but also other complex and non–linear forms of non–stationarity. The paper also includes the examples of application of the described method.

Keywords: non–stationarity, task–oriented dynamic resampling, instantaneous frequency.

WYKORZYSTANIE ZORIENTOWANEGO ZADANIOWO DYNAMICZNEGO PRZEPRÓBKOWANIA W REDUKCJI NIESTACJONARNOŚCI SYGNAŁU

Streszczenie

W artykule przedstawiono metodę redukcji niestacjonarności sygnału bazującą na zorientowanym zadaniowo dynamicznym przepróbkowaniu sygnału. Charakterystyczną cechą metody jest brak wstępnych założeń co do zjawiska wywołującego eliminowaną niestacjonarność. Dzięki temu metoda pozwala na usunięcie nie tylko niestacjonarności wywołanej liniową zmianą częstotliwości sygnału ale także i innych, złożonych i nieliniowych form niestacjonarności. W pracy zamieszczono także przykłady zastosowania opisanej metody.

Słowa kluczowe: niestacjonarność, zorientowane zadaniowo dynamiczne przepróbkowanie, chwilowa częstotliwość.

1. INTRODUCTION

In diagnostic practice we often come across the need for analyzing non–stationary signals. There can be various reasons of non–stationarity of signal. Non–stationarity can result from a special mode of operation of an object, including such elements as start–up and coasting of a machine. From diagnostic point of view the observation of the non–stationary mode of operation, which occurs during start–up and coasting, can provide a lot of diagnostically–interesting information since it enables the analysis of the behavior of object during operation at variable rotational speed, which leads to additional excitation of the signal as it encounters resonance of the structure. Moreover, during start–up there usually occur loads which enable observation of flexibility and play in bearings and clutches [2]. Non–stationarity can also be the outcome of unstable operation of the object, resulting from variability of operating conditions and demonstrating itself as fluctuation of the basic revolutions frequency of a machine. Non–stationarity of analyzed signals can be also the consequence of the adopted method of registration of these signals. Such a situation occurs when we deal with use of a stationary system for acquisition of sound signals generated by objects which are in motion. The signal registered in the conditions of relative motion of the transmitter (source of signal) and the receiver (microphone) is burdened with disturbance being the outcome of Doppler's effect [3].

Analysis of non–stationary signals generally requires special method of signal preprocessing [5], since the frequency of signal structure which changes in time, effectively hinders effective use of classical methods of signal analysis. It is important, however, that the methods applied remove only the undesirable non–stationarity of a signal while not causing at the same time the unintended, by us, elimination of other, diagnostically–useful properties of the signal. Thus a need emerges for having as useful as possible, but at the same time precisely selective, methods of reducing signal non–stationarity.

The paper will present the method of reducing signal non–stationarity which relies on task–oriented, dynamic resampling of signals. We will also present the examples of use of this method.

2. TASK–ORIENTED DYNAMIC SIGNAL RESAMPLING

The basis of a dynamic (variable in time) signal resampling is the use of variable time resolution in such a way that each subsequent (variable) period contains a permanent number of time samples [3]:

$$n = \frac{T_1}{dt_1} = \ldots = \frac{T_N}{dt_N} = \text{const} = \frac{T_W}{dt} \quad (1)$$

which is equivalent to the following relationship:

$$n = \frac{1}{f_1 \cdot dt_1} = \ldots = \frac{1}{f_N \cdot dt_N} = \text{const} = \frac{1}{f_w \cdot dt} \quad (2)$$

where:

- f_1, ..., f_N – represents the subsequent, instantaneous frequencies of the signal,
- dt_1, ..., dt_N – represents the subsequent, instantaneous sampling rate of the signal,
- f_w – denotes the "resultant" frequency of the modified signal,
- dt – denotes the sampling rate of the original signal.

Due to the fact that the instantaneous sampling rate of the signal can be calculated from the following relationship:

$$dt_{chw}(i) = \frac{f_w \cdot dt}{f_{chw}(i)} \quad (3)$$

where: $dt_{chw}(i)$, $f_{chw}(i)$ represent respectively the instantaneous sampling rate and the instantaneous frequency of the signal.

