ABOUT NEED FOR USING NONLINEAR MODELS IN VIBROACOUSTIC DIAGNOSTICS

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Summary

The paper discussed the thesis that in vibroacoustic diagnostics using the nonlinear mathematical model is necessary. Natural specification of machine degradation is the frequency response function change and "additional" inputs related to defects remain usually as self-exited vibrations, which require the application of nonlinear description. Very often a new machine can be described with an adequate accuracy by a linear model. During exploitation certain nonlinear disturbances related to wear and tear - occur. Thus, an observation of nonlinear effects allows solving a diagnostic task.

Keywords: nonlinear effects, vibroacoustic diagnostic.

O KONIECZNOŚCI STOSOWANIA W DIAGNOSTYCE WIBROAKUSTYCZNEJ MODELI NIELINIOWYCH

Streszczenie

W artykule przedyskutowano potrzebę stosowania w diagnostyce wibroakustycznej modeli nieliniowych. Naturalnym opisem degradacji maszyny jest zmiana funkcji odpowiedzi częstotliwościowej, a „dodatkowe" wymuszenia związane z uszkodzeniami pozostają z reguły jako drgania samowzbudne, co z założenia wymaga zastosowania opisu nieliniowego. Bardzo często nową maszynę możemy opisać z dobrą dokładnością modelem liniowym. W trakcie eksploatacji pojawiają się i wzrastają nieliniowe zaburzenia związane ze zużyciem. Tym samym obserwacja efektów nieliniowych pozwala na rozwiązanie zadania diagnostycznego.

Słowa kluczowe: efekty nieliniowe, diagnostyka wibroakustyczna.

The problem of assessment of the machine state by means of vibrations and noise analysis is based – from the theoretical side – on the statement that, the vibroacoustic energy dissipation increases during the machine exploitation. Therefore a certain vibration or noise measure should exist, which in the moment - when further exploitation is dangerous – exceeds the permissible value. Such reasoning results from the adaptation of a model assuming an increase of dissipated energy during wear and tear as well as on the assumption that energy of parasitic vibroacoustic processes is proportional to the total dissipation of energy. Now-a-days the assumption of a general increase of energy consumed is not doubtful. We may assume that this is the proved law of nature. However, there is still a problem of developing the easiest mathematical notation and looking for the "optimal" model.

It is well known, that the statement of proportionality of vibroacoustic energy to the total parasitic energy is a simplification from which might be – and actually are – exemptions. Examples, where a periodical lowering of the vibration level indicates a dangerous defect and where the "waving" of a trend of changing values being the measure of the vibroacoustic process occurs, are considered in the paper. It should be assumed that those phenomena are accompanied by increases of dissipation energy in other processes, mainly thermal ones, but also electric and hydraulic (e.g. an oil leakage from the damaged bearing can cause damping of vibrations). In the complicated structure of the mechanical system the effect of an apparent "self repairing" can occur and it may periodically lower the amount of dissipated energy and change proportions in between dissipation forms. It is presented pictorially in Figure 1.

If, according to this short reasoning, we assume that there are cases when an increased level of vibrations and noises (in the whole observable range or in the selected bands) is not proportional to a wear, it will still not indicate that vibroacoustic processes are insensitive to it. **The assumption arises, that the change of proportion in between individual forms of the dissipated energy can have its representation in the observed form of vibrations (even when the level remains constant or lowered).** This phenomenon was investigated at analysing defects of rolling bearings, where – at the constant general level – the proportions between the dominating amplitudes in the spectrum were changing. Similar results were obtained at checking hydraulic elements when investigating a multi-symptom index (vibrations + heat) and at diagnostics of a tool wear in the machining process. The mentioned phenomena have been observed during passive as well as active diagnostic experiments. Having an accurate recording of several symptoms – during the whole life-time – for the statistically representative sample of specimens one can establish significant dependencies. However, the difficult problem of assuming the proper model still remains to be solved.