While having the vector of the subsequent, determined temporal resolutions $dt_{chw} = [dt_{chw}(1), dt_{chw}(2), \ldots, dt_{chw}(m)]$, while referring to the cumulative sum we can define the modified time vector t_{chw} by means of the following relationship:

$$t_{chw}(i) = dt_{chw}(1) + dt_{chw}(2) + \ldots + dt_{chw}(i) \quad (4)$$

The last step is to define the value of the signal at the moments in time defined by the values of the modified time vector t_{chw}. We do this through interpolation of the value of the original signal by means of polynomial splines. In the presented solution we used the cubic splines. This way we receive the modified signal whose sampling rate is identical to the sampling rate of the original signal.

Correct performance of dynamic resampling of a signal requires one to know the course of the instantaneous frequency of the signal, which defines the removed non–stationarity of the signal. Depending on the specific task there exist various possibilities of defining the required course of the instantaneous frequency. The course of frequency can be learned while relying on the knowledge of the phenomenon which causes the undesirable non–stationarity of the signal, which enables definition of the function defining the variability of the instantaneous signal frequency. The course of the instantaneous frequency can be also defined based on measuring a defined value which describes the eliminated non–stationarity of the signal (e.g. tachometric signal). There also exist methods which define the course of the instantaneous frequency based on the information contained in the signal subjected to resampling. An example of this method can be the algorithm, used by Bruel & Kjaer, which relies on the implementation of Bayes' statistical method [6, 7]. Another example of such an approach can be the method relying on the Hilbert transform. The course of the instantaneous frequency of a signal is then determined by means of the following relationship [1]:

$$f_{chw}(t) = \frac{1}{2 \cdot \pi} \cdot \frac{d\,\varphi(t)}{dt} \quad (5)$$

where:

- $\varphi(t)$ – denotes an argument of the analytic signal:

$$x_a(t) = x(t) + j \cdot H\left[x(t)\right] = \left|x_a(t)\right| \cdot e^{j \cdot \varphi(t)} \quad (6)$$

where H[·] denotes the Hilbert transform.

3. NUMERICAL EXPERIMENT – CHIRP TYPE SIGNAL

An example of a non–stationary signal generated during a start–up of machine can be a swept–frequency signal (chirp). In this experiment we used signals with two types of chirps: linear and square. Then, while relying on the presented method of dynamic resampling, we performed reduction of non–stationarity of these signals. In the case of linear chirp, the course of the instantaneous frequency of the signal was defined by a linear function while in the case of square chirp, the course of the instantaneous frequency of the signal was defined by a method relying on Hilbert transform.

The results of processing of the linear chirp are presented in figures 1, 2 and 3 (Fig. 1 – the course of instantaneous frequency as well as the original and modified time; Fig. 2 – the amplitude spectra; Fig. 3 – the Wigner–Ville time–frequency distributions, respectively for the original and the modified, dynamically resampled signal). The results of processing of square chirp are presented in figures 4, 5 and 6 (Fig. 4 – the course of instantaneous frequency as well as the original and

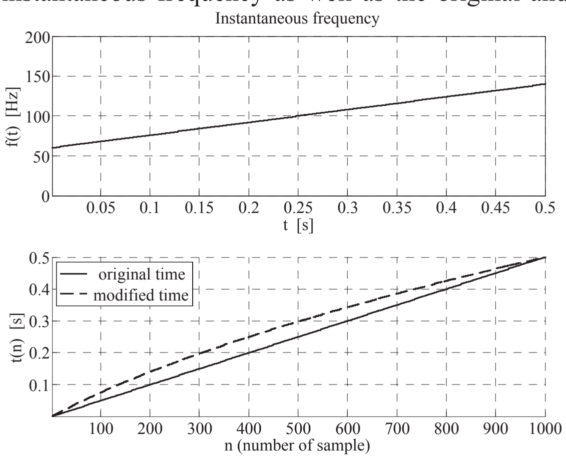

Fig. 1. Instantaneous frequency and times (linear chirp)

modified time; Fig. 5 – the amplitude spectra; Fig. 6 – the Wigner–Ville time–frequency distributions, respectively for the original and the modified, dynamically resampled signal).

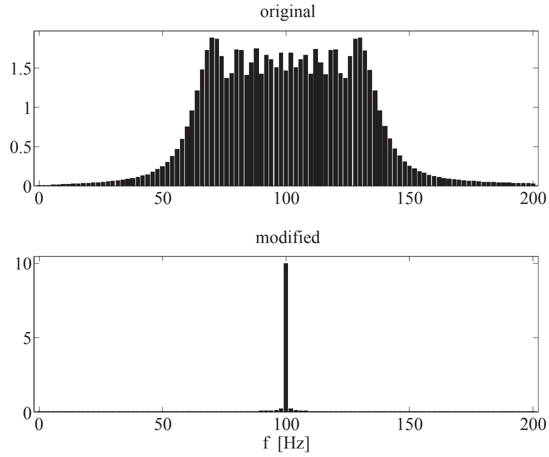

Fig. 2. Amplitude spectra (linear chirp)

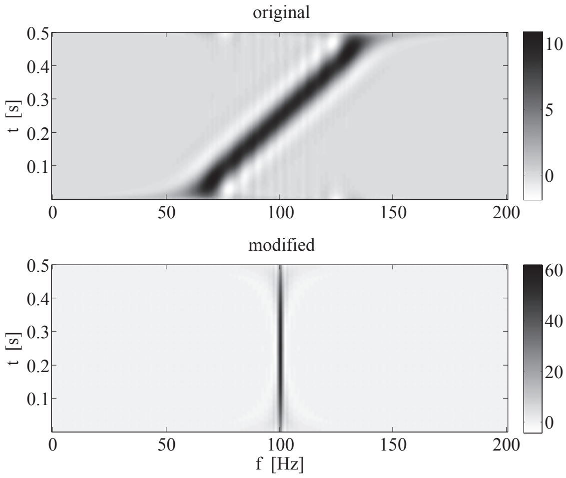

Fig. 3. Wigner–Ville distributions (linear chirp)

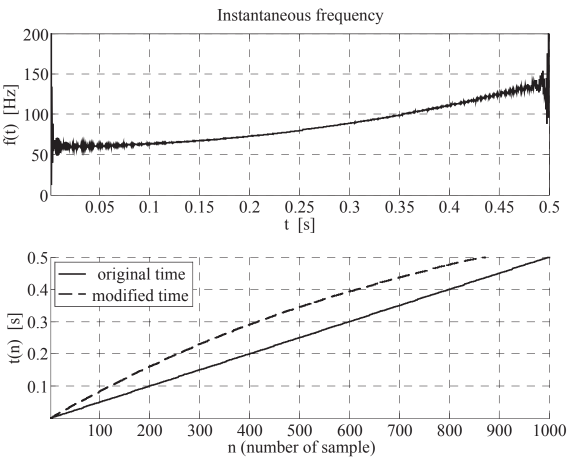

Fig. 4. Instantaneous frequency and times (square chirp)

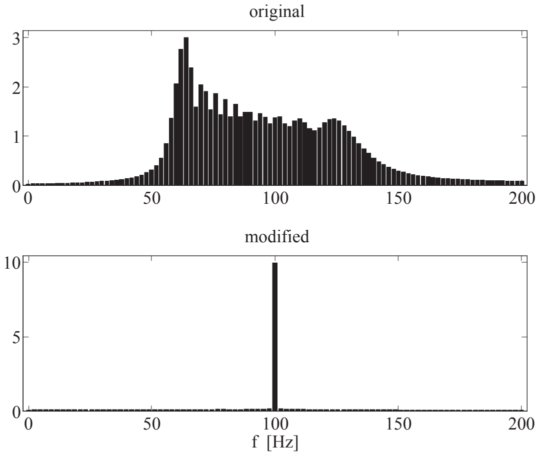

Fig. 5. Amplitude spectra (square chirp)