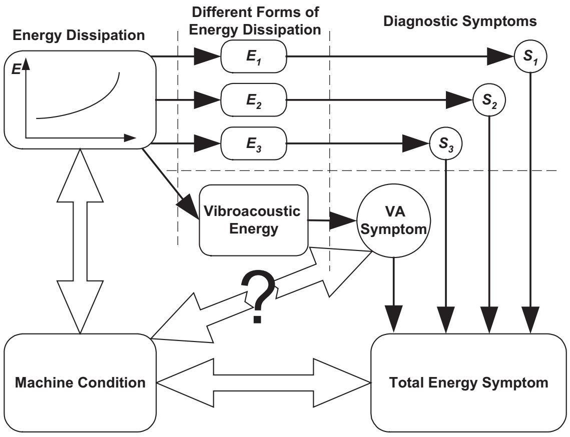


Fig. 1. Utilising the diagnostic symptom in a machine diagnostics.

Let us discuss the example presented in Figure 2 showing the spectrum of vibration accelerations of the rolling bearing. An average value and average square value are the same as measurement accuracy.

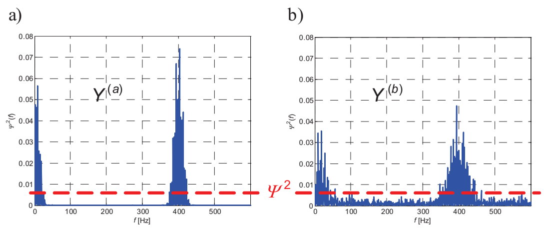


Fig. 2. Example of „iso-energy" evolution of a spectral power density of the vibration acceleration during the machine wear (rolling bearing).

A frequency distribution is – of course – completely different. Taking into account only the module, or more precisely the spectral power density and disregarding phase shifts, let us discuss which transformation should be applied to the dynamic system in order to transform spectrum "a" into spectrum "b". Let us assume, at first, that the system is linear and discrete. Both spectra were obtained from the observation of the machine steady state, it means under conditions when inputs are stable processes of the dominating participation of determined and periodical components. Let us neglect – for the time being – random disturbances. Let the system has n degrees of freedom. Both spectra, "a" and "b", concern "normal" working conditions. Thus, neither a loss of stability nor a loss of continuity of solution as a function of parameter changes occurs. Such situation would correspond to the essential defect of a device. During the time elapsing between both observations the system evolution could have occurred and probably have done so. This evolution manifested itself by small changes of coefficients; it means elements of matrix of inertia, damping and rigidity and by an occurrence of new – generally weak – excitations and also by eventual decreasing of previous forces. This last phenomenon can be omitted as being of a low probability. Let us assume further, that there are m poly-harmonic inputs (where $m < n$). The amplitude spectrum for a linear system is formed as the result of the following transformation:

$$Y(f) = \sum_{i=1}^{m} P_i(f) \cdot H_i(f, m_j, k_j, c_j) \quad j = 1 \div n \quad (1)$$

where:

P_i – Fourier transform of excitation moments,
H_i – Transmittance.

Each transmittance, and more accurate its module – it means the coefficient of amplification, has as many extremes as degrees of freedom of the system (natural frequencies). The set of transmittances is explicitly determined by $3 \cdot m$ parameters. Equation (1) must be satisfied for each frequency of the output spectrum. At assuming a spectrum discretisation into k elements we have k+2 equations. An addition of number 2 results from the postulate of the average and average square value conservation. At the assumption that the system inputs were not changed and that all coefficients (parameters) of mass, rigidity and dissipation could have changed, the minimum number of degrees of freedom of the linear system enabling transformation of the response $Y^{(a)}(f) \to Y^{(b)}(f)$ equals:

$$n = \frac{k+2}{3} \quad (2)$$

Equation (2) regardless of the fact, that it requires generating a huge number of equations, has only a formal meaning. It determines precisely the minimum number of degrees of freedom enabling – at the preserved model structure – the possibility of the given change of the frequency response structure without changing inputs and at assuming the application of outside forces in each degree of freedom and the possibility of a free selection of all coefficients. Physical realisability of such system is practically impossible. In the actual diagnostic tasks, investigating an evolution of a device, the change of a dynamic response structure depends on changes of the insignificant part of parameters (the ones responsible for the defect). Thus, the number of the necessary equations should be multiplied by coefficient λ, defined by the ratio of invariable elements (degrees of freedom), to the ones in which certain number of parameters are changing (usually not all) and by coefficient λ_1, which denotes the ratio of all degrees of freedom to the ones receiving energy from outside. When the number of spectral lines, taken into account, is limited to 100 the number of degrees of freedom of the system should amount to several thousands. This is the condition for the identificability of the system; it means that changes of actual parameters (in our example: state variables) should be transformable by the determined and mutually explicit procedure into the model parameters changes. However, regardless of the mentioned difficulties, there is a possibility of obtaining such solution. Therefore the application of the linear software FEM has allowed to solve several problems described analytically as nonlinear. Simplifying a little the consideration, we can state that the presented operations are based on **reducing the globally nonlinear influences to locally linear ones** and the increase of the number of degrees of freedom of the system results from decreasing the zone considered to be the "local" area – up to the determined limit of error. This linearisation corresponds de facto to the approximation of a curve by a certain number of segments.

Let us return now to our assumptions. We have assumed at the beginning, that the response $Y^{(a)}(f)$ was the result of linear transformations of inputs, which means that the sum of inputs and outputs from the system should fill the same frequency bandwidth. Thus, each harmonic component of the input corresponds to one and only one harmonic component of the response, which usually has different amplitude, slightly changed frequency (because of damping) and different phase shift (what was not taken into account in our reasoning) but surely will not decompose into a sequence of components. In the discussed example no spectrum of the system response $Y^{(a)}(f)$ nor $Y^{(b)}(f)$ can be obtained at inputs of a smaller number of components. In diagnostic tests – during machine exploitation - new harmonic components appear very often and the value of their amplitude is considered the symptom. Such transformation in a linear system requires applying a new excitation, which – in the diagnostic test – would need a postulate that each defect (wear) constitutes such force. This is an evident contradiction. We assumed at the beginning, that the transformation of spectral concentration $Y^{(a)}(f) \to Y^{(b)}(f)$ is isoenergetic (Ψ^2=const), whereas each defect would have been described by the energy "inflow" from outside. However, supporters of a linear description for any cost (which really means: for the cost of a tremendously increased number of motion equations) could find the solution of the problem by assuming simultaneous decrease of primary inputs caused by other means of energy loss due to wear, but the model obtained in such manner would not be identifiable.

Natural specification of machine degradation is the frequency response function (transmittance) change and "additional" inputs related to defects remain usually as self-exited vibrations, which require the application of nonlinear description, etc.

The presented above considerations lead to the conclusion that a classic diagnostic task very often requires application of a nonlinear description. The following postulate can be formulated on the basis of numerous papers: A new ("after an initial usage") machine can be described with an adequate accuracy by a linear model. During exploitation certain nonlinear disturbances related to wear and tear – occur. Thus, **an observation of nonlinear effects allows solving a diagnostic task.**

This postulate is also true for technical devices, which operations require a nonlinear description from the 'very beginning' (e.g. piston-and-crank mechanism). In such situation we will observe an increase of nonlinear disturbances. Besides, "a diagnostic" model does not need to be a fully "dynamic" model.

Let us solve now a simple example. An ordinary differential equation of the 2nd order in a form of a simple harmonic oscillator – is given:

$$\ddot{x} + \omega_0^2 x = P(t)$$

with an input:

$$P(t) = P\cos 2\Omega t\,.$$

An evident singular solution is a well-known "school type" dependency:

$$x(t) = \frac{P}{\omega_0^2 - \Omega^2}\cos 2\Omega t \qquad (3)$$

Let us check whether finding the singular solution of a frequency being equal e.g. to the half of the input frequency Ω in the form: $x = A\cos\Omega t$ is possible. By substitution we obtain the following equation:

$$A\cos\Omega t(\omega_0^2 - \Omega^2) = P\cos 2\Omega t\,, \qquad (4)$$

which satisfaction for each t requires zeroing of the input amplitude at the arbitrary amplitude of response. Thus, it leads to an obvious triviality. Let us assume the possibility of modification of the basing equation by introducing an arbitrary nonlinear function of variable $x(t)$ and let us check whether now obtaining the response of the frequency equal half of the input frequency is possible. The task is as follows:

$$P\cos 2\Omega t \rightarrow x = A\cos\Omega t : \ddot{x} + \omega_0^2 x + f(x) = P\cos 2\Omega t \quad f(x) = ? \;. \quad (5)$$