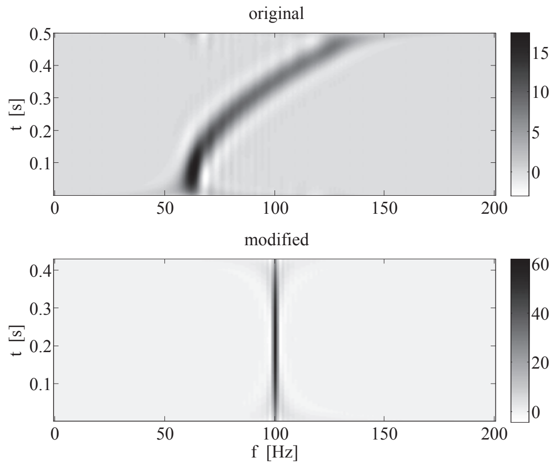

Fig. 6. Wigner–Ville distributions (square chirp)

4. NUMERICAL EXPERIMENT – A SIGNAL WITH FREQUENCY FLUCTUATION

A signal with fluctuating frequency can be an example of a non–stationary signal generated during unstable mode of an operation of object. In this experiment we used a signal with harmonic fluctuation of frequency. Then, with the use of the presented method of dynamic resampling, we performed reduction of the non–stationarity of signal. The course of the instantaneous frequency of the signal was defined with the use of the method relying on Hilbert transform.

The results of processing of the signal with the use of harmonic frequency fluctuation are presented in figures 7, 8, 9 (Fig. 7 – the course of the instantaneous frequency as well as the original and modified time; Fig. 8 – amplitude spectra; Fig. 9 – the Wigner–Ville time–frequency distributions, respectively for the original and the modified, dynamically resampled signal).

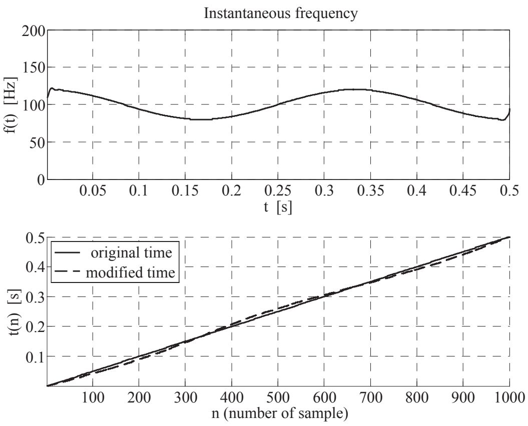

Fig. 7. Instantaneous frequency and times (frequency fluctuation)

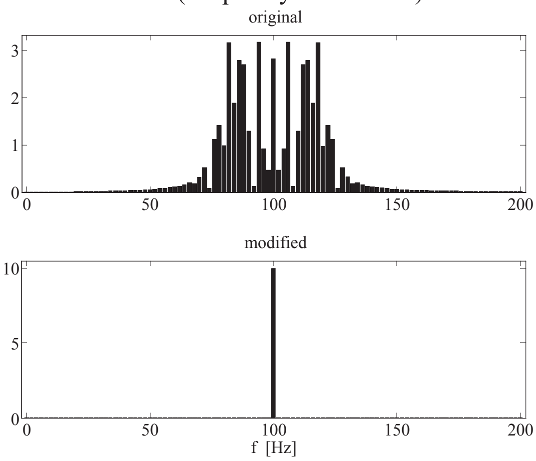

Fig. 8. Amplitude spectra (frequency fluctuation)

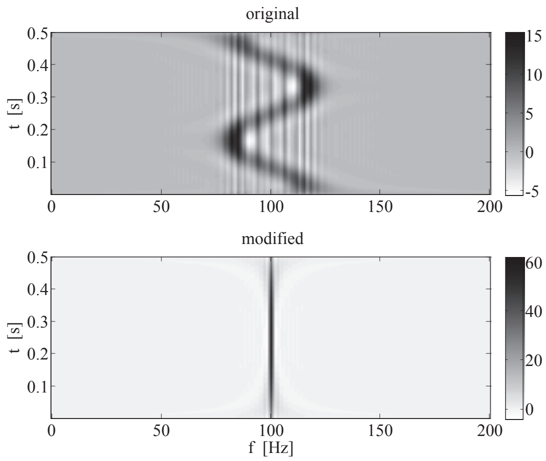

Fig. 9. Wigner–Ville distributions (frequency fluctuation)

5. NUMERICAL EXPERIMENT – A SIGNAL WITH AMPLITUDE–FREQUENCY (PHASE) MODULATION

In this experiment we have used a signal with amplitude–frequency modulation. The signal was then dynamically resampled. The course of the instantaneous frequency of the signal was defined with the use of the method relying on Hilbert transform.

The results of processing of the signal with the use of amplitude–frequency modulation are presented in figures 10, 11 and 12 (Fig. 10 – the course of the instantaneous frequency as well as the original and modified time; Fig. 11 – amplitude spectra; Fig. 12 – the Wigner–Ville time–frequency distributions, respectively for the original and the modified, dynamically resampled signal).

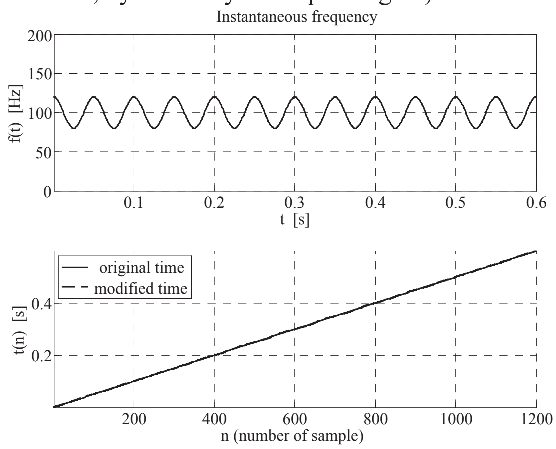

Fig. 10. Instantaneous frequency and times (amplitude–frequency modulation)

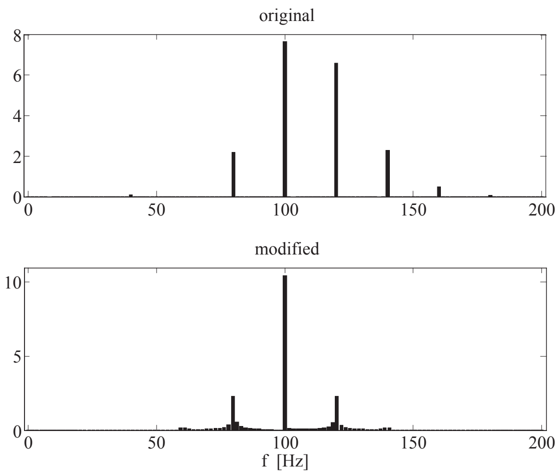

Fig. 11. Amplitude spectra (amplitude–frequency modulation)

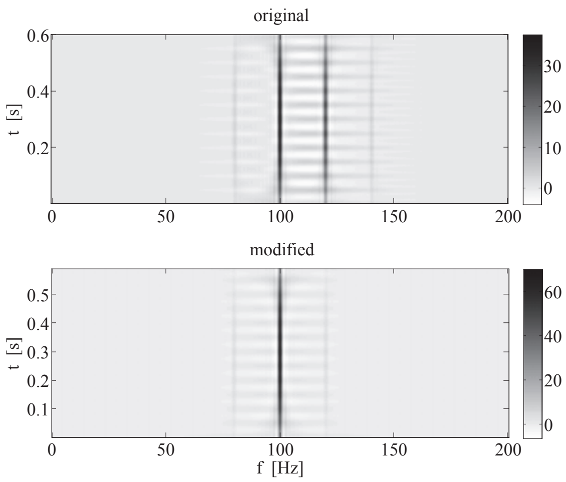

Fig. 12. Wigner–Ville distributions (amplitude–frequency modulation)

6. NUMERICAL EXPERIMENT – A SIGNAL WITH DOPPLER'S EFFECT

In this experiment we modeled the operation of a stationary system of acquisition of sound signals generated by an object in motion. An assumption was made that the object moves at a constant speed in a straight line. The diagram of the modeled situation is presented in Fig. 13.