Proceeding in an identical fashion as previously and transforming the input we will obtain:

$$-A\Omega^2 \cos 2\Omega t + A\omega_0^2 \cos 2\Omega t + f(A\cos\Omega t) = \frac{1}{2}P(\cos^2\Omega t + 1) \;. \quad (6)$$

This equation is much better and at the proper selection of $f(x)$ function its identity satisfaction is possible. E.g. assuming:

$$f(x) = k\left(x^2 + 1\right)$$

we will obtain:

$$\bigwedge_t \; (\omega_0^2 - \Omega^2)A\cos 2\Omega t + kA(\cos^2\Omega t + 1) = \frac{1}{2}P(\cos^2\Omega t + 1) \Leftrightarrow \Omega^2 = \omega_0^2 \quad P = 2kA \;. \quad (7) \quad P = 2k \quad A = 1$$

This is a special case, relatively difficult – however possible at the appropriately selected kinematics – for the technical realisation. It corresponds to the situation when the properly selected input force amplitude of the resonance frequency instead of increasing the first harmonic "releases" vibrations of a different frequency. This, in turn, corresponds to changing the phase trajectory into a certain limiting cycle and constitutes a certain form of self-exited oscillations, in which a restitution function described by an even function ("full" parabola) becomes an "amplifier". However, the discussion of the obtained results is not significant in this case. The example should be treated as a mathematical "plaything", which indicates – in a very simple manner – the possibility of generating, by a nonlinear system, the response of the frequency different than the input frequency.

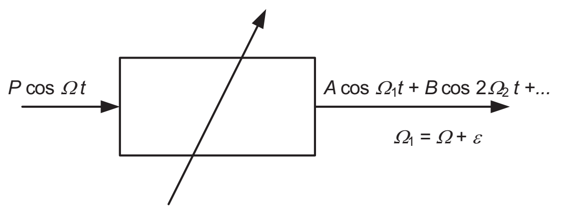


Fig. 3. Nonlinear system as frequency transformer.

Much more physically real would be an example of a double frequency response – according to the schematic presentation from Figure 3, but finding function $f(x)$ is not simple, similarly as a function causing a sub-harmonic response, for various amplitudes and frequencies of inputs. A solution of such problem requires an application of analytical approximate methods or simulation methods, exactly the same as applied at solving nonlinear differential equations. Mathematical laws are not to be avoided. A solution of nonlinear problem is obtained in an approximate form "in both ways", the most often in a series and infinite form. The fact, that the first and the second approximation are usually confirmed by experiments indicates that nonlinear models are worth to be applied now a days when calculation tools are highly developed. Features of nonlinear models are as follows:

1. Principle of superposition is not binding;
2. The system response can have and generally has the different frequency distribution that the input;
3. System transmittances depend on themselves and on inputs (they are not system invariables);
4. Local transient states can occur (example of „bended" amplitude-frequency characteristics is known from each text-book on the vibration theory);
5. Resonance responses can occur at frequencies being an arbitrary linear combination of input and natural frequencies, including an effect of the so-called internal resonance;
6. Self-exited vibrations can occur.

Diagnostics specialists know all mentioned above features from the observation of actual objects. The situation presented in Fig. 2 is a typical example of a frequency conversion.

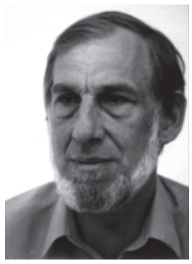
Resonant increases of the amplitude in bands, in which the vibration level in a new machine was low, local losses of motion stability, generation of self-exited vibrations, as well as strong dependence of parasitic vibration processes on the load – are typical symptoms frequently utilised in vibroacoustic diagnostics.

Thus, purposefulness of application nonlinear descriptions seems to be doubtless similarly as looking for vibration measures (more precisely: methods and techniques of signal analysis) sensitive to the nonlinear disturbances evolution.

REFERENCES

Many examples of the applications of this thesis and theoretical discuss as well you can find in book:

1. Batko W., Dąbrowski Z., Kiciński J.: *Nonlinear Effects in Vibroacoustic Diagnostic*, Institute for Sustainable Technologies – National Research Institute, 2008.



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