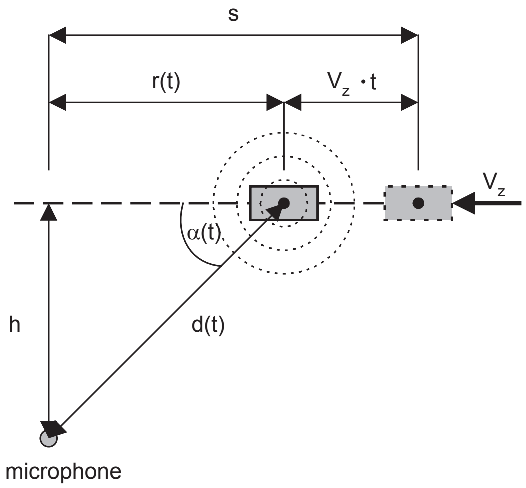

Fig. 13. Diagram presenting the modeled measuring situation

The following notation has been adopted:

- V_z – speed of the object (source of signal),
- h – distance separating the microphone from the path of object,
- s – distance defining the moment when signal registration starts (the distance between the object and the place in which the object is closest to the microphone).

The signal registered by a microphone in the conditions of relative motion of the object (the source of the signal) is burdened with noise being the result of the Doppler's effect [4]:

$$f_R = f_0 \cdot \frac{1}{1 - \frac{V_z}{V_d} \cdot \cos \alpha(t)} \quad (7)$$

where:

- f_0 – the frequency of the source of the signal,
- f_R – the frequency of the registered signal,
- V_d – velocity of propagation of sound in the air (V_d = 343 m/s was assumed),
- $\alpha(t)$ – the angle between the vector of the speed of object and the line between the object and the microphone.

While taking into account that:

$$\cos \alpha(t) = \frac{r(t)}{d(t)} = \frac{s - V_z \cdot t}{\sqrt{h^2 + (s - V_z \cdot t)^2}} \quad (8)$$

we obtain:

$$f_R(t) = f_0 \cdot \frac{1}{1 - \frac{V_z}{V_d} \cdot \frac{s - V_z \cdot t}{\sqrt{h^2 + (s - V_z \cdot t)^2}}} \quad (9)$$

Thus one can see that in the presented measurement situation we will be registering a signal whose frequency will change during the measurement.

As part of the measurement we performed a computer simulation of signal registration based on the following assumptions f_0=100Hz, s=8.3m, h=1m, V_z=33.3m/s (120km/h). Then, with the use of the presented method of dynamic resampling, we performed reduction of the non–stationarity of signal. To determine the course of the instantaneous frequency of the signal we used the functional relationship describing the change of the frequency of the signal when caused by a Doppler's effect (formula 9).

The results of processing are presented in figures 14, 15, 16 (Fig. 14 – the course of the instantaneous frequency as well as the original and modified time; Fig. 15 – amplitude spectra; Fig. 16 – the Wigner–Ville time–frequency distributions, respectively for the original and the modified, dynamically resampled signal).

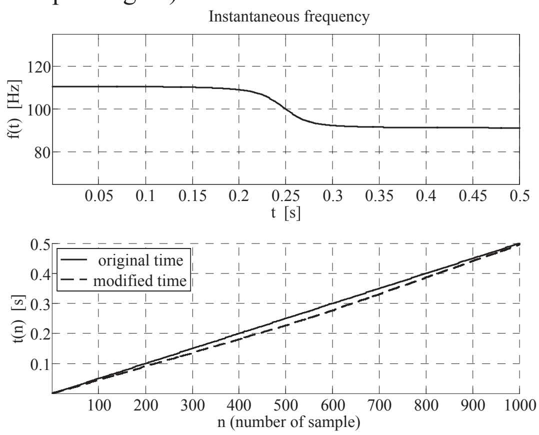

Fig. 14. Instantaneous frequency and times (Doppler's effect)

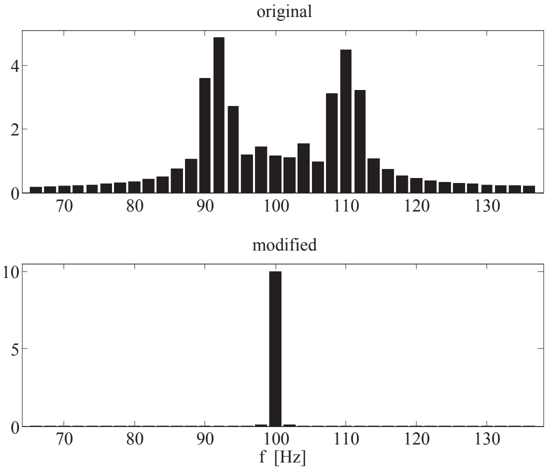

Fig. 15. Amplitude spectra (Doppler's effect)

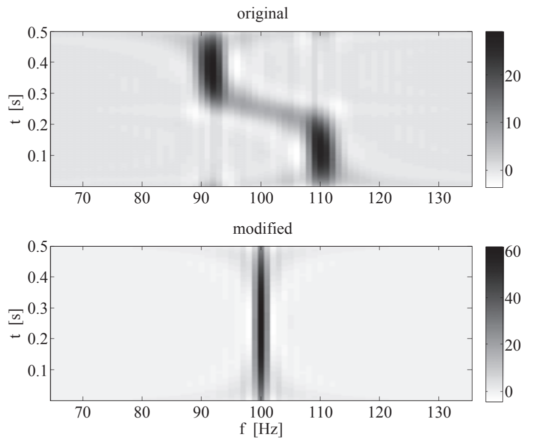

Fig. 16. Wigner–Ville distributions (Doppler's effect)

7. SUMMARY AND CONCLUSIONS

The computer simulations we conducted demonstrate that the method of task–oriented dynamic signal resampling, that was presented in this paper, effectively reduces the non–stationarity of a signal. The characteristic feature of the method is lack of preliminary assumptions as regards the phenomena causing non–stationarity. The simulations we conducted demonstrate that the possibilities of this method are not restricted to eliminating the non–stationarity caused by linear change of a frequency of signal only. The method also enables elimination of other, complex and non–linear forms of non–stationarity.

Effective reduction of various forms of non–stationarity is conditioned by the availability of the course of instantaneous frequency of which defines the eliminated non–stationarity. It is this very course that defines the task while leading to a situation where the method becomes oriented on eliminating the non–stationarity indicated this way.

The method of determining the course of the instantaneous frequency of signal is discretionary (free), independent of the method itself. Usually, however, it is conditioned by a specific task and form of the removed non–stationarity. One of the applied methods can include determination of the course of a instantaneous frequency by means of a method relying on the Hilbert's transform. This method is effective particularly in the case of signals characterized by non–complex frequency structure. If the method is used for more complex signals, then the course of instantaneous frequency should be defined in a specifically selected frequency band [1].

This restriction is not necessary since the method seeks all the forms of changes of the instantaneous frequency and not only the changes of the frequency we intend to eliminate. Thus, the outcome of applying this method of determining the frequency could be elimination of the frequency modulation of signal which was not intended by us (see the experiment with the amplitude and frequency modulated signal). Hence, if the method relying on the Hilbert' transform is used, then we should select carefully the frequency band to be used for determining the course of the instantaneous frequency, so that the bandwidth contains the form of the non–stationarity of signal that we really want to eliminate.

8. LITERATURE

